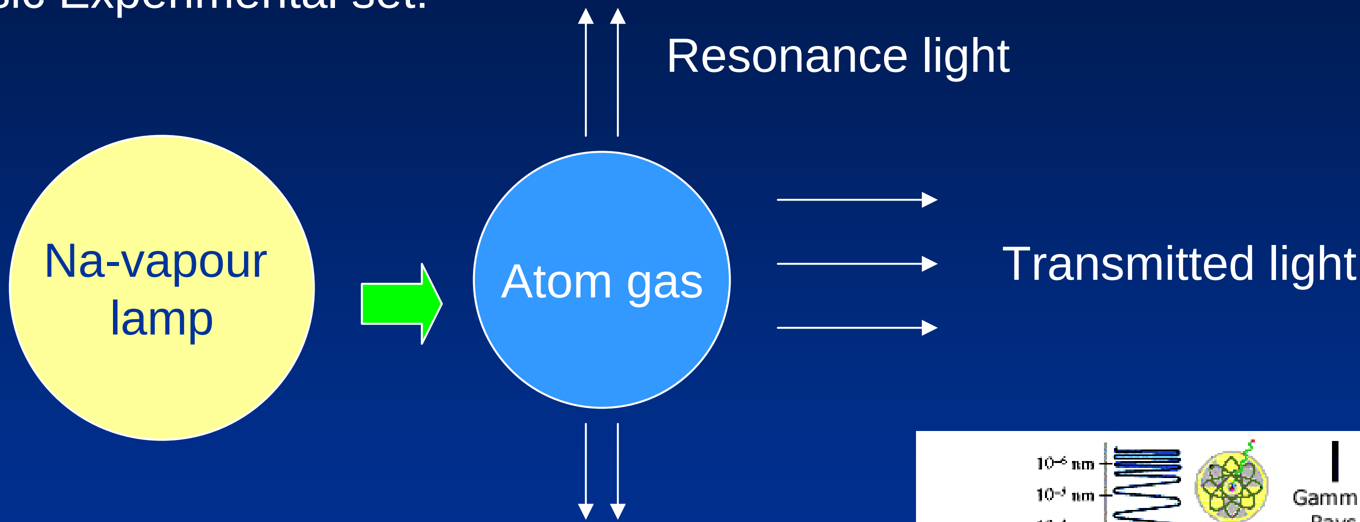
Atom and Quanta: Bohr's Theory

Phys 460, Fall 2009, JP Hu

Optical Spectrum of Atom

- Basic Experimental set:

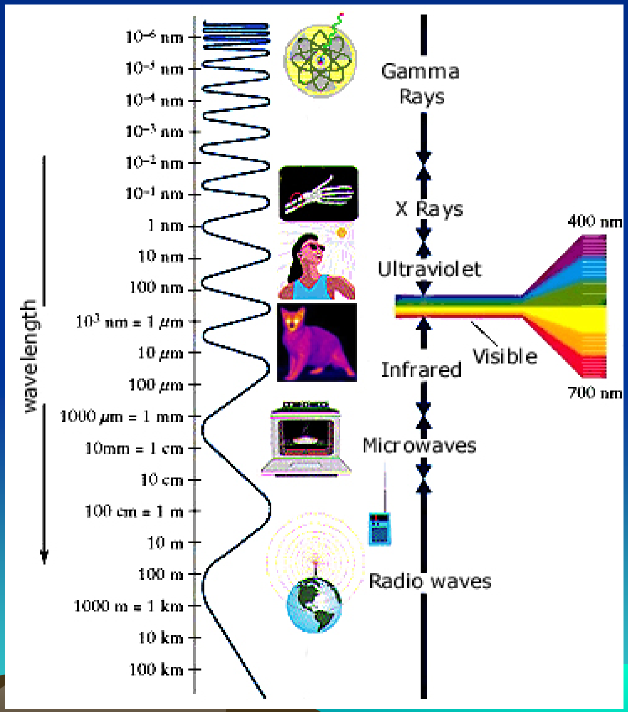


- Basic knowledge of electromagnetic spectrum

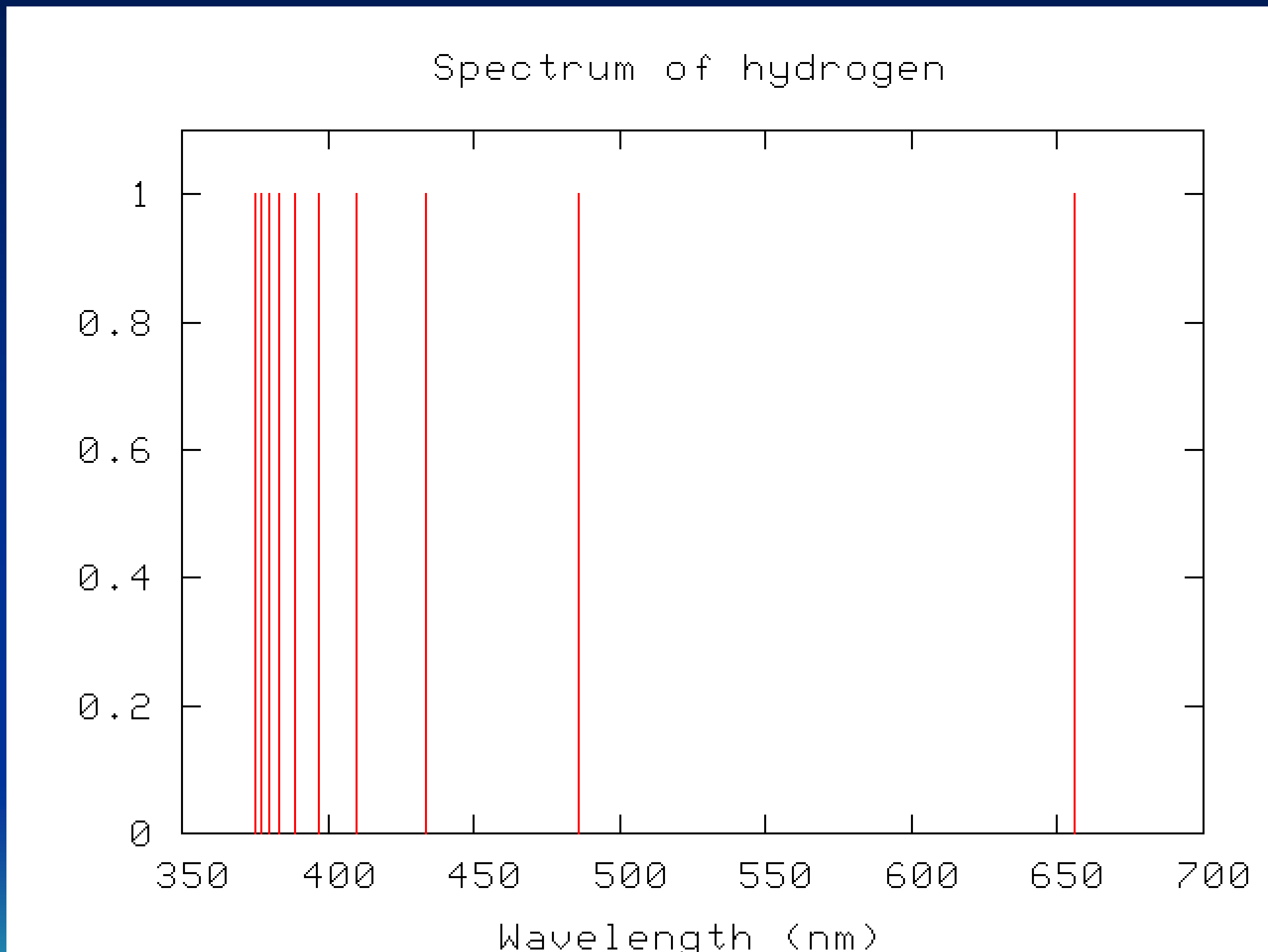
Wavelength: λ

Wavenumber: $\nu=1/\lambda$

Frequency: $\omega=c/\lambda$



Optical Spectra of Hydrogen atom



- Discrete numbers: 656.3, 486.1, 434(nm),…

Balmer's formula (1885)

$$\lambda = \left(\frac{n^2}{n^2 - 4}\right)G$$

$$\nu = 1/\lambda = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

$$R_H = 109677.5810 cm^{-1}, n = 3,4,...$$

R_H: Rydberg constant

- How good is the formula?

  Example: 15233.21(exp) vs 15233.00 (th) for n=3

  20564.77(exp) vs 20564.55 (th) for n=4

More spectra

$$\lambda = \left(\frac{n^2}{n^2 - n'^2}\right) G$$

$$\nu = 1/\lambda = R_H \left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$$

$$R_H = 109677.5810 cm^{-1}$$

$$n' = 1, 2 \ldots, n = n' + 1, \ldots$$

n', n: principal quantum numbers

1906 Lyman, 1908, Paschen, Brackett 1922

Bohr's Solution

- Classical orbits:

$$mr\omega^2 = \frac{e^2}{4\pi\varepsilon_0}\frac{1}{r^2}$$

- Energy of electron:

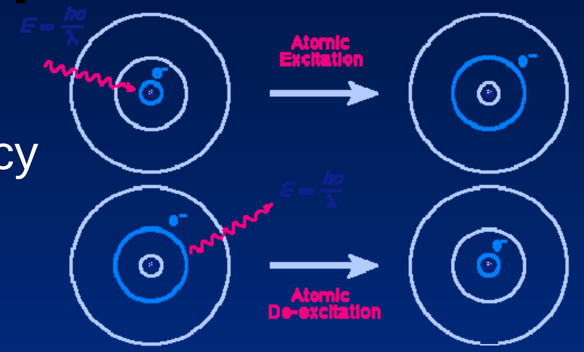
$$E = \frac{1}{2}mr\omega^2 - \frac{e^2}{4\pi\varepsilon_0}\frac{1}{r} = -\frac{e^{4/3}}{2(4\pi\varepsilon_0)^{2/3}}(m\omega^2)^{1/3}$$

- Bohr's bold postulates:
  - The classical motion of electron in atom is still valid. However, only discrete orbits with certain energy E_n is allowed.
  - The motion of the electrons in these quantized orbits is radiationless.
  - The light is emitted or absorbed when the electron transfers from one orbit to the other.

$$E_n = -Rhc/n^2$$
$$E_n - E_{n'} = h\Omega$$

  - With increasing orbital radius r, the law should become identical to classical physics ---- Bohr's Correspondence Principle

Bohr solution



- Classical:
  light frequency = classical electron orbiting frequency

- From Bohr correspondence principle, we have

$$E_n - E_{n-1} \to 2Rhc/n^3 = h\Omega = h\omega$$

$$E_n = -Rhc/n^2 = -\frac{e^{4/3}}{2(4\pi\varepsilon_0)^{2/3}}(m(2Rc/n^3)^2)^{1/3}$$

$$R = me^4/(8\varepsilon_0{}^2h^3c) = 109737.318cm^{-1}$$

- Quantization of angular momentum:

$$2Rhc/n^3 = h\omega$$

$$l = m\omega^2 r = n\hbar$$

Beyond Bohr's Theory

- Sommerfeld's extention of the Bohr Model (Relativistic mass change)

$$E_{n,k} = \frac{-Rhc}{n^2}[1 + \frac{\alpha^2 z^2}{n^2}(n/k - 3/4) + ...)$$

$$\alpha = 1/137 = \frac{e^2}{2\varepsilon_0 hc}$$

- Semiclassical quantization rule:

$$\oint pdq = n\hbar$$

De Broglie's Particle-wave

- Classical waves have certain modes with fixed boundary

- From Plank-Einstein: a light with frequency ω

$$E = h\omega$$

$$p = h\omega / c = h / \lambda$$

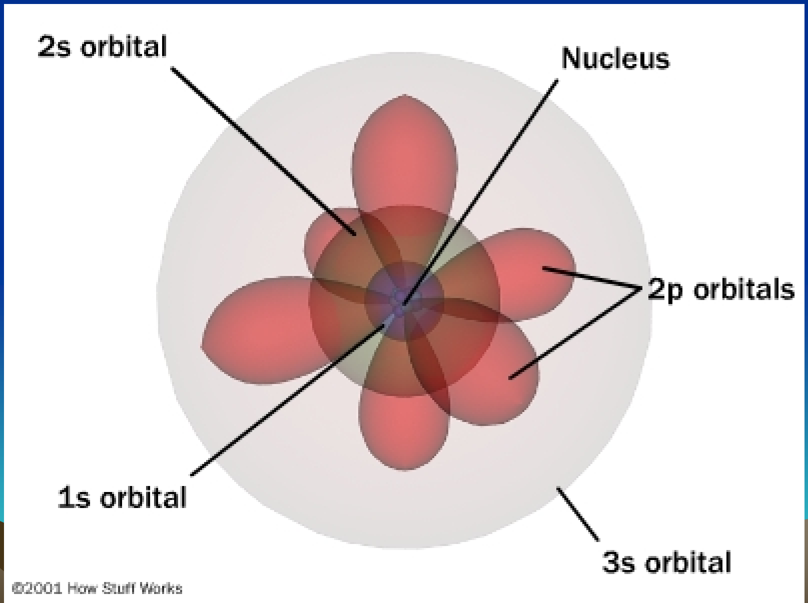
- Consider a particle as wave with wavelength defined as

$$\lambda = h / p$$

De Broglie's interpretation of Bohr's orbit

$$2\pi r = n\lambda = nh / p$$

$$pr = l = \hbar n$$



Limitation of Bohr's Theory

- It is a theory with conjectures
- Lack of real calculation power
- Limit to hydrogen-type atoms
- Do not know how to extend it to more complicated system

Anyway, Bohr's theory fundamentally changes our view of world.

- System is characterized by states.
- The physics is determined by final and initial states.
- Energy is quantized.