Vortex Liquid Crystals in Anisotropic Type II Superconductors

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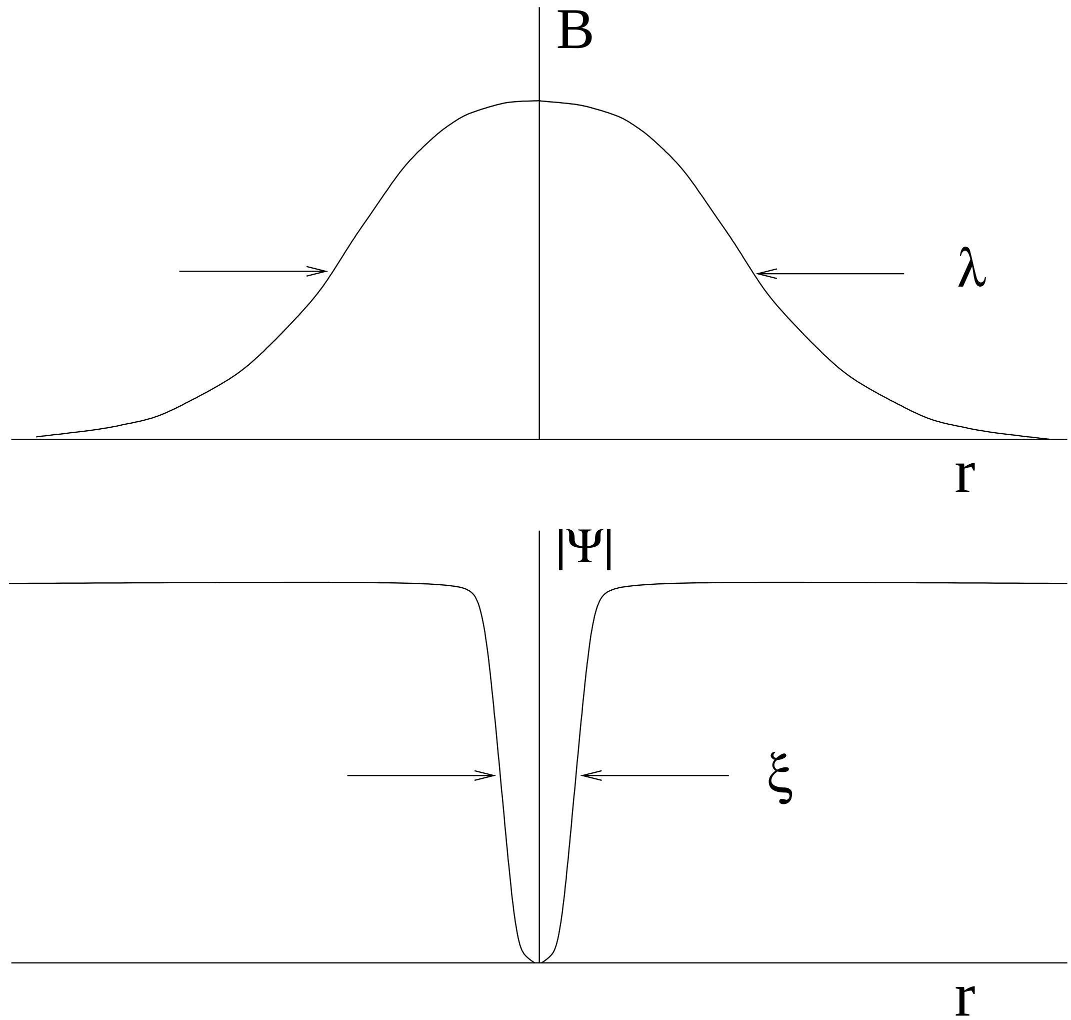
Vortex

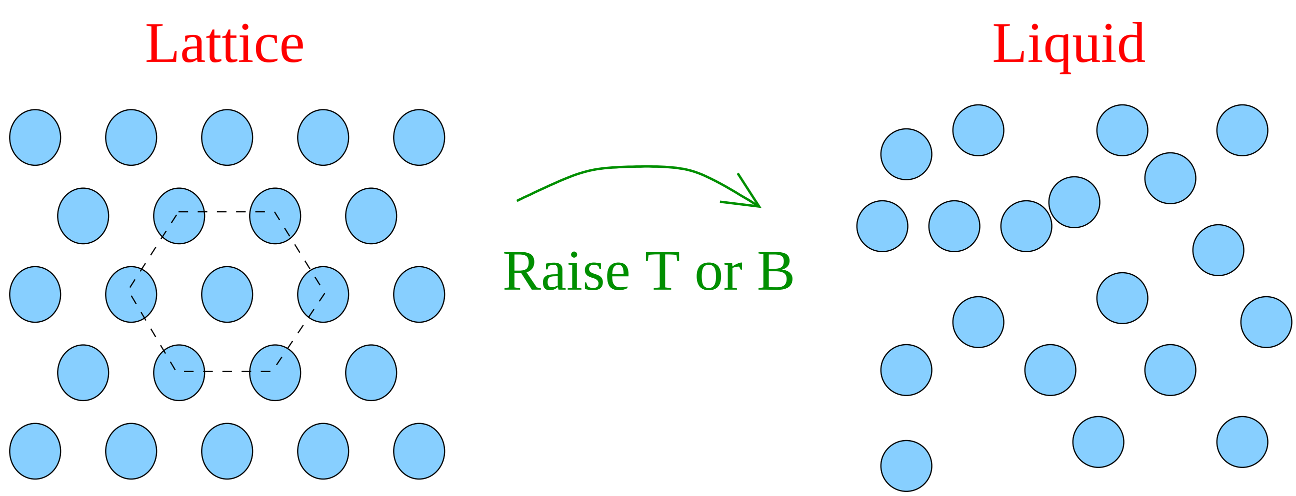


In the high temperature superconductors,
$\kappa \equiv \frac{\lambda}{\xi} \approx 100$

λ: screening currents and magnetic field
ξ: the normal "core"

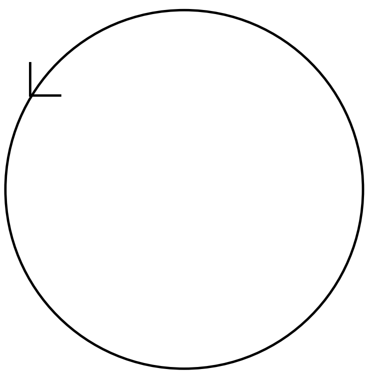
Vortex Lattice Melting

Circular Cross Sections:



Cross Section of a Vortex

ISOTROPIC



CircularProfile

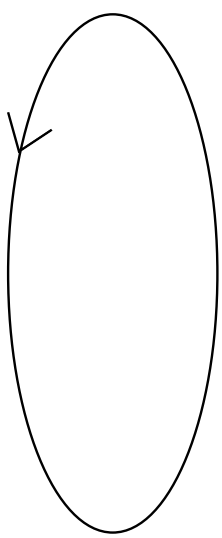
$$\overline{m} = \begin{pmatrix} m & & \\ & m & \\ & & m \end{pmatrix}$$

$$\lambda^2 = \frac{mc^2}{4\pi e^2 n_s}$$

$$\lambda^2 \nabla^2 \overrightarrow{B} - \overrightarrow{B} = \Phi_o \delta_2(r)$$

$$\overrightarrow{B} = \frac{\Phi_o}{2\pi\lambda^2} K_o(r/\lambda)$$

ANISOTROPIC



EllipticalProfile

$$\overline{m} = \begin{pmatrix} m_x & & \\ & m_y & \\ & & m_z \end{pmatrix}$$

$$\lambda_x^2 = \frac{m_x c^2}{4\pi e^2 n_s} \qquad (B || \hat{z})$$

$$\lambda^2 \nabla^2 \overrightarrow{B} - \overrightarrow{B} = \Phi_o \delta_2(r)$$

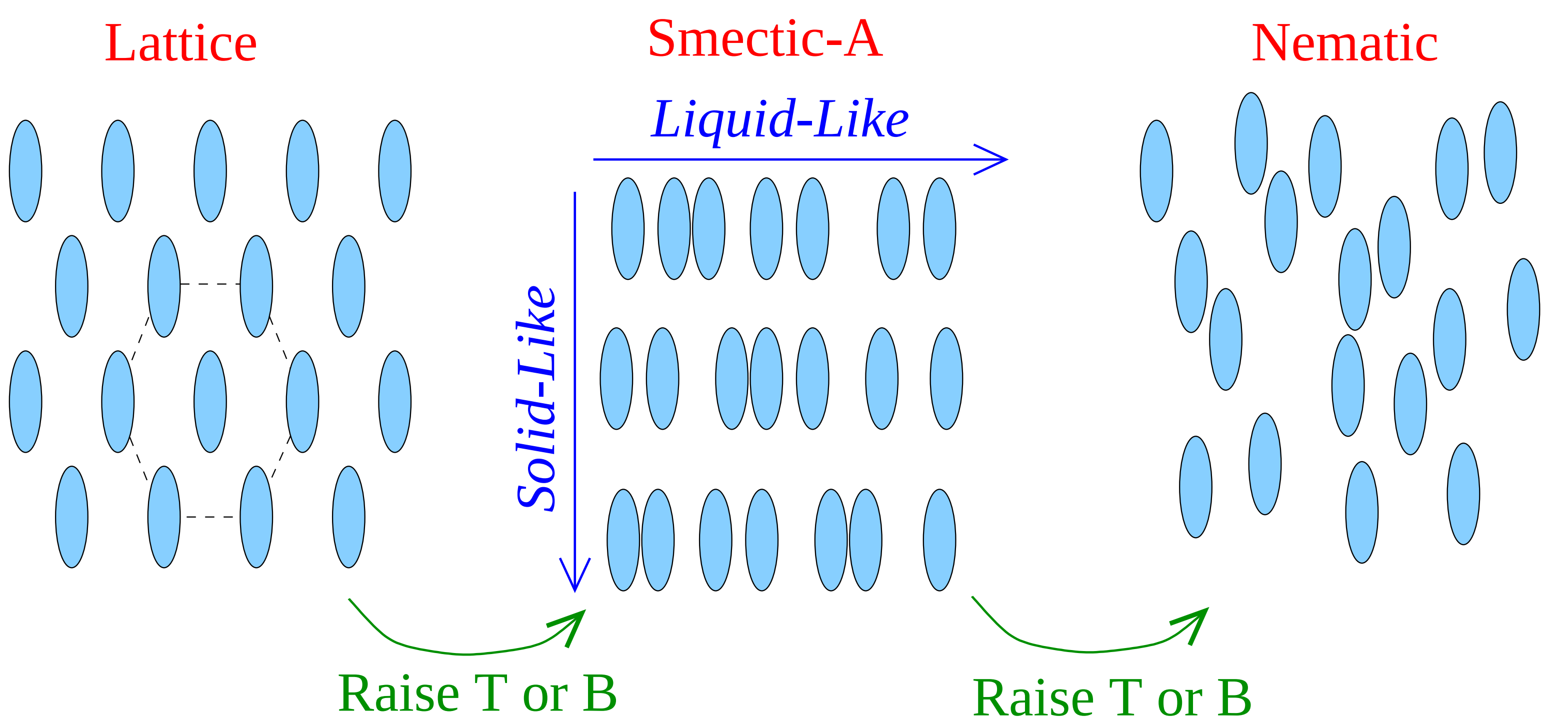
$$\overrightarrow{B} = \frac{\Phi_o}{2\pi\lambda^2} K_o(r/\lambda)$$

Anisotropic Vortex Lattice Melting

Elliptical Cross Sections:

Anisotropic Interacting "Molecules"
$\longrightarrow$ Liquid Crystalline Phases

Abrikosov Liquid Crystals?

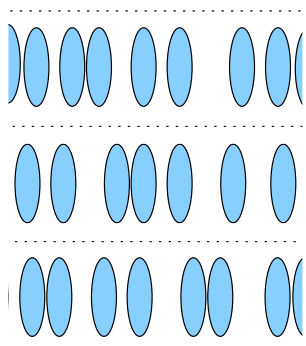


Symmetry of Phases

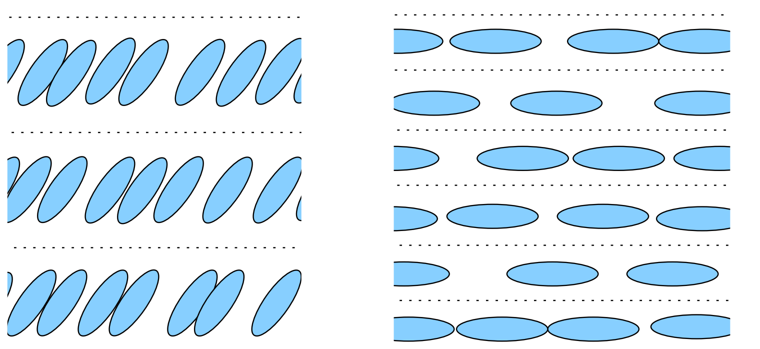
CRYSTALS:	Break continuous rotational and translational symmetries of 3D space
LIQUIDS:	Break none
LIQUID CRYSTALS:	Break a subset

Hexatic:	6-fold rotational symmetry unbroken translational symmetry
Nematic:	2-fold rotational symmetry unbroken translational symmetry
Smectic:	breaks translational symmetry in 1 or 2 directions

Smectics



Smectic-A



Smectic-C

Full translational symmetry in at least one direction
Broken translational symmetry in at least one direction
(Broken rotational symmetry)

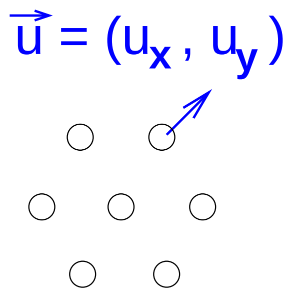
Many Vortex Phases

Abrikosov Lattice	Abrikosov (1957)
Entangled Flux Liquid	Nelson (1998)
Chain States	Ivlev, Kopnin (1990)
Hexatic	Fisher (1980)
Smectic-C	Efetov (1979)
	Balents, Nelson (1995)
Driven Smectic	Balents, Marchetti, Radzihovsky (1998)

Our Assumption:

- ☐ Explicitly broken rotational symmetry

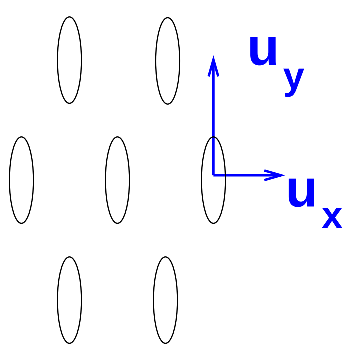
Instability of Ordered Phase:
Lindemann Criterion for Melting



$$< u^2 >=< u_x^2 > + < u_y^2 > \ \geq \ c^2 a^2$$

Typically, lattice melts for $c \approx .1$
Houghton, Pelcovits, Sudbo (1989)

Extended to Anisotropy:



$$< u_x^2 > \ \geq \ \frac{1}{2} c^2 a_x^2$$

$$< u_y^2 > \ \geq \ \frac{1}{2} c^2 a_y^2$$

Look for one to be exceeded
well before the other.

Method

Calculate $< u_x^2 >$ and $< u_y^2 >$

- □ based on elasticity theory of the ordered state
- □ using k-dependent elastic constants
- □ from Ginzburg-Landau theory

$$F = \frac{1}{(2\pi)^3} \int d\vec{k} \vec{u} \cdot \bar{C} \cdot \vec{u}$$

where $\vec{u} = (u_x, u_y) =$ vortex displacement

$$\bar{C} = \begin{pmatrix} c_{11}(\vec{k})k_x^2 + c_{66}^e k_y^2 + c_{44}^e(\vec{k})k_z^2 & c_{11}(\vec{k})k_x k_y \\ c_{11}(\vec{k})k_x k_y & c_{11}(\vec{k})k_y^2 + c_{66}^h k_x^2 + c_{44}^h(\vec{k})k_z^2 \end{pmatrix}$$

... for $B||ab$

Elastic constants are known for uniaxial superconductors:

$$\bar{m} = \begin{pmatrix} m_{ab} & & \\ & m_{ab} & \\ & & m_c \end{pmatrix}$$

ANISOTROPY: $\gamma^4 = \frac{m_{ab}}{m_c} = (\frac{\lambda_{ab}}{\lambda_c})^2 = (\frac{\xi_c}{\xi_{ab}})^2$

Elastic Constants

TILT MODULI

$$c_{44}^{e}(\overrightarrow{q}) = \frac{B^2}{4\pi}\frac{1-b}{2b\kappa^2}\left[\frac{1}{m_\lambda^2+(q_x^2+q_y^2)+\gamma^{-2}q_z^2}\right] + \frac{B^2}{4\pi}\frac{5}{2b\kappa^2} ln\left(\widetilde{\kappa} + \frac{1-b}{2}\right)$$

$$c_{44}^{h}(\overrightarrow{q}) = \frac{B^2}{4\pi}\frac{1-b}{2b\kappa^2}\left[\left(\frac{m_\lambda^2+\left(\gamma^{-4}q_x^2+q_y^2+\gamma^{-2}q_z^2\right)}{m_\lambda^2+\left(q_x^2+\gamma^4 q_y^2+\gamma^2 q_z^2\right)}\right)\left(\frac{1}{m_\lambda^2+q_x^2+q_y^2+\gamma^{-2}q_z^2}\right)\right] + \frac{B^2}{4\pi}\frac{\gamma^{-4}}{2b\kappa^2} ln\left(\widetilde{\kappa} + \frac{1-b}{2}\right)$$

BULK MODULI

$$c_{11}^{e}(\overrightarrow{q}) = c_{11}^{h}(\overrightarrow{q}) = c_{44}^{e,o}(\overrightarrow{q})$$

SHEAR MODULI

$$c_{66} = \frac{\Phi_o B}{(8\pi\lambda_{ab})^2} = \frac{B^2}{4\pi}\gamma^2\frac{(1-b)^2}{8b\kappa^2}$$

$$c_{66}^{e} = \gamma^6 c_{66}$$

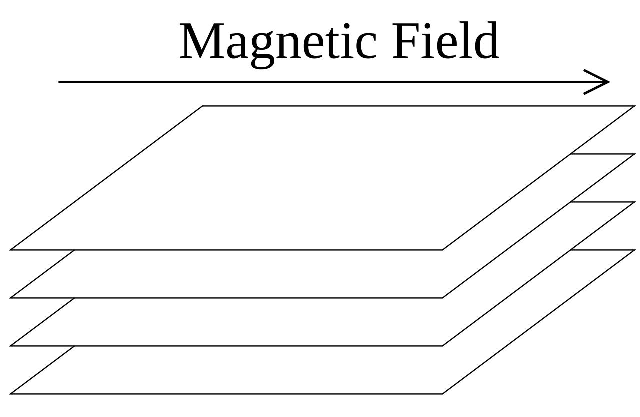
$$c_{66}^{h} = \gamma^{-2} c_{66}$$

where:

$$m_\lambda^2 = \frac{1-b}{2b\kappa^2}$$

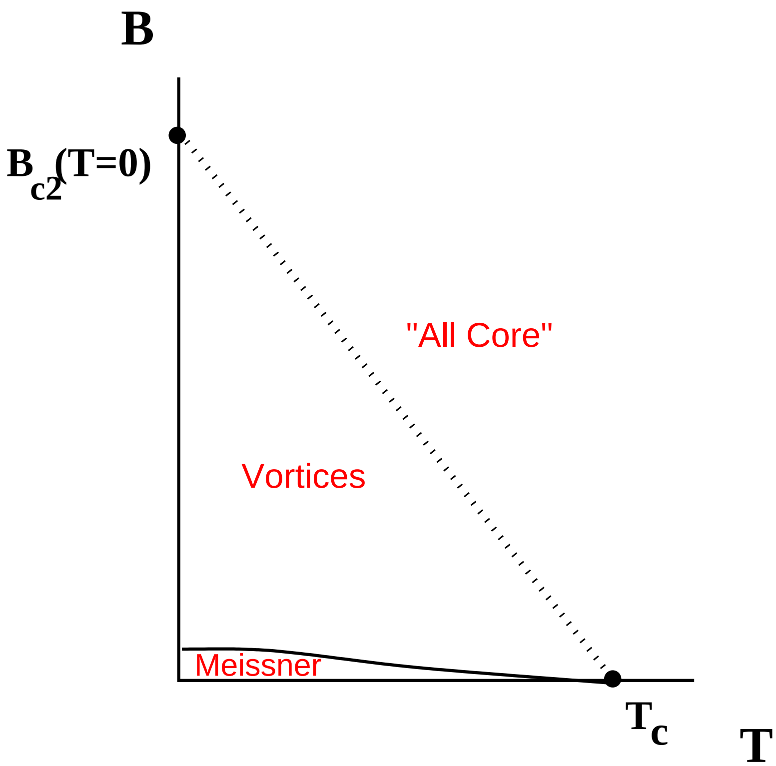
$$\widetilde{\kappa} = \sqrt{\frac{1+\gamma^{-4}\kappa^2+2b\kappa^2\gamma^{-2}q_z^2}{1+b\kappa^2+2b\kappa^2\gamma^{-2}q_z^2}}$$

$$b \equiv \frac{B}{B_{c2}^{ab}(T)} = \frac{B}{B_{c2}^{ab}(T=0)(1-t)}$$



Input parameters: b, γ, $\kappa = \frac{\lambda_{ab}}{\xi_{ab}}$

Elastic Constants Vanish at B_{c2}



$$\mathrm{B}^{c}_{c2} = \frac{\phi_o}{2\pi\xi^2_{ab}(T)}$$

$$B^{ab}_{c2} = \frac{\phi_o}{2\pi\xi_c(T)\xi_{ab}(T)}$$

$$\xi(T) = \frac{\xi(T)}{\sqrt{1-t}} \qquad \Rightarrow \qquad B^{ab}_{c2} = \frac{\phi_o}{2\pi\xi_c(0)\xi_{ab}(0)}(1-t)$$

$$t = T/T_c$$

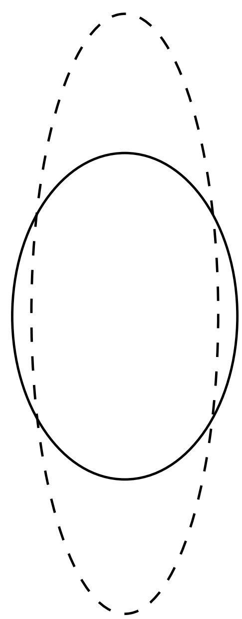
Which way will it melt?

- ☐ Elastic constants scale on short length scales.
- ☐ Scaling breaks down at long length scales, $c_1 1, c_4 4 \to \frac{B^2}{4\pi}$

Lindemann criterion

$< u_x^2 > \geq \frac{1}{2} c^2 a_x^2$

$< u_y^2 > \geq \frac{1}{2} c^2 a_y^2$



Lindemann ellipse follows eccentricity of the lattice

Fluctuations are less eccentric

Anisotropy favors smectic-A

Short wavelengths: Elastic constants soften
Long wavelengths: Low energy cost
Both are important.

YBCO with $B||ab$

We can compare to the uniaxial case experimentally.

Add pinning:

$$\bar{C} = \begin{pmatrix} c_{11}(\vec{k})k_x^2 + c_{66}^e k_y^2 + c_{44}^e(\vec{k})k_z^2 & c_{11}(\vec{k})k_x k_y \\ c_{11}(\vec{k})k_x k_y & c_{11}(\vec{k})k_y^2 + c_{66}^h k_x^2 + c_{44}^h(\vec{k})k_z^2 + \Delta \end{pmatrix}$$

Using Lawrence-Doniach model, $\Delta = \frac{8\sqrt{\pi}B_{c2}^2(b-b^2)\xi_c\gamma^2}{s^3\beta_A\kappa^2} e^{-8\xi_c^2/s^2}$ where $\beta_A \approx 1.16$

Pinning vanishes exponentially as the $B_{c2}(T)$ line is approached.

$\rightarrow$ Pinning favors smectic-C

$\rightarrow$ Anisotropy favors smectic-A

With Planar Pinning

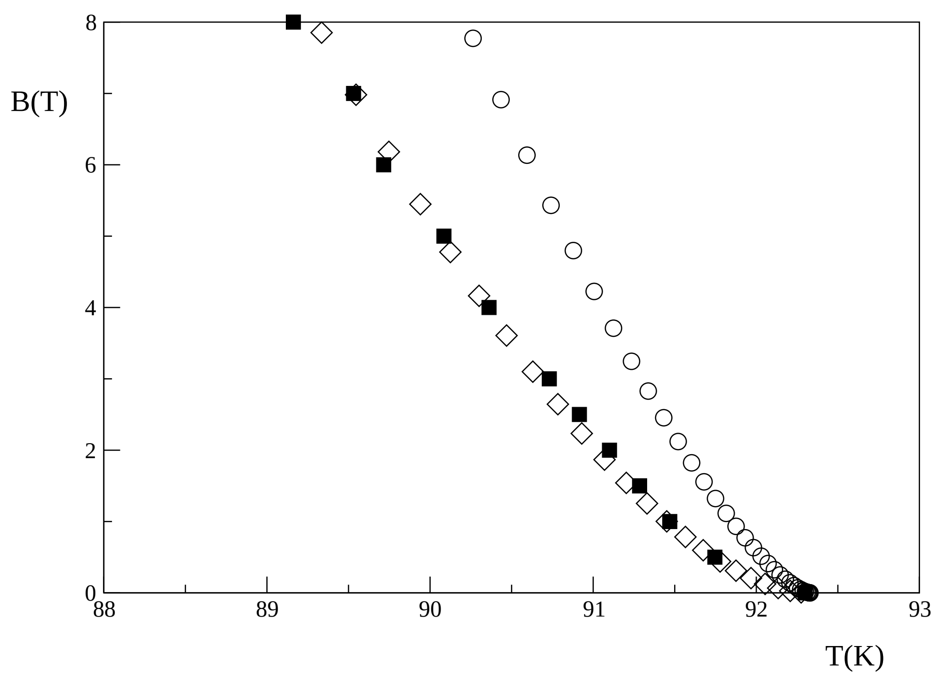
Integrate $< u_x^2 >$ and $< u_y^2 >$ numerically to obtain melting curves

Compare to data on YBCO with $B||ab$

Parameters:

$T_c = 92.3K \quad \frac{m_c}{m_{ab}} = 59 \quad \kappa = \frac{\lambda_{ab}}{\xi_{ab}} = 55 \quad H_{c2}^{ab} = 842T$

Lindemann parameter $c = .19$ (only free parameter)



Data is from Kwok *et al.*, PRL **69** 3370(1992)

Grigera *et al.* PRB (1998) find smectic-C in optimally doped YBCO with $B||ab$

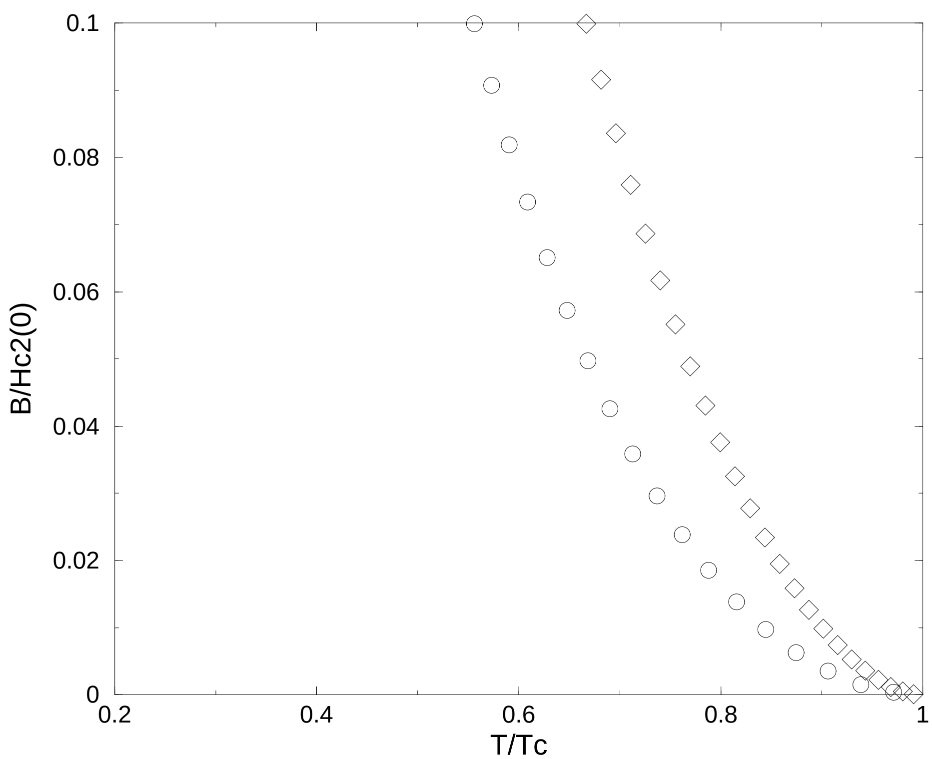
But we are really interested in the case without pinning.

Now consider the effect of anisotropy alone...

In the absence of pinning:

Parameters: $\frac{m_c}{m_{ab}} = 10 \qquad \kappa = \frac{\lambda_{ab}}{\xi_{ab}} = 100$

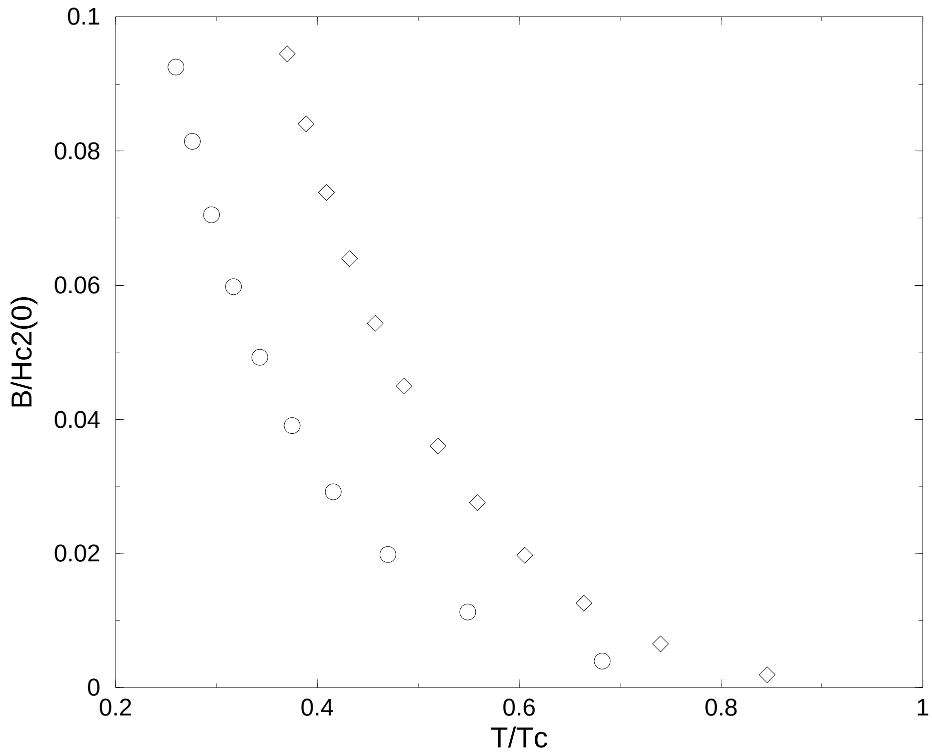
Lindemann parameter: $c = .2$



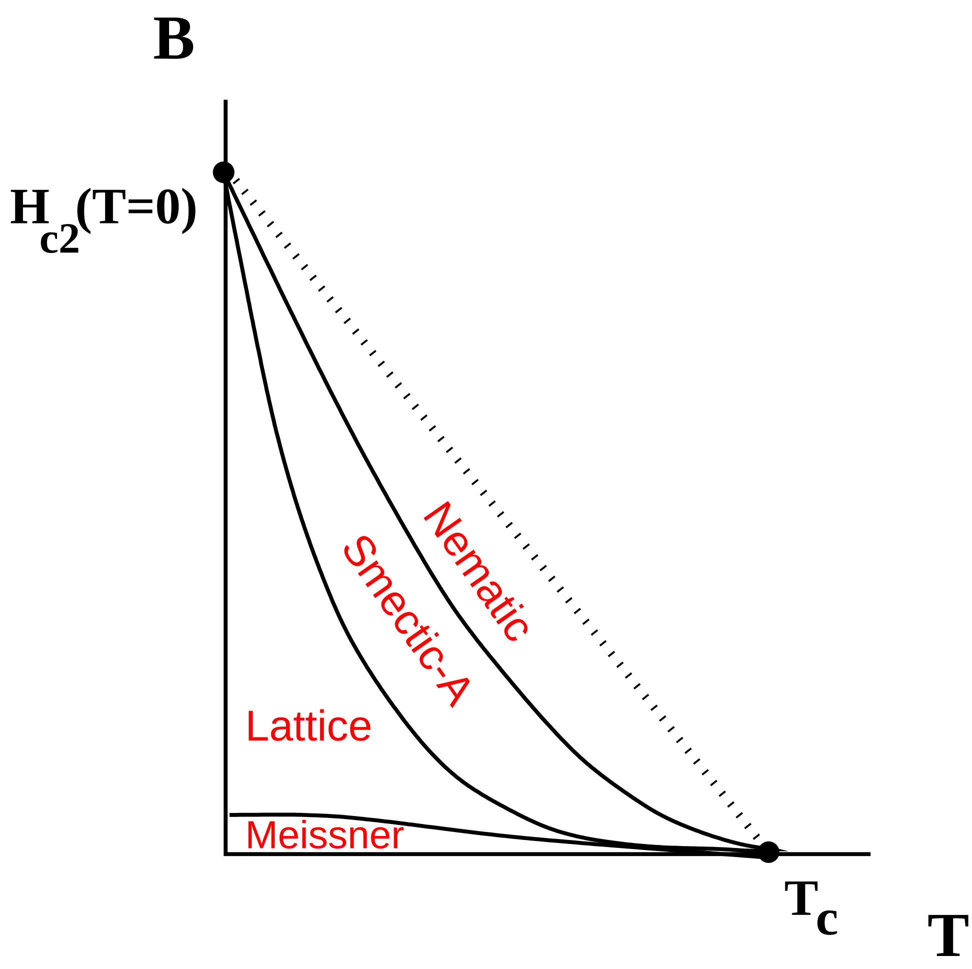
Stronger anisotropy:

$\frac{m_c}{m_{ab}} = 100 \qquad \kappa = \frac{\lambda_{ab}}{\xi_{ab}} = 100$

Lindemann parameter $c = .2$



Theoretical Phase Diagram



Long Range Order?

Smectic + *spontaneously broken rotational symmetry*

Has at most quasi long-range order

Smectic order parameter: $\rho = |\rho| e^{i\theta}$

$$F = \frac{1}{2} \int d\mathbf{r} [\alpha (\nabla_{||} \theta)^2 + \beta (\nabla_{\perp}^2 \theta)^2]$$

Smectic + *explicitly broken rotational symmetry*

Our assumption: $m_{ab} \neq m_c$

Explicitly broken rotational symmetry

Costs energy to rotate the vortex smectic.

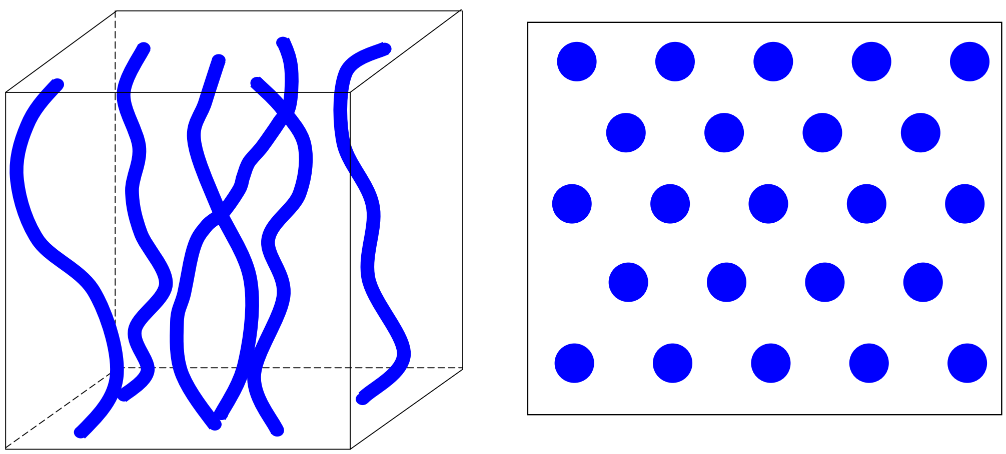
$$F = \frac{1}{2} \int d\mathbf{r} [\alpha' (\nabla_x \theta)^2 + \beta' (\nabla_y \theta)^2 + \beta'' (\nabla_z \theta)^2]$$

Just like a 3D crystal.

> 3D smectic + explicitly broken rotational symmetry
>
> $\rightarrow$ Long Range Order

Other methods point to smectic-A

2D boson mapping: (Nelson, 1988)

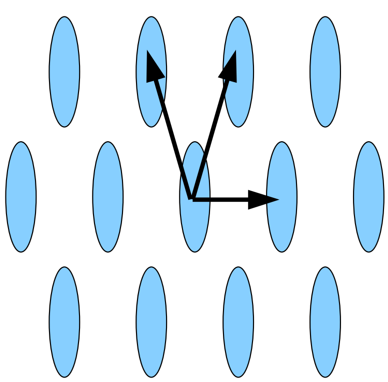


L $\to \tau \to \infty$

3D vortices

2D bosons $T = 0$

2D melting $T > 0$: (Ostlund Halperin, 1981)



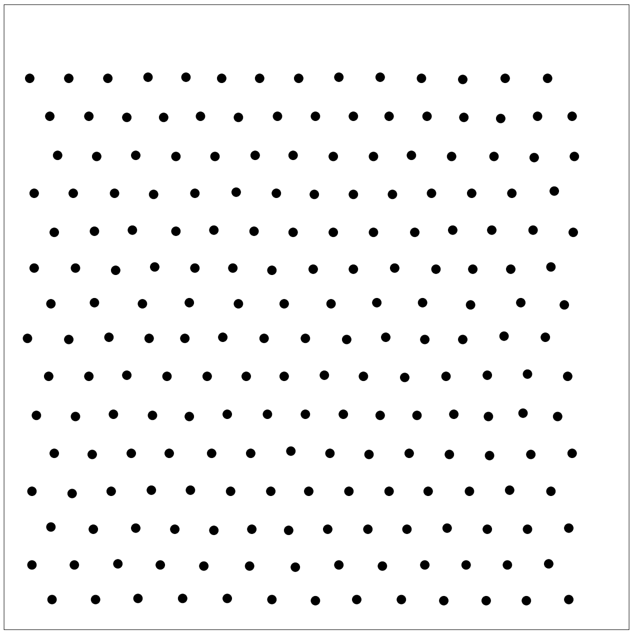
Short Burgers' vector dislocations unbind first

$\to$ Smectic-A
Quasi-Long-Range Order

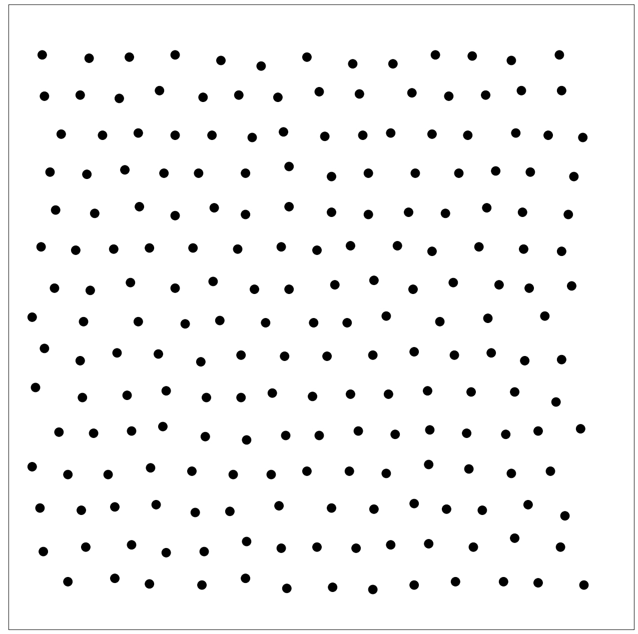
Our case: $T = 0$ Smectic can be long-range ordered.

C. Reichhardt and C. Olson

Numerical simulations on 2D vortices with anisotropic interactions



a



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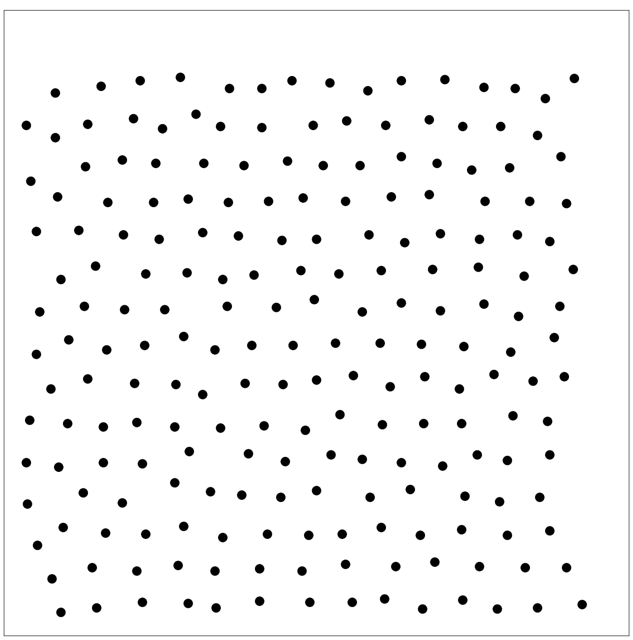

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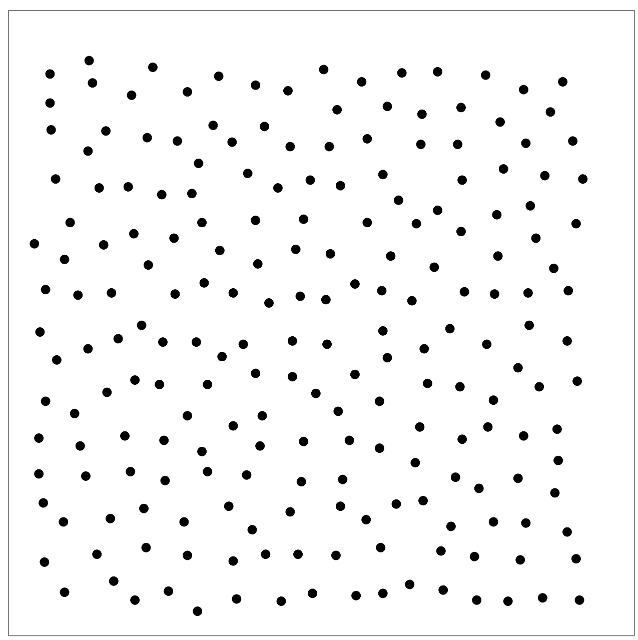

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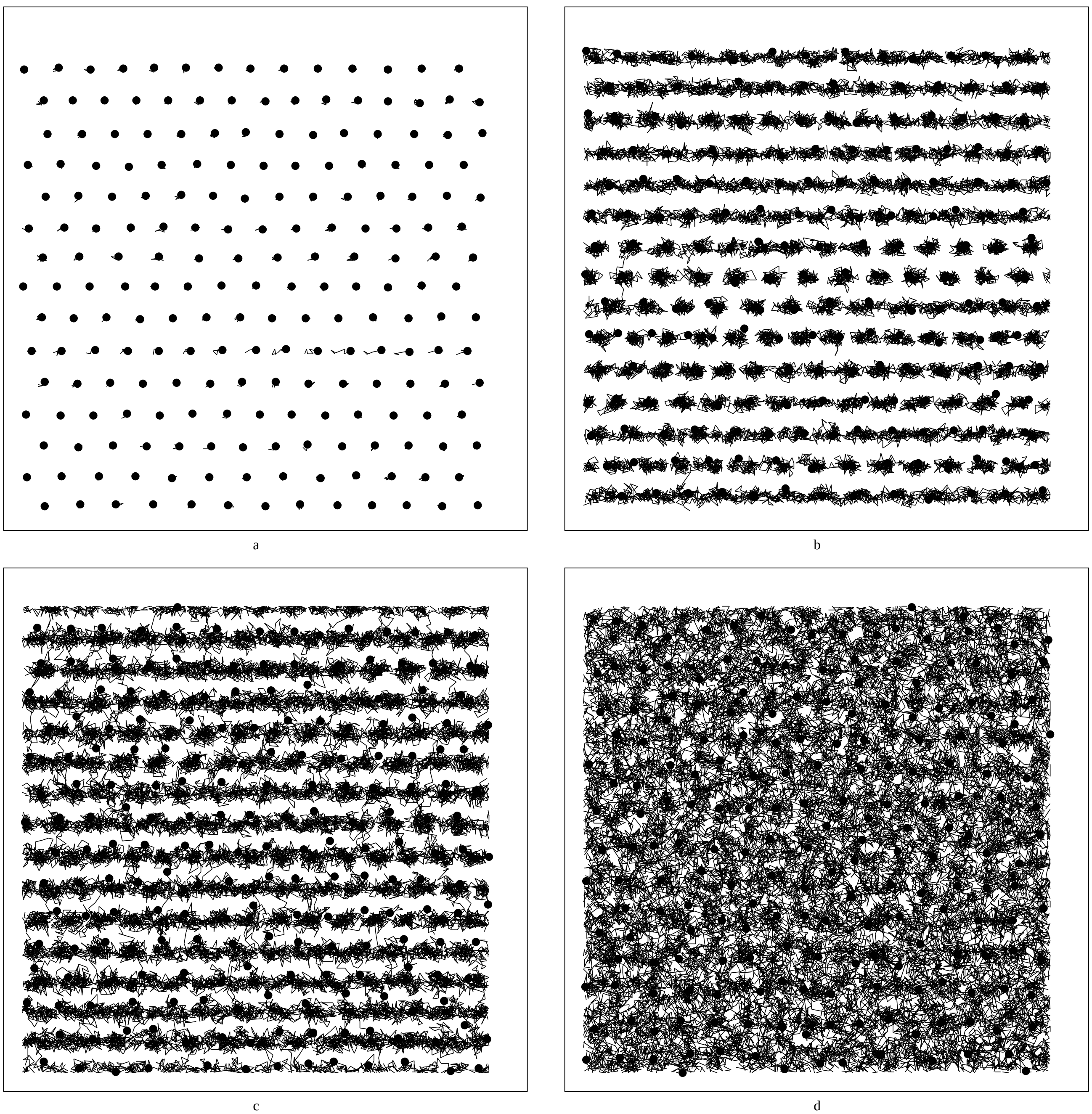
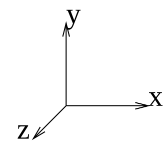
C. Reichhardt and C. Olson

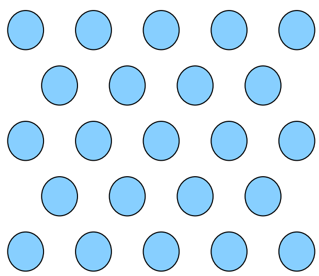
Numerical simulations on 2D vortices with anisotropic interactions



Distinguishing the Smectics: Lorentz



Lattice



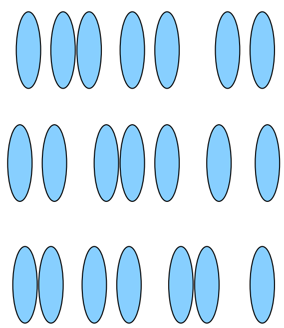
$\rho_x = 0$

$\rho_y = 0$

$\rho_z = 0$ $(F_L = 0)$

3D Superconductivity

Smectic-A



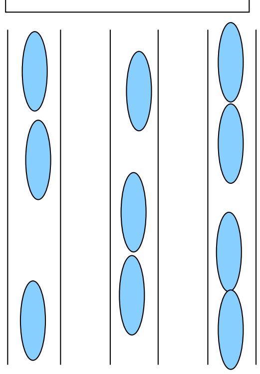
$\rho_x = 0$

$\rho_y \neq 0$ $(F_L \parallel$ liquid-like)

$\rho_z = 0$ $(F_L = 0)$

2D Superconductivity between smectic layers

Smectic-C



$\rho_x \neq 0$ $(F_L \parallel$ liquid-like)

$\rho_y = 0$

$\rho_z = 0$ $(F_L = 0)$

2D Superconductivity between smectic layers

Experimental Signatures

Resistivity:

- □ Smectic may retain 2D superconductivity
- □ $\rho = 0$ along liquid-like smectic layers
- □ $\rho \neq 0$ along the density wave

Structure Factor:

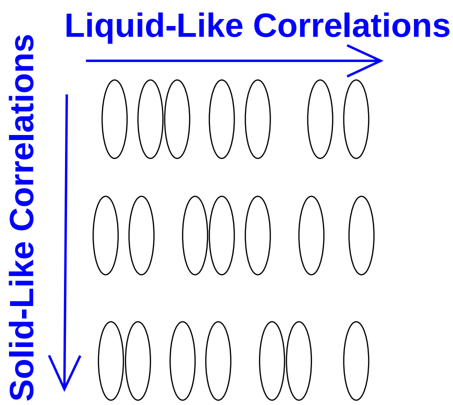
(Neutron scattering or Bitter decoration)

- □ Liquid-like correlations in one direction
- □ Solid-like correlations in the other

μSR:

- □ Signature at both melting temperatures

Where to look for the Smectic-A



Our Assumptions:

- ☐ Explicitly broken rotational symmetry
- ☐ No explicitly broken translational symmetry

Cuprate Superconductors: $\rightarrow$ Stripe Nematic

- ☐ Yields anisotropic superfluid stiffness
- ☐ Does not break translational symmetry

$B||c$: Look for vortex smectic-A in the presence of a stripe nematic

Conclusions

□ Anisotropy favors intermediate melting

(Lattice $\rightarrow$ Smectic-A $\rightarrow$ Nematic)

- Lindemann criterion
- 2D boston mapping
- 2D numerical simulations (Reichhardt and Olson)

□ Smectic-A has Long Range Order

- 3D smectic + explicit symmetry breaking
- 2D boson mapping

□ Experimental Signatures:

- Resistivity anisotropy
- μSR
- Bitter decoration
- Neutron scattering