LEAI-42 Apparatus of Millikan's Experiment - Advanced Instructional Manual

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## CONTENTS

1. Introduction ................................................................................................................................. 1

2. Specifications ................................................................................................................................ 1

3. Key Features of Apparatus ........................................................................................................ 1

4. Apparatus Structure .................................................................................................................. 2

5. Theory .......................................................................................................................................... 4
   A. Millikan’s oil drop ................................................................................................................... 4
   B. Brownian motion (optional) .................................................................................................... 5

6. Experiment of Millikan’s Oil Drop ............................................................................................... 8

7. Experiment of Brownian Motion (optional) ............................................................................... 10

8. List of Parts .................................................................................................................................. 13
1. Introduction

This LEAI-42 apparatus of Millikan's oil drop experiment is an advanced model with CCD image acquisition and LCD monitor display, it is designed to verify the quantum nature of electrical charges, measure the elementary charge of an electron, and observe the Brownian motion. It is an ideal teaching apparatus for physics laboratories in colleges and universities.

2. Specifications

<table>
<thead>
<tr>
<th>Feature</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage Between Electrode Plates</td>
<td>DC 0 ~ 700 V, continuously adjustable</td>
</tr>
<tr>
<td>Elevation Voltage</td>
<td>200 ~ 300 V</td>
</tr>
<tr>
<td>Digital Voltmeter</td>
<td>0 ~ 999 V ± 1 V</td>
</tr>
<tr>
<td>Digital Timer</td>
<td>0 ~ 99.9 9 s ± 0.01 s</td>
</tr>
<tr>
<td>Spacing Between Electrode Plates</td>
<td>5.00 ± 0.01 mm</td>
</tr>
<tr>
<td>Magnification of Microscope</td>
<td>60 X and 120 X</td>
</tr>
<tr>
<td>Electronic Scale</td>
<td>Type A: 8×3 grid, 2 mm in vertical direction with 8 divisions and 0.25 mm/div when the 60× objective is used</td>
</tr>
<tr>
<td></td>
<td>Type B: 15×15 grid, 0.08 mm/div with 60× objective and 0.04 mm/div when 120× objective is used</td>
</tr>
<tr>
<td>Relative Measurement Error (Mean)</td>
<td>&lt;3%</td>
</tr>
<tr>
<td>Operation Temperature Range</td>
<td>-10 to +40 °C</td>
</tr>
<tr>
<td>Relative Humidity</td>
<td>≤ 85% (at 40 °C)</td>
</tr>
</tbody>
</table>

3. Key Features of Apparatus

Although the Millikan oil drop experiment is very classical, the conventional oil drop apparatus is very hard to operate. To address this issue, we developed this advanced oil drop apparatus consisting of CCD sensing microscope, illumination lamp, LCD monitor display, oil chamber, high-voltage supply, and microprocessor electronics. Hence, this modular oil drop apparatus possesses the following features:

Multiple integrated display technology: by displaying voltage, time, and oil drop images on one monitor screen, the experimental operation is intuitive and simplified.

Electronic scale on screen: this is a dark-field method, i.e. optical illumination on reticle is not needed, so the image of oil drops is bright and clear. It is more accurate than a conventional scale mask without distortion and viewing error.

Multifunctional: a high magnification objective lens is provided for the observation of tiny oil drops that cannot be seen with conventional oil drop apparatus. Using a higher magnification objectives lens, this apparatus can be used to perform “Brownian motion” experiment.

Synchronized voltage and timing operation: voltage applying can be synchronized with the electronic timer for the simplification of manual operation.
Reliable and safe design: Microprocessor offers high accuracy for both time and voltage measurements. LED light source has a long lifetime. High voltage protective switch provides safety to users.

4. Apparatus Structure

This oil drop apparatus consists of oil drop chamber, CCD TV microscope, monitor screen, and electrical control unit. Among them, the oil drop chamber is a critical part with a schematic diagram shown in Figure 1.

As seen in Figure 1, the upper and lower electrodes are parallel plates with a bakelite ring placed between them to ensure the parallelism between electrodes with high spacing accuracy (better than 0.01 mm). There is an oil mist hole (0.4 mm in diameter) in the center of the upper electrode. On the wall of the bakelite ring, there are three through holes, one for microscope objective, one for light passing and one as a spare hole.

![Figure 1 Schematic diagram of oil drop chamber](image)

The oil drop cavity is covered by a wind proof screen on which a removable oil mist cup is placed. There is an oil drop aperture at the bottom of the oil mist cup and the oil drop aperture can be blocked with a stop.

The upper electrode is secured with a clamp which can be toggled left and right for the removal of the upper electrode. An illumination lamp is located at the middle of the oil channel base with light focusing mechanism. The intersection angle between the illumination light and the microscope objective is about 150° ~ 160° that is designed based on light scattering theory to achieve high brightness. Unlike a conventional Tungsten filament bulb that can be blown out over time, a focusing LED is used in this apparatus with extended lifetime. The CCD camera microscope is designed in an integral structure with stable and reliable operation. A schematic
diagram of the upper panel of the main unit is given in Figure 2. Electronic scale is generated in synchronization with the video signal of the CCD camera, thus ensuring stable scale reading independent of monitor settings.

There are two sets of electronic scales, i.e. A and B. The former has 8 divisions in vertical direction and each division is 0.25 mm when the $60 \times$ objective is used; the latter is designed to observe Brownian movement of oil drops, with 15 small divisions in both $X$ and $Y$ directions. The division is 0.08 mm when the $60 \times$ objective is used; and 0.04 mm when the $120 \times$ objective is used. Scale B can be toggled on or off by pressing button “Timing Start/Stop” for more than 5 seconds.

![Figure 2 Schematic of the upper panel of main unit](image)

There are two groups of push selector switches ($K_2$ has 3 selection buttons) for controlling the voltage between the parallel electrodes. $K_1$ controls the voltage polarity applied to the upper electrode while $K_2$ controls the voltage value. When “Balance” button is pressed down, the potentiometer $W$ can be used to adjust the balance voltage (adjustable range 0~400 V); when “Elevation” is selected, a fixed elevating voltage between 200 V and 300 V is added to the balance voltage; when “0 V” is selected, voltage on the electrode is set to 0 V.

To increase measurement accuracy, “Balance” and “0 V” can be operated simultaneously with timing switch “Timing Start/Stop” when the “Gang Switch” is pressed down. Under this case, when selector switch $K_2$ changes from “Balance” to “0 V”, oil drop begins to fall at uniform velocity and timing starts simultaneously. When oil drop reaches the preset distance, $K_2$ should be changed from “0 V” to “Balance”. Immediately, the oil drop stops falling and timing ends simultaneously. The distance the oil drop travelled and the time spent are shown on the screen. Alternatively, $K_2$ and $K_3$ can be operated independently if “Gang Switch” is released.
Due to air resistance, oil drops fall with variable speed initially before moving at uniform speed. Since the duration of the initial motion is very short (< 0.01 s), oil drops can be considered to undertake motion with uniform speed immediately from a stationary state. On the other hand, when the balance electric field is applied, the motion of oil drops will stop immediately so that oil drops go back to the stationary state.

To time the motion of oil drops, the “Timing” button should be pressed once (reset and start); if the button is pressed again, then timing is stopped.

5. Theory

A. Millikan’s oil drop

If an oil drop with mass $m$ and charge $e$ enters the oil chamber, it falls down freely under the force of gravity when no voltage is applied between the parallel polar plates. When the force of gravity is balanced against air resistance (neglecting air buoyancy), the oil drop falls down at a uniform speed of $V_g$, as described by the following equation:

$$mg = f_a$$  \hspace{1cm} (1)

where $f_a$ is the air resistance when the oil drop falls down at a uniform speed of $V_g$, and $g$ is the gravitational constant. According to Stokes’ law, the air resistance exerted on the oil drop is

$$f_a = 6\pi\eta aV_g$$  \hspace{1cm} (2)

While the force of gravity exerted on the oil drop is

$$G = mg = \frac{4}{3} \pi a^3 \rho g$$  \hspace{1cm} (3)

where $\eta$ is the coefficient of viscosity for air; $\rho$ is the density of oil drop; and $a$ is the radius of the oil drop. By combining equations (1) to (3), the radius of the oil drop can be derived as:

$$a = \frac{\eta V_g}{2g\rho}$$  \hspace{1cm} (4)

When an electric field is applied across the parallel polar plates, the oil drop is subject to the force of the electric field. If the strength and polarity of the electric field are controlled and selected properly, the oil drop can be elevated under the influence of the electric field. If the force of the electric field exerted on the oil drop is balanced against the sum of the force of gravity and the resistance of air exerted on the oil drop, the oil drop will rise at a uniform speed of $V_e$. Hence, we have

$$mg + f_a = qE$$  \hspace{1cm} (5)

where $E$ is the strength of the electric field applied to the parallel polar plates and $q$ is the electric charge on the oil drop.

From equations (1) and (2), equation (5) can be rewritten as

$$mg + f_a = f_a + f_a = 6\pi\eta a(V_g + V_e) = qE = q \frac{V}{d}$$  \hspace{1cm} (6)

Thus,
\[ q = \frac{6\pi \eta ad(V_g + V_e)}{V} \]  \hspace{1cm} (7)

where \( V \) and \( d \) are the voltage and separation between the parallel polar plates, respectively.

Equation (7) is derived assuming the oil drop is in a continuous medium. In this experiment, the radius of the oil drop is as small as \( 10^{-6} \) m and therefore it is comparable to air molecules. Thus, air is no longer considered as a continuous medium, and viscosity coefficient \( \eta \) of the air needs to be corrected as

\[ \eta' = \frac{\eta}{1 + \frac{b}{pa}} \]  \hspace{1cm} (8)

where \( b \) is a correction constant; \( p \) is the atmospheric pressure; \( a \) is the radius of oil drop as given in equation (4). By assuming the falling and rising distance of an oil drop with uniform speed are identical as \( l \) and the spent time are \( t_g \) and \( t_e \), respectively, we have \( V_g = lt_g \) and \( V_e = lt_e \).

By substituting (8) and (4) into (7), we have:

\[ q = \frac{18\pi d}{V\sqrt{2\rho g}} \left( \frac{\eta l}{1 + \frac{b}{pa}} \right)^{3/2} \left( \frac{1}{t_e} + \frac{1}{t_g} \right)^{1/2} \]  \hspace{1cm} (9)

where \( K = \frac{18\pi d}{\sqrt{2\rho g}} \left( \frac{\eta l}{1 + \frac{b}{pa}} \right)^{3/2} \left( \frac{1}{t_e} + \frac{1}{t_g} \right)^{1/2} \)

The above formula is used to calculate charge quantity in dynamic (unbalanced) measurement method. In static (balanced) measurement method, we adjust the voltage between the parallel electrodes to hold the oil drop stationary (\( V_e = 0, t_e \rightarrow \infty \)). Now equation (9) becomes:

\[ q = K \left( \frac{1}{t_g} \right)^{3/2} \]  \hspace{1cm} (10)

**B. Brownian motion (optional)**

Particles suspended in a fluid undertaking irregular motion are known as Brownian motion. In theory, the probability of displacement between \( \xi \) and \( \xi + d\xi \) of a Brownian particle is:

\[ \phi (\xi) \, d\xi = \frac{1}{\lambda \sqrt{2\pi}} e^{-\xi^2/2\lambda^2} \, d\xi \]  \hspace{1cm} (11)

where \( \xi \) is the displacement of a particle rather than the coordinate of a particle at a certain time, and \( \phi (\xi) \) is known as displacement probability density function of a Brownian particle.

If the Brownian motion of a particle is projected in \( x \) axis, we have:
\[ \varphi(x) \Delta_s = \frac{1}{\sqrt{2\pi}} e^{-x^2/2\Delta_s^2} \Delta_s \]

where \( \varphi(x) \Delta_s \) is the probability of the particle located between \( x \) and \( x + \Delta_s \) within a certain time period; \( \lambda^2 \) is equal to the average value of the square of Brownian particle displacement \( x \), i.e. \( \lambda^2 = \frac{\lambda_x}{2} \).

\[
\frac{x^2}{\lambda^2} = \int_{-\infty}^{\infty} x^2 \varphi(x) dx \int_{-\infty}^{\infty} \varphi(x) dx = \int_{-\infty}^{\infty} x^2 \varphi(x) dx = \frac{\sqrt{\pi}}{\lambda \sqrt{2\pi}} e^{-\frac{x^2}{2\lambda^2}} \Delta_s \]

As \( \int_{-\infty}^{\infty} e^{-Ax^2} dx = \frac{\sqrt{\pi}}{A} \) and \( \int_{-\infty}^{\infty} x^2 e^{-Ax^2} dx = \frac{\sqrt{\pi}}{2} (A)^{3/2} \) while assuming \( A = 1/2 \lambda^2 \), equation (13) can be rewritten as:

\[
\frac{x^2}{\lambda^2} = \frac{1}{\lambda \sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 e^{-x^2/2\lambda^2} dx = \frac{1}{\lambda \sqrt{2\pi}} \left( 2\lambda^2 \right)^{3/2} = \lambda^2
\]

Brown particles are impacted by three forces: the collision of irregular motions of molecules, \( F = (X, Y, Z) \); the motion resistance in fluid, \( -\beta \nu \), which is proportional to the particle speed in reversal direction; the gravity and buoyancy, \( +mg(1-\rho/\rho_0)K \), where \( m \) and \( \rho \) are the mass and density of the Brownian particle, respectively, \( \rho_0 \) is the density of the fluid medium, and \( K \) is the unit vector in gravity field direction.

The particle motion in this experiment is considered as the projection in horizontal direction \( x \) perpendicular to \( K \), so gravity and buoyancy have no effect. Thus, particle motion equation is:

\[
m \ddot{x} = -\beta \dot{x} + X
\]

As \( \ddot{x} = \frac{1}{2} \frac{d^2}{dt^2} (x^2) \), we get \( \frac{1}{2} \frac{d^2}{dt^2} (mx^2) - \dot{m}x^2 = m\ddot{x} \). Thus, equation (15) can be rewritten as:

\[
\frac{1}{2} \frac{d^2}{dt^2} (mx^2) - \dot{m}x^2 = x\dddot{x} - \frac{1}{2} \beta \frac{d}{dt} x^2
\]

Assuming many particles with identical radius \( a \) and mass \( m \), they all share equation (16). By summing these equations divided by the number of particles, we get:

\[
\frac{1}{2} \frac{d^2}{dt^2} (m\overline{x}^2) - \dot{m}\overline{x}^2 = \overline{x}\dddot{x} - \frac{1}{2} \beta \frac{d}{dt} \overline{x}^2
\]

Since the probability of positive and negative signs of same value \( X \) are equal, we have \( \overline{Xx} = 0 \). Furthermore, \( m\overline{x}^2 \) of many particles should be equal to that of the average value \( \overline{m\dot{x}^2} \) of a single molecule, i.e., the molecule and a large particle is in thermal equilibrium with each other. In the equilibrium state of irregular movement, the average kinetic energies of molecules and particles are equal, so particles can be considered as giant molecules but the motion velocity is inversely proportional to the square root of its mass. Since \( m\overline{x}^2/2 = kT/2 \), equation (17) can be rewritten as:
\[
\frac{d^2 \vec{x}}{dt^2} = -\frac{\beta}{m} \frac{d \vec{x}}{dt} + \frac{2kT}{m} = 0
\]  

(18)

Thus

\[
\vec{x} = \frac{2kT}{\beta} t + C_1 e^{\frac{\beta}{m} t} + C_2
\]  

(19)

where \( k \) is Boltzmann's constant, \( T \) is temperature in Kelvin, \( C_1 \) and \( C_2 \) are integral constants, \( \beta/m \) is a great value which can be determined by fluid mechanics. Since Brownian particles are all small, they can be considered as spheres having same radius \( a \). As a result, we can assume that the motion resistance they are subject to is the same as that of a ball slowly moving in a viscous fluid. Based on Stokes law, the force on the particle is:

\[
f = 6\pi \eta a v = \beta v.
\]

Since \( a \approx 10^{-4} \text{ cm} \) and the viscosity of water \( \eta \approx 10^{-2} \text{ Pa} \), we estimate \( \frac{\beta}{m} = \frac{6\pi a \eta}{4\pi a^3 \rho} = \frac{9\eta}{2a^2 \rho} \approx 10^7 \).

After a short period of time (e.g. \( t > 10^{-6} \text{ s} \)), the exponential term in equation (19) can be ignored (\( e^{-10} \approx e^{-10} \)). Thus, equation (19) can be simplified as:

\[
\vec{x} = \frac{2kT}{\beta} t + C_2
\]  

(20)

Integral constant \( C_2 \) depends on the coordinate location. When \( t = 0, \vec{x} = C_2 \). By assuming \( t_0 = 0, x_0 = 0 \), we can rewrite equation (20) as:

\[
\vec{x} = \frac{2kT}{\beta} t = 2D t
\]  

(21)

This relationship was first derived by Einstein, so equation (21) is called Einstein’s formula. It is a rigorous proof of Brownian motion originated from micro molecular motion.

In this experiment, user needs to record displacement of oil drop motion over specific time interval and plot displacement distribution curve \( \phi(x) \) of Brownian particle as seen in Figure 3.

![Displacement distribution curve of Brownian particle](image-url)
6. Experiment of Millikan’s Oil Drop

1) Adjustment of apparatus
   a. Connect AV/BNC cable between the apparatus and the monitor.
   b. Adjust the base feet while monitoring the leveling bubble to level the apparatus.
   c. Turn on power. There is no need to align the illumination light.
   d. To focus the microscope, slide the objective lens through the hole on the windscreen until the front edge of the objective lens aligns with the inner surface of the windscreen. There is no need to use a focusing needle to focus the microscope.
   e. The focusing range of the microscope should not exceed 1 mm.
   f. Turn on the power of the monitor and the Millikan apparatus, after about 5 seconds, the monitor will display information of the standard scale plate, voltage (V) and time (s).
   g. Press “Timing Start/Stop” button once to trigger measurement state.
   h. Selector switch K₁ is used to select voltage polarity of upper electrode.

2) Measurement rehearsal
   a. Spray oil: spray oil drops into the oil drop channel through the hole with the sprayer while observing the oil drops on the monitor. Do not spray too much oil into the oil drop channel. Spray once or twice should be sufficient. Then, close the oil mist aperture (item 10 in Fig. 1) with the oil aperture switch (item 2 of Fig. 1) to avoid disturbance of ambient air flow.
   b. Select oil drops: one of the key challenges in this experiment is to select proper oil drops. If the oil drops are too large, their uniform falling speed will be too fast, resulting in more charges to be carried by these large oil drops and higher voltage to be applied to the polar plates. As a result, measurement accuracy will be decreased; on the other hand, if the oil drops are too small, they will be vulnerable to thermal motion and hence be difficult to control. In general, it is preferred to select oil drops in medium size and with slow rising or falling speed. Usually, if 200 ~ 300 V is applied to the electrodes (select “Balance” for K₂ and adjust “Balance Voltage” W to achieve this voltage), oil drops that move across 6 divisions (1.5 mm) in 8 ~ 20 s are the suitable oil drops for this experiment. Then change K₂ from “Balance” to “0 V”, observe the falling speed of these oil drops, and select one proper oil drop as the measurement target. For reference, when viewing on a 10” display, the size of the oil drop image should be 0.5~1.0 mm.
   c. Time uniform motion of oil drops: select several oil drops at different moving speeds, and use the same criterion to start/stop timing by looking at the scale on the monitor.
   d. Balance oil drops: adjust the balance voltage carefully to ensure that the oil drop truly stops moving and repeat the procedure several times to ensure measurement accuracy.

3) Formal measurement
   a. Spray oil and close oil mist aperture
   b. The static method:
      (1) Press down “Balance” button, carefully balance an oil drop to be stationary by adjusting the balance voltage.
      (2) Press down “Elevation” button, bring the balanced oil drop to the 2nd scale line.
      (3) Press down “Gang Switch” and press “Timing Start/Stop” button to stop timing (the
time readout may not be zero at this time). Next, press down “0 V” button, the balance voltage is turned off immediately as it is synchronized with the timing button. Now timing starts, and the oil drop starts to fall down. When the same oil drop reaches the 8th scale line \((l=1.5 \text{ mm})\), press down “Balance” button immediately. Now the same oil drop stops falling and timing stops simultaneously.

(4) Use the balance voltage and time displayed on the monitor screen to calculate the elementary charge by using equation (10).

(5) Repeat steps (1) to (4) for different oil drops (5 to 10 drops) to get accurate results.

c. The dynamic method:

(1) Select a proper oil drop and balance it by adjusting the balance voltage \((K_2 \text{ is set at “Balance”})\).

(2) Press down “Elevation” button, bring the balanced oil drop above the 2nd scale line, used as the starting line.

(3) Release “Gang Switch” to void “gang” mode. Press down “0 V” button, the voltage on the electrodes is turned off, now the oil drop starts to fall down. When this oil drop reaches the starting line (i.e. the 2nd scale line), press “Timing Start/Stop” button to start timing immediately. When this oil drop reaches the ending line (e.g. the 8th scale line, \(l=1.5 \text{ mm}\)), press “Timing Start/Stop” button again to stop timing. The falling time for distance \(l\) is recorded on the screen. In the meantime, press “Balance” button to stop the same oil drop before losing it out of the viewing field. Write down the falling time \(t_g\).

(4) Press down “Elevation” button, the same oil drop starts to rise. When the oil drop reaches the ending line (i.e. the 8th scale line), press “Timing Start/Stop” button to start timing; when the oil drop reaches the starting line (e.g. the 2nd scale line), press “Timing Start/Stop” button again to stop timing. The rising time \(t_e\) for distance \(l\) is recorded. Write down the rising time \(t_e\) and the elevation voltage \(V\).

(5) Repeat the above measurements for different oil drops (5 to 10 drops) to calculate the elementary charges using equation (9).

4) Data Processing

The relationship between oil density and temperature is given in the table below:

<table>
<thead>
<tr>
<th>T (°C /°F)</th>
<th>0/32</th>
<th>10/50</th>
<th>20/68</th>
<th>30/86</th>
<th>40/104</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho) (kg/m³)</td>
<td>991</td>
<td>986</td>
<td>981</td>
<td>976</td>
<td>971</td>
</tr>
</tbody>
</table>

Take the static method as example, the formula is:

\[
q = \frac{18\pi d}{V\sqrt{2\rho g}} \left[ \frac{\eta l}{t_g \left( 1 + \frac{b}{pa} \right)} \right]^{3/2}
\]

where, \(a = \sqrt{\frac{9\eta l}{2\rho gt_g}}\).
Density of oil: \( \rho = 981 \text{ kg/m}^3 \) (at 20 °C)  
Gravitational constant: \( g = 9.8 \text{ m/s}^2 \)  
Air viscosity: \( \eta = 1.83 \times 10^{-5} \text{ kg/m} \cdot \text{s}^{-1} \)  
Distance of oil drop falling: \( l_1 = 1.5 \text{ mm} \)  
Correction constant: \( b = 6.17 \times 10^{-6} \text{ m/cmHg} \)  
Atmosphere pressure: \( p = 76.0 \text{ cmHg} \)  
Distance of parallel electrodes: \( d = 5 \text{ mm} \)

Multiple oil drops should be measured in this experiment and the charge quantities carried by oil drops are always multiples of a minimum value. This minimum fixed value of charges is the elementary charge \( e \) as accepted as \( e \approx 1.6021892 \times 10^{-19} \) coulombs.

When processing the experimental data, the data should be verified in a reverse order: divide the experimental data (charge quantity \( q \)) by the accepted electric elementary charge value \( e \approx 1.6021892 \times 10^{-19} \) C. The obtained quotient is a value close to a certain integer, which is the charge quantity \( n \) carried by the oil drop. Further, by dividing \( q \) by \( n \), the obtained quotient is the experimental value of the elementary charge \( e \).

The elementary charge can also be derived by drawing plot method. Let’s assume the measured charge quantities of \( m \) oil drops are \( q_1, q_2, \ldots, q_m \), respectively. Considering the quantization characteristic of the charge quantity, we have \( q_i = n_i e \), which is a linear equation, where \( n_i \) is the independent variable, \( q_i \) is the dependent variable, and \( e \) is the slope. Therefore, the data points of \( m \) oil drops will form a common straight line passing the origin point of the \( n \sim q \) coordinates plane. If this straight line is found, the elementary charge \( e \) is the slope.

### 7. Experiment of Brownian Motion (optional)

1) Apparatus adjustment
   a) Move the objective barrel backward to separate it from the oil chamber, and replace the 60× objective with the 120× one;  
   b) Keep “Timing Start/Stop” button pressed for 5 seconds to change the scale plate to type B (0.04 mm/div);  
   c) Adjust objective barrel and balance voltage, make balance of buoyancy, electric field force and gravity for an oil drop;  
   d) Record the positions (only horizontal direction) of the oil drop on the scale plate in every 5 s or 10 s. Take at least 500 data.

2) Process the data and plot \( \varphi(x) \sim x \) graph
   Find the maximum and minimum values from these recorded data, determine step size, group numbers, and boundary points of all groups, calculate data number in every group, list in a frequency distribution table, make frequency histogram, and finally draw distribution curve.

The following is an example of data set with a total number \( n = 651 \).

   a. Distribution table of particle displacement frequency

   The step size in the following table is \( \Delta x = 3 \), i.e. the times of displacement in range -1 to 1 is 201, in -1 to -4 is 128, … \( \varphi(x) \Delta x \) equals the times in one group divided by the total times of the records.
b. Histogram of displacement frequency

Draw the histogram of displacement frequency according to the distribution table.

<table>
<thead>
<tr>
<th>Displacement</th>
<th>Frequency $v_i$</th>
<th>$v_i / n$</th>
<th>$\sum_{i=1}^{n} v_i / n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-15</td>
<td>2</td>
<td>0.0031</td>
<td>0.0031</td>
</tr>
<tr>
<td>-12</td>
<td>9</td>
<td>0.0138</td>
<td>0.0170</td>
</tr>
<tr>
<td>-9</td>
<td>23</td>
<td>0.0353</td>
<td>0.0522</td>
</tr>
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<td>-6</td>
<td>70</td>
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<td>-3</td>
<td>128</td>
<td>0.1966</td>
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</table>

Figure 4 Histogram of displacement frequency

c. Plot straight line using normal probability paper

On the normal probability paper, the vertical axis is the distribution function $\phi(x)$ and the horizontal axis is the particle displacement $x$, that is,

$$\phi(x) = \sum_{i=1}^{n} \frac{v_i}{n}$$

where $v_i$ is frequency, $n$ is the total data points, and $v_i/n$ is the probability of displacement $x$ in the $i^{th}$ group.
If a straight line is obtained on the normal probability paper, \( x \) follows the normal distribution. Use the data in the above Table to plot the graph, a straight line is obtained as shown in Figure 5 indicating that \( x \) follows a normal distribution.

![Figure 5 Displacement frequency on normal probability paper](image)

3) Post-experiment cleaning

After experiment, the electrodes and oil channel should be cleaned throughout with dry cloth (\textbf{Warning:} turn off the electricity before cleaning the electrodes and oil channel).

\textbf{User instruction of oil sprayer}

a) Suck oil into a burette or syringe from the oil bottle.

b) Add oil into the oil storage of the sprayer. Oil height 3-5 mm is enough (do not add oil to flow over the top of the small nozzle inside).

c) The outlet of the sprayer is fragile, place the outlet tip outside the oil spray aperture of the oil drop chamber with a distance 1-2 mm to spray oil drop into the chamber (do not need to insert the tip into the aperture hole).

d) If there is oil remaining in the storage, keep the sprayer in upward direction (e.g. place it in a cup) to avoid oil leakage.
e) At the end of each semester, pour out the oil from the sprayer and pinch a few times to empty the sprayer.

8. List of Parts

<table>
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<th>Name</th>
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<tr>
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<td>3</td>
<td>Oil Bottle</td>
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<tr>
<td>4</td>
<td>LCD Monitor with 12V DC power supply</td>
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