Vortex Liquid Crystals in Anisotropic Type II Superconductors

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In an isotropic type II superconductor in a moderate magnetic field, the transition to the normal state occurs by vortex lattice melting. In certain anisotropic cases, the vortices acquire elongated cross sections and interactions. Systems of anisotropic, interacting constituents generally exhibit liquid crystalline phases. We examine the possibility of a two step melting in homogeneous type II superconductors with anisotropic superfluid stiffness from a vortex lattice into first a vortex smectic and then a vortex nematic at high temperature and magnetic field. We find that fluctuations of the ordered phase favor an instability to an intermediate smectic-*A* in the absence of intrinsic pinning.

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Recently, there has been much interest in high temperature superconductors in a magnetic field. Various experiments have studied the interplay of superconductivity with coexistent magnetic orders in the presence of an external field. Experiments using both neutron scattering [1,2] and STM [3] show that there is significant local electronic inhomogeneity. These and other experiments lend credence to the idea that there may be electronic liquid crystalline phases in strongly correlated systems, leading to anisotropy even within a CuO_2 plane.

The cuprate superconductors are also ideal for studying vortex physics, due to the large values of $\kappa \equiv \lambda_{ab}/\xi_{ab}$ (where λ_{ab} and ξ_{ab} are the London penetration depth within a plane, and the coherence length within a plane, respectively) and small critical depinning current [4]. In this Letter, we consider the effects of an anisotropic superfluid stiffness on the vortex phases in superconductors in the continuum limit. We account for this anisotropy by allowing different effective masses in the three crystalline directions, which we will call m_a, m_b, and m_c.

In a biaxial superconductor (with m_a, m_b, and m_c all distinct), or a uniaxial superconductor ($m_a = m_b$) with magnetic field oriented perpendicular to the symmetry axis, vortices acquire elliptical cross sections, whether measured by the shape of the core, or by the profile of the screening currents or magnetic field which penetrates beyond the core. Systems of anisotropic interacting constituents generically lead to liquid crystalline phases. In such a system, we expect the melting to proceed from the body centered rectangular lattice, to a smectic, to a nematic, as in Fig. 1. (In this case, the high temperature phase is trivially nematic due to the explicitly broken rotational symmetry introduced by the mass anisotropy.)

Liquid crystals lie somewhere between the full translational and rotational symmetry of a liquid, and that of a 3D crystal, which has broken rotational symmetry, and retains only discrete translational symmetry in the three directions of the crystal axes. In a superconductor, the application of an external magnetic field **B** to produce vortices explicitly breaks rotational symmetry. We choose axes such that $\mathbf{B} \| \hat{z}$. In the vortex system, smectic phases correspond to liquidlike correlations (and unbroken translational symmetry) in one direction in the xy plane, and simultaneous solidlike correlations (and only discrete translational symetry) in the other direction in the xy plane. Smectics may be further classified by which direction the "elongated molecule" is pointing on average with respect to the orientation of the liquidlike layers. Call θ the angle that the long axis of the molecule makes with respect to the normal of the liquidlike layers. For $\theta = 0$, the phase is smectic-*A*, illustrated in Fig. 2. For all other values of θ the phase is a smectic-*C*. A nematic phase is characterized by unbroken translational symmetry, with

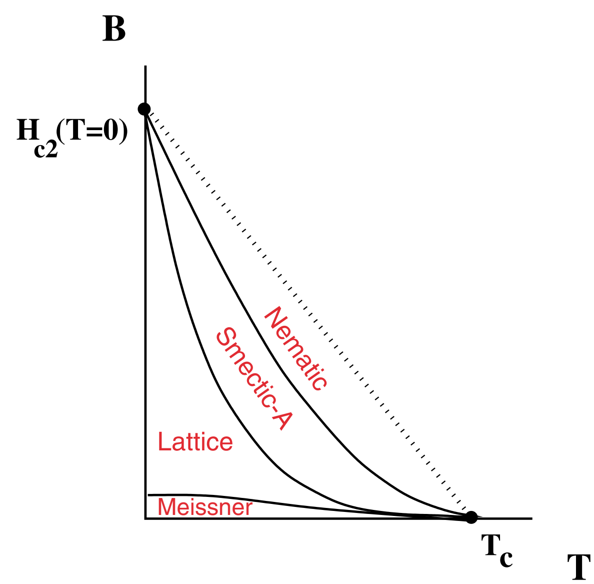

FIG. 1 (color online). Schematic phase diagram of vortex matter with anisotropic interactions. Solid lines represent phase transitions, and the dotted line represents the crossover at H_{c2}. There may also be melted or partially melted phases near H_{c1}, between the regions marked Meissner and Lattice.

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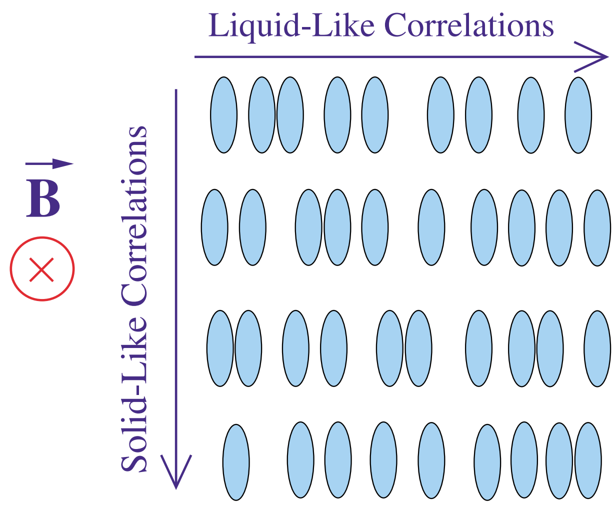

FIG. 2 (color online). Smectic-A. Cross sections of vortices in an anisotropic superconductor are represented schematically above as filled ellipses, for a magnetic field oriented perpendicular to the page. There may be an intermediate melting from the elongated lattice first to a smectic-A, shown above, and then to a nematic phase at high temperature.

broken orientational symmetry. Correlations in this phase are liquidlike in all directions, but the constituent molecules have a preferred orientation.

When the magnetic field is oriented parallel to the planes [5–7] in a layered superconductor, the explicit translational symmetry breaking of the planes may cause the vortex lattice to melt first along the direction of the planes, leading to a smectic-C with $\theta = \frac{\pi}{2}$. Smectic phases have also been predicted in the presence of a driving current [8], as well as chain states which may arise when the field is tilted sufficiently away from a crystalline axis [9]. Here we explore the possibility of liquid crystalline phases due to explicit rotational symmetry breaking (mass anisotropy), with no explicit translational symmetry breaking in the problem (i.e., no intrinsic pinning). We find that thermal fluctuations of the anisotropic lattice favor an instability to a vortex smectic-A. Such a phase, if it exists, should be detectable by several experimental probes, including Bitter decoration, neutron scattering, muon spin resonance (μSR), resistivity, and peak effect measurements.

We use continuum elasticity theory to describe the thermal fluctuations of an ordered vortex lattice in three dimensions [10]. The displacement of a vortex from its equilibrium position is denoted by the vector $\mathbf{u} = (u_x, u_y)$, which is a function of the position z along a given vortex. To second order in $\mathbf{u}$, the free energy is

$$F = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\mathbf{u}\cdot\mathbf{C}\cdot\mathbf{u}, \tag{1}$$

where the matrix $\mathbf{C}$ contains the elastic constants [4]:

$$\mathbf{C} = \begin{pmatrix} c_{11}k_y^2 + c_{66}^h k_x^2 + c_{44}^h k_z^2 & c_{11}k_x k_y \\ c_{11}k_x k_y & c_{11}k_x^2 + c_{66}^e k_y^2 + c_{44}^e k_z^2 \end{pmatrix}. \tag{2}$$

Here we orient the magnetic field perpendicular to the axis of symmetry: $\mathbf{B} \perp \hat{c}||\hat{z}$. We have also assumed a uniaxial superconductor, $m_a = m_b \equiv m_{ab} \neq m_c$; the elastic constants are not yet known for the fully anisotropic, biaxial case. Nonetheless, this uniaxial geometry captures the physics we are interested in, namely, anisotropic interactions. We use elastic constants derived from Ginzburg-Landau theory [4,10–12]. The bulk modulus, $c_{11}(\mathbf{k})$, describes the compressibility of the lattice. The hard tilt modulus, $c_{44}^h(\mathbf{k})$, corresponds to tilts along the symmetry axis $\hat{c}$, and the (smaller) easy tilt modulus, $c_{44}^e(\mathbf{k})$, corresponds to tilts perpendicular to $\hat{c}$. Similarly, it is easier to shear vortices (c_{66}^e) along the major axis of the cross sectional ellipse, rather than perpendicular to it (c_{66}^h). Note that since the magnetic field is oriented along a crystal symmetry axis, there is no mixing between the bulk, tilt, and shear moduli. The bulk and tilt moduli are highly momentum dependent. In fact, the two soften significantly at the Brillouin zone edge, so that their momentum dependence is important in the physics of melting. The shear moduli are approximately independent of the wavelength of the distortion, and we neglect their weak momentum dependence.

We use an extension of the Lindemann criterion to the case of anisotropy [6] and allow for the possibility that the lattice may melt in one direction before the other. In this case, fluctuations in the x direction compete with the lattice spacing in the x direction, and fluctuations in the y direction with the lattice spacing in the y direction:

$$\langle u_x^2\rangle = \frac{1}{2}c^2a^2\gamma^2, \qquad \langle u_y^2\rangle = \frac{1}{2}c^2a^2/\gamma^2, \tag{3}$$

where $\gamma^4 \equiv (m_{ab}/m_c)$, $a = \sqrt{2\Phi_o/3^{1/2}B}$ is the lattice spacing for the triangular lattice of the isotropic case at the same magnetic field strength B, and Φ_o is the quantum of flux. We look for this criterion to be significantly violated in one direction before the other. The factor of $1/2$ allows the Lindemann parameter c to recover the usual definition in the isotropic case.

For short wavelengths, the physics of an anisotropic superconductor can be mapped onto an isotropic superconductor by a scaling procedure introduced by Blatter *et al.* [13]. Were this true at all wavelengths, there would be no reason to expect a breakdown of the Lindemann criterion in one direction before the other. While the short wavelength interactions are essentially controlled by the screening currents, there is also an electromagnetic interaction among vortices so that the bulk and tilt moduli at long wavelength correspond to $B^2/4\pi$. The length scale of the crossover is direction-dependent, and is determined by the London penetration depth, the anisotropy, and κ. The result is that the spatial profile of the fluctuations of the vortices is less eccentric than the equilibrium lattice, suggesting an instability to partially melted (liquid crystalline) phases.

Using the scaled momenta, $\mathbf{q} = (\gamma k_x/\Lambda, k_y/\gamma\Lambda, k_z/\Lambda)$, the average fluctuations may be written as follows:

$$\langle u_x^2 \rangle = \frac{\Lambda}{B^2} \frac{k_B T}{(2\pi)^3} \int d\mathbf{q} \mathbf{C}_{xx}^{-1}(\mathbf{q}), \qquad (4)$$

$$\langle u_y^2 \rangle = \frac{\Lambda}{B^2} \frac{k_B T}{(2\pi)^3} \int d\mathbf{q} \mathbf{C}_{yy}^{-1}(\mathbf{q}), \qquad (5)$$

where the matrix $\mathbf{C}(\mathbf{q})$ and the elastic constants therein are functions of $\mathbf{q}$ and the cutoff $\Lambda = \sqrt{4\pi B/\Phi_o} \propto \frac{1}{a}$ is set by the vortex lattice spacing. The integrals are functions only of κ, γ, and $b \equiv [B/H_{c2}(T)]$. We compute the integrals numerically to obtain the melting curves.

We first compare to data on optimally doped YBCO, with $\mathbf{B} \perp \hat{c}$. It is believed that in this geometry, the intrinsic pinning of the planes leads to a partially melted phase which is a smectic-C, in which vortices have melted along the planes. In the case of a lattice which is commensurate with the planes, the explicit symmetry breaking of the planes adds a momentum-independent pinning term to the matrix element C_{yy} [7] and tends to encourage melting along the planes, into a smectic-C. In this sense, pinning competes with the aforementioned tendency of the anisotropy to encourage a smectic-A. We find for this material that the Lindemann criterion is violated in one direction well before the other, as shown in Fig. 3, and the instability favors a smectic-C. Note that our approach is only capable of calculating the first melting curve, from solid to smectic. To calculate the melting curve for smectic to nematic, it is necessary to first derive elastic constants for the smectic phase, which is beyond the scope of the present paper.

In Fig. 4, we plot the results *in the absence of intrinsic pinning*. The instability now favors a smectic-A, in which the long direction of each constituent "molecule" (in this case, the major axis of each cross sectional ellipse of a vortex) is oriented perpendicular to the melted layers, as in Fig. 2. Although this means that the vortices have melted first along the direction of harder shear, c_{66}^h, this is the most common smectic geometry observed for oblong molecules. We show the schematic phase diagram in Fig. 1. The smectic region may be pinched off by first order transitions from lattice directly to nematic near $H_{c2}(T=0)$ and near T_c.

The problem of vortices in a 3D superconductor may be mapped to that of 2D bosons at zero temperature, by approximating the vortex interactions as "local" in the coordinate $\hat{z}$, where the path of the vortex represents a bosonic world line. The theory of anisotropic 2D melting in the presence of explicitly broken rotational symmetry predicts a region of quasi-long-range ordered (QLRO)

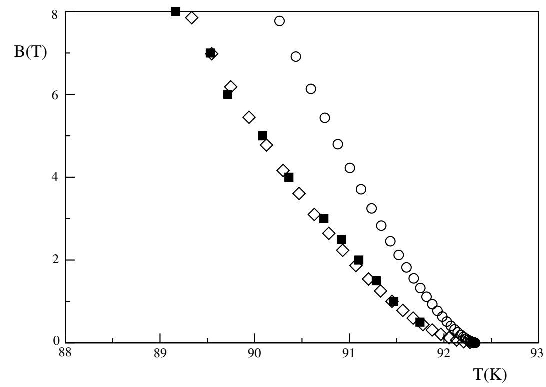

FIG. 3. Solid squares are vortex lattice melting data on optimally doped YBCO with $B \perp c$ [14]. Open symbols are the results of the numerical integration of Eq. (3), taking into account the pinning of the planes. Circles refer to melting in the "short" direction, and diamonds to melting in the "long" direction. We have taken the following parameters, appropriate for optimally doped YBCO with $B \perp c$: $T_c = 92.3K$, $\gamma^{-4} = (m_c/m_{ab}) = 59$, $\kappa = (\lambda_{ab}/\xi_{ab}) = 55$, and $H_{c2}^{ab} = 842T$. The figure is plotted for $c = 0.19$.

smectic-A at finite temperature [15,16]. At zero temperature (to which the current case maps), a long-range ordered (LRO) smectic-A is possible. The intermediate smectic-A has also been seen in recent numerical simulations of a two dimensional vortex system [17].

For the present case, the soft rotational modes usually responsible for preventing translational LRO in the smectic are absent because the mass tensor introduces explicit rotational symmetry breaking. It costs energy to rotate

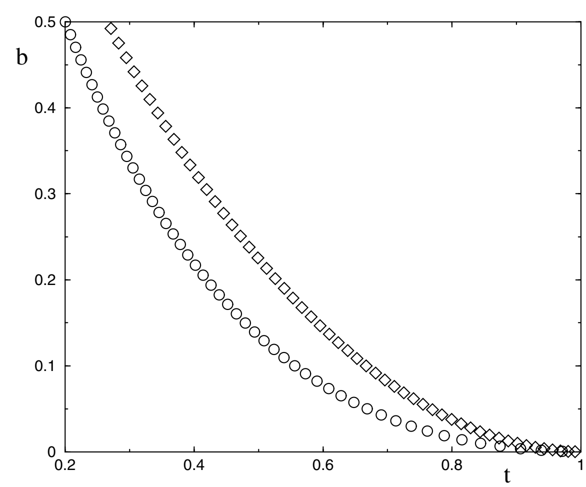

FIG. 4. Results of the numerical integration of Eq. (3), in the absence of pinning. The circles refer to melting in the "short" direction, and the diamonds to melting in the "long" direction. We have taken the following parameters: $\gamma^{-4} = (m_c/m_{ab}) = 10$, $\kappa = (\lambda_{ab}/\xi_{ab}) = 100$, and $H_{c2}^{ab}(T=0) = 100T$. The figure is plotted for $c = 0.2$, as a function of $b = [B/H_{c2}(0)]$, and $t = (T/T_c)$.

the vortex smectic, and the system exhibits gradient elasticity. It follows that a 3D smectic with explicitly broken rotational symmetry can have translational LRO. An interesting consequence of this is that the rigidity of the vortex smectic preserves superconductivity between the liquidlike layers, so that the transition from vortex lattice to vortex smectic is also a transition from 3D superconductivity to 2D superconductivity.

Although we have presented results for a (homogeneous) uniaxial superconductor, we also expect the results to apply for a biaxial superconductor, with three different entries in the mass tensor. The cuprates certainly exhibit anisotropy between the c direction and the planar directions, but they often also exhibit anisotropy within the ab plane. In particular, our assumptions of mass anisotropy with no explicit translational symmetry breaking in the electronic degrees of freedom [18] are in principle satisfied for the geometry $\mathbf{B}||\hat{c}$ in the cuprates in the presence of an electron nematic phase [19] within the planes. Our assumptions may also be satisfied in stripe ordered phases, provided the mutual pinning is not too strong and thermal depinning of vortices from stripes occurs at a lower temperature than that at which the vortex lattice melts.

The smectic-A has clearest implications for experimental probes that are capable of measuring the structure factor of the vortex order, such as Bitter decoration (which is surface sensitive) or neutron scattering (which is a bulk probe). There are also distinctive implications of the double melting for μSR. Muon spin rotation detects the distribution of local magnetic fields. In the nematic phase, the time-averaged magnetic field density is uniform. In partially freezing from the nematic to the smectic, μSR would exhibit a new inhomogeneity in the magnetic field in the smectic state. Upon freezing further into the vortex lattice, the μSR signal would reveal another transition to further magnetic inhomogeneity. The changes in the μSR signal are expected to coincide with the onset of highly anisotropic resistivity in going to the smectic, and with the onset of 3D superconductivity upon entering the lattice phase. Quenched disorder [20], neglected in our treatment, may produce related effects, such as a two-stage peak effect, a disorder-induced smectic, or glassiness.

Resistivity measurements are sensitive to smectic order as well. When the Lorentz force is along a liquidlike direction, vortices move easily and the resistivity is large. When the Lorentz force is along the solidlike direction, the rigidity of the smectic resists vortex motion, and the resistivity vanishes (although the movement of defects may provide some small amount of dissipation) [6]. If the current is along the magnetic field direction, the Lorentz force and dissipation are negligible. The vortex smectic-A retains 2D superconductivity between the liquidlike layers of vortices, with the resistivity $\rho_{||}$ vanishing parallel to the layers, but $\rho_{\perp} \neq 0$ for currents applied perpendicular to the smectic layers.

In conclusion, we have studied the problem of vortex lattice melting in anisotropic superconductors in the continuum limit. The introduction of anistropy in the mass tensor leads to elongation of vortex cross sections and interactions. We have demonstrated that interacting elongated vortices can form liquid crystalline phases. Using elasticity theory, with momentum-dependent elastic constants derived from Ginzburg-Landau theory, we have calculated the thermal fluctuations of the vortex lattice. Comparing these results to an anisotropic Lindemann criterion, we argue that there is an instability to an intermediate smectic phase. In the absence of intrinsic pinning, we find an instability favoring a smectic-A, wherein the lattice has melted along the direction of the shorter lattice constant.

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