Vortex Liquid Crystals in Anisotropic Type II Superconductors

E. W. Carlson
A. H. Castro Neto
D. K. Campbell

Boston University

cond-mat/0209175
In the high temperature superconductors, 
\[ \kappa \equiv \frac{\lambda}{\xi} \approx 100 \]

\( \lambda \): screening currents and magnetic field
\( \xi \): the normal “core”
Vortex Lattice Melting

Circular Cross Sections:

Lattice

Liquid

Raise T or B
Cross Section of a Vortex

**ISOTROPIC**

Circular Profile

\[
\overline{m} = \begin{pmatrix} m \\ m \\ m \end{pmatrix}
\]

\[
\lambda^2 = \frac{mc^2}{4\pi e^2 n_s}
\]

\[
\lambda^2 \nabla^2 \overrightarrow{B} - \overrightarrow{B} = \Phi_0 \delta_2(r)
\]

\[
\overrightarrow{B} = \frac{\Phi_0}{2\pi \lambda^2} K_0(r/\lambda)
\]

**ANISOTROPIC**

Elliptical Profile

\[
\overline{m} = \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix}
\]

\[
\lambda_x^2 = \frac{m_x c^2}{4\pi e^2 n_s} \quad (B \parallel \hat{z})
\]

\[
\lambda^2 \nabla^2 \overrightarrow{B} - \overrightarrow{B} = \Phi_0 \delta_2(r)
\]

\[
\overrightarrow{B} = \frac{\Phi_0}{2\pi \lambda^2} K_0(r/\lambda)
\]
Anisotropic Vortex Lattice Melting

Elliptical Cross Sections:

Anisotropic Interacting “Molecules”
→ Liquid Crystalline Phases

Abrikosov Liquid Crystals?

Lattice

Smectic-A

Liquid-Like

Nematic

Solid-Like

Raise T or B

Raise T or B
Symmetry of Phases

CRYS TALS: Break continuous rotational and translational symmetries of 3D space

LIQUIDS: Break none

LIQUID CRYSTALS: Break a subset

**Hexatic:** 6-fold rotational symmetry unbroken translational symmetry

**Nematic:** 2-fold rotational symmetry unbroken translational symmetry

**Smectic:** breaks translational symmetry in 1 or 2 directions
Smectics

Smectic-A

Smectic-C

Full translational symmetry in at least one direction
Broken translational symmetry in at least one direction
(Broken rotational symmetry)
Many Vortex Phases

Abrikosov Lattice: Abrikosov (1957)
Chain States: Ivlev, Kopnin (1990)
Smectic-C: Efetov (1979)

Our Assumption:

- Explicitly broken rotational symmetry
Instability of Ordered Phase: Lindemann Criterion for Melting

\[ \vec{u} = (u_x, u_y) \]

\[ < u^2 > = < u_x^2 > + < u_y^2 > \geq c^2 a^2 \]

Typically, lattice melts for \( c \approx 0.1 \)
Houghton, Pelcovits, Sudbo (1989)

Extended to Anisotropy:

\[ < u_x^2 > \geq \frac{1}{2} c^2 a_x^2 \]
\[ < u_y^2 > \geq \frac{1}{2} c^2 a_y^2 \]

Look for one to be exceeded well before the other.
Method

Calculate \( < u_x^2 > \) and \( < u_y^2 > \)

- based on elasticity theory of the ordered state
- using \( k \)-dependent elastic constants
- from Ginzburg-Landau theory

\[
F = \frac{1}{(2\pi)^3} \int dk u \cdot \bar{C} \cdot \bar{u}
\]

where \( \bar{u} = (u_x, u_y) = \) vortex displacement

\[
\bar{C} = \begin{pmatrix}
    c_{11}(\bar{k})k_x^2 + c_{66}^e k_y^2 + c_{44}^e (\bar{k})k_z^2 & c_{11}(\bar{k})k_x k_y \\
    c_{11}(\bar{k})k_x k_y & c_{11}(\bar{k})k_y^2 + c_{66}^h k_x^2 + c_{44}^h (\bar{k})k_z^2
\end{pmatrix}
\]

... for \( B || ab \)

Elastic constants are known for uniaxial superconductors:

\[
\bar{m} = \begin{pmatrix}
    m_{ab} & m_{ab} \\
    m_{ab} & m_{c}
\end{pmatrix}
\]

ANISOTROPY: \( \gamma^4 = \frac{m_{ab}}{m_{c}} = \left( \frac{\lambda_{ab}}{\lambda_c} \right)^2 = \left( \frac{\xi_c}{\xi_{ab}} \right)^2 \)
Elastic Constants

**TILT MODULI**

\[
c_{44}(\vec{q}) = \frac{B^2}{4\pi} \frac{1-b}{2b\kappa^2} \left[ \frac{1}{m_\lambda^2 + (q_x^2 + q_y^2)} + \gamma^{-2} q_z^2 \right] + \frac{B^2}{4\pi} \frac{5}{2b\kappa^2} \ln \left( \hat{\kappa} + \frac{1-b}{2} \right)
\]

\[
c_{44}^h(\vec{q}) = \frac{B^2}{4\pi} \frac{1-b}{2b\kappa^2} \left[ \left( \frac{m_\lambda^2 + (q_x^2 + q_y^2)}{m_\lambda^2 + (q_x^2 + q_y^2 + \gamma^2 q_z^2)} \right) \left( \frac{1}{m_\lambda^2 + q_x^2 + q_y^2 + \gamma^2 q_z^2} \right) \right] + \frac{B^2}{4\pi} \frac{\gamma^{-4}}{2b\kappa^2} \ln \left( \hat{\kappa} + \frac{1-b}{2} \right)
\]

**BULK MODULI**

\[
c_{11}^e(\vec{q}) = c_{11}^h(\vec{q}) = c_{44}^{e,o}(\vec{q})
\]

**SHEAR MODULI**

\[
c_{66}^e = \frac{\Phi_e B}{(8\pi \lambda_{ab})^2} = \frac{B^2}{4\pi} \gamma^2 \frac{(1-b)^2}{8b\kappa^2}
\]

\[
c_{66}^c = \gamma^6 c_{66}
\]

\[
c_{66}^h = \gamma^{-2} c_{66}
\]

where:

\[
m_\lambda^2 = \frac{1-b}{2b\kappa^2}
\]

\[
\hat{\kappa} = \sqrt{\frac{1 + \gamma^{-4} \kappa^2 + 2b\kappa^2 \gamma^{-2} q_z^2}{1 + b\kappa^2 + 2b\kappa^2 \gamma^{-2} q_z^2}}
\]

\[
b = \frac{B}{B_{c2}^o(T)} = \frac{B}{B_{c2}^o(T=0)(1-t)}
\]

Input parameters: \(b, \gamma, \kappa = \frac{\lambda_{ab}}{\xi_{ab}}\)

**Magnetic Field**
Elastic Constants Vanish at $B_{c2}$

$$B_{c2}(T=0)$$

"All Core"

Meissner

Vortices

$$B_{c2}^c = \frac{\phi_o}{2\pi \xi_{ab}(T)}$$

$$B_{c2}^{ab} = \frac{\phi_o}{2\pi \xi_c(T) \xi_{ab}(T)}$$

$$\xi(T) = \frac{\xi(T)}{\sqrt{1-t}}$$

$$B_{c2}^{ab} = \frac{\phi_o}{2\pi \xi_c(0) \xi_{ab}(0)} (1 - t)$$

$$t = \frac{T}{T_c}$$
Which way will it melt?

- Elastic constants scale on short length scales.
- Scaling breaks down at long length scales, \( c_{11}, c_{44} \rightarrow \frac{\beta^2}{4\pi} \)

\[
\begin{align*}
\langle u_x^2 \rangle &\geq \frac{1}{2} c^2 a_x^2 \\
\langle u_y^2 \rangle &\geq \frac{1}{2} c^2 a_y^2
\end{align*}
\]

Lindemann ellipse follows eccentricity of the lattice

Fluctuations are less eccentric

Anisotropy favors smectic-A

Short wavelengths: Elastic constants soften
Long wavelengths: Low energy cost
Both are important.
YBCO with $B \parallel ab$

We can compare to the uniaxial case experimentally.

Add pinning:

$$\tilde{\mathcal{C}} = \begin{pmatrix}
    c_{11}(\vec{k})k^2_x + c_{66}^e k^2_y + c_{44}^e (\vec{k})k^2_z & c_{11}(\vec{k})k_xk_y \\
    c_{11}(\vec{k})k_xk_y & c_{11}(\vec{k})k^2_y + c_{66}^h k^2_x + c_{44}^h (\vec{k})k^2_z + \Delta
\end{pmatrix}$$

Using Lawrence-Doniach model, $\Delta = \frac{8\sqrt{\pi}B^2_{c2}(b-b^2)\xi_\gamma^2}{s^3\beta_Ak^2}e^{-8\xi^2_\gamma/s^2}$

where $\beta_A \approx 1.16$

Pinning vanishes exponentially as the $B_{c2}(T)$ line is approached.

$\rightarrow$ Pinning favors smectic-C

$\rightarrow$ Anisotropy favors smectic-A
With Planar Pinning

Integrate $<u_x^2>$ and $<u_y^2>$ numerically to obtain melting curves

Compare to data on YBCO with $B||ab$

Parameters:

$T_c = 92.3K \quad \frac{m_c}{m_{ab}} = 59 \quad \kappa = \frac{\lambda_{ab}}{\xi_{ab}} = 55 \quad H_{c2}^{ab} = 842T$

Lindemann parameter $c = .19$ (only free parameter)

Data is from Kwok et al., PRL 69 3370(1992)

Grigera et al. PRB (1998) find smectic-C in optimally doped YBCO with $B||ab$
But we are really interested in the case without pinning.

Now consider the effect of anisotropy alone...
In the absence of pinning:

Parameters: $\frac{m_c}{m_{ab}} = 10 \quad \kappa = \frac{\lambda_{ab}}{\xi_{ab}} = 100$

Lindemann parameter: $c = 0.2$
Stronger anisotropy:

\[
\frac{m_c}{m_{ab}} = 100 \quad \kappa = \frac{\lambda_{ab}}{\xi_{ab}} = 100
\]

Lindemann parameter \( c = 0.2 \)
Theoretical Phase Diagram

- Lattice
- Nematic
- Smectic-A
- Meissner

$B$

$H_{c2}(T=0)$

$T_c$
**Long Range Order?**

Smectic + *spontaneously broken rotational symmetry*

Has at most quasi long-range order

Smectic order parameter: $\rho = |\rho|e^{i\theta}$

$$F = \frac{1}{2} \int dr[\alpha(\nabla_{\parallel} \theta)^2 + \beta(\nabla_{\perp} \theta)^2]$$

Smectic + *explicitly broken rotational symmetry*

Our assumption: $m_{ab} \neq m_c$

Explicitly broken rotational symmetry

Costs energy to rotate the vortex smectic.

$$F = \frac{1}{2} \int dr[\alpha'(\nabla_x \theta)^2 + \beta'(\nabla_y \theta)^2 + \beta''(\nabla_z \theta)^2]$$

Just like a 3D crystal.

---

3D smectic + explicitly broken rotational symmetry

→ Long Range Order
Other methods point to smectic-A

2D boson mapping: (Nelson, 1988)

\[ L \to \tau \to \infty \]

3D vortices 2D bosons \( T = 0 \)

2D melting \( T > 0 \): (Ostlund Halperin, 1981)

Short Burgers' vector dislocations unbind first

\[ \to \text{Smectic-A} \]

Quasi-Long-Range Order

Our case: \( T = 0 \) Smectic can be long-range ordered.
C. Reichhardt and C. Olson

Numerical simulations on 2D vortices with anisotropic interactions
C. Reichhardt and C. Olson

Numerical simulations on 2D vortices with anisotropic interactions
Distinguishing the Smectics: Lorentz

Lattice

\[ \rho_x = 0 \]
\[ \rho_y = 0 \]
\[ \rho_z = 0 \quad (F_L=0) \]

Smectic-A

\[ \rho_x = 0 \]
\[ \rho_y \neq 0 \quad (F_L \parallel \text{liquid-like}) \]
\[ \rho_z = 0 \quad (F_L=0) \]

Smectic-C

\[ \rho_x \neq 0 \quad (F_L \parallel \text{liquid-like}) \]
\[ \rho_y = 0 \]
\[ \rho_z = 0 \quad (F_L=0) \]
Experimental Signatures

Resistivity:
- Smectic may retain 2D superconductivity
- $\rho = 0$ along liquid-like smectic layers
- $\rho \neq 0$ along the density wave

Structure Factor:
(Neutron scattering or Bitter decoration)
- Liquid-like correlations in one direction
- Solid-like correlations in the other

$\mu$SR:
- Signature at both melting temperatures
Where to look for the Smectic-A

Our Assumptions:
- Explicitly broken rotational symmetry
- No explicitly broken translational symmetry

Cuprate Superconductors: → Stripe Nematic
- Yields anisotropic superfluid stiffness
- Does not break translational symmetry

\[ B||c: \]
Look for vortex smectic-A in the presence of a stripe nematic
Conclusions

- Anisotropy favors intermediate melting (Lattice $\rightarrow$ Smectic-A $\rightarrow$ Nematic)
  - Lindemann criterion
  - 2D boston mapping
  - 2D numerical simulations (Reichhardt and Olson)

- Smectic-A has Long Range Order
  - 3D smectic + explicit symmetry breaking
  - 2D boson mapping

- Experimental Signatures:
  - Resistivity anisotropy
  - $\mu$SR
  - Bitter decoration
  - Neutron scattering