CH.4) SUSYGUTS

4.0. Introduction. Spontaneous Symmetry Breaking

Finally, before we begin our study of the SU(5) model, let's discuss the spontaneous breaking of SUSY and gauge invariance. In general since

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu \tag{4.0.1}$$

we have for $\alpha = 1 = \dot{\alpha}$

$$Q_1\bar{Q}_1 + \bar{Q}_1 Q_1 = 2(P_0+P_3) \tag{4.0.2}$$

and $\alpha = 2 = \dot{\alpha}$

$$Q_2\bar{Q}_2 + \bar{Q}_2Q_2 = 2(P_0-P_3) \ . \tag{4.0.3}$$

The Hamiltonian is given by

$$H = P_0 = \frac{1}{4}[\bar{Q}_1Q_1+Q_1\bar{Q}_1+\bar{Q}_2Q_2+Q_2\bar{Q}_2] \ . \tag{4.0.4}$$

Thus H is the sum of squares and is non-negative, i.e. $\langle\psi|H|\psi\rangle \geq 0$ for all states $|\psi\rangle$. Further the supersymmetric vacuum, being the lowest energy state, has zero energy $\langle 0|H|0\rangle = 0$, it is supersymmetric because $\langle 0|H|0\rangle = 0$ implies $Q|0\rangle = \bar{Q}|0\rangle = 0$. Vacuum states with positive energy then must break SUSY spontaneously.

We saw in the W-Z model that the potential of the theory $V \sim F^\dagger F$. This is true in any chiral model with fields ϕ_i

$$\Gamma = Z_i\int dV\phi_i\bar{\phi}_i + 4m_{ij}\int dS\phi_i\phi_j + 4m_{ij}\int d\bar{S}\bar{\phi}_i\bar{\phi}_j + g_{ijk}\int dS\phi_i\phi_j\phi_k$$
$$+ g^*_{ijk}\int d\bar{S}\bar{\phi}_i\bar{\phi}_j\bar{\phi}_k + f_i\int dS\bar{\phi}_i + f^*_i\int d\bar{S}\bar{\phi}_i \ . \tag{4.0.5}$$

Then

$$V = 16Z_iF^\dagger_iF_i \tag{4.0.6}$$

where

$$16Z_iF^\dagger_i = 4f_i + 32m_{ij}A_j + 12g_{ijk}A_jA_k \ . \tag{4.0.7}$$

[no summation over i] .

Thus the supersymmetric vacuum corresponds to the case where $\langle F_i\rangle = 0 = \langle F^\dagger_i\rangle$. The vacuum values of $\langle A_i\rangle \equiv a_i$ must satisfy the quadratic equation

$$0 = f_i + 8m_{ij}a_j + 3g_{ijk}a_ja_k \tag{4.0.8}$$

given the parameters m, g, f for a supersymmetric ground state. If we can arrange the parameters f, m, g such that no solution exists then SUSY is spontaneously broken. The action can be rewritten in terms of shifted fields $A_i \to A_i + a_i$; $F_i \to F_i + v_i$

$$\Gamma \to \Gamma - \int d^4 x \, [2g_{ijk}v_iA_jA_k + g_{ijk}a_i\psi_j\psi_k - 4g_{ijk}a_iA_jF_k + \text{h.c.}] \qquad (4.0.9)$$

and we see that the scalar and pseudoscalar fields have a mass shifted from that of the fermions.

The O'Raifeartaigh model is the simplest such example of no solution existing. It consists of 3 fields ϕ_0, ϕ_1, ϕ_2 and $f_0 = f$, $m_{12} = m_{21} = m$, $g_{011} = g_{101} = g_{110} = g$, all other parameters = 0.

We can also consider the potential in gauge models to find solutions that (1) break SUSY but not the gauge inv. (2) break both gauge inv. and SUSY (3) break gauge inv. but not SUSY.

Let's turn to our U(1) case to be specific, where we now study these cases. We allow a term linear in V in the action (since the gauge group is abelian this is both guage and SUSY invariant; but parity non-invariant since V is a pseudo-scalar). The potential can always be written as

$$V = +64Z(\tfrac{1}{2} D^2) + 16Z_+F_+F_+^\dagger + 16Z_-F_-F_-^\dagger \qquad (4.0.10)$$

where

$$64ZD = 4g[Z_+A_+A_+^\dagger - Z_-A_-A_-^\dagger] + d \qquad (4.0.11)$$

and

$$Z_+F_+^\dagger = mA_-$$
$$Z_-F_-^\dagger = mA_+ \qquad (4.0.12)$$

with

$$\Gamma = \Gamma^{SQED} + \frac{d}{16} \int dVV . \tag{4.0.13}$$

Now SUSY remains unbroken only if $V = 0 \rightarrow \langle D \rangle = 0 = \langle F_\pm \rangle$ thus $Z_+ |\langle A_+ \rangle|^2 - Z_- \ |\langle A_- \rangle|^2 + d = 0$ from $\langle D \rangle = 0$, but $\langle F_\pm \rangle = 0 \rightarrow \langle A_\pm \rangle = 0$, thus as long as $d \neq 0$ SUSY is spontaneously broken.

We can rewrite the potential in terms of the scalar fields to find

$$V = \frac{d^2}{128Z} + \frac{16}{Z_-} [m^2 + \frac{Z_+ Z_-}{256Z} gd] A_+ A_+^\dagger + \frac{16}{Z_+} [m^2 - \frac{Z_+ Z_-}{256Z} gd] A_- A_-^\dagger$$

$$+ \frac{g^2}{8Z} [Z_+ A_+ A_+^\dagger - Z_- A_- A_-^\dagger]^2 \tag{4.0.14}$$

Hence we have two cases

$$m^2 > \frac{Z_+ Z_-}{256Z} gd \quad \text{or} \quad m^2 < \frac{Z_+ Z_-}{256Z} gd .$$

First is

$$m^2 > \frac{Z_+ Z_-}{256Z} gd .$$

Thus $\langle A_\pm \rangle = 0$ minimizes the potential since it is a sum of squares and $\langle D \rangle = d/64Z$.

Then $A_\pm$ have real masses with equal but opposite splitting while the $\psi_\pm$ remain at m^2. In addition the gauge boson V_μ and the gaugino λ remain massless, however for different reasons, the V_μ still mediates the unbroken U(1) gauge interaction, so it is massless; while the λ becomes the Goldstone spinor associated with the spontaneous breakdown of SUSY. Recall the SUSY transformation of λ:

$$Q\lambda_\alpha = i\xi_\alpha D + \ldots$$

Thus

$$\langle 0|Q\lambda|0\rangle = i\xi \langle 0|D|0\rangle + \ldots$$

$$= i\xi \frac{d}{64Z}$$

So $Q|0\rangle \neq 0$ and λ is a Goldstone fermion since it transforms inhomogeneously when $\langle D\rangle \neq 0$. Once again whenever the auxiliary fields, in this case D, develop a vacuum value SUSY is broken.

Next we can consider the minimum of the potential when $m^2 < \frac{Z_+Z_-}{256Z} gd$. Then we find

$$\frac{\partial V}{\partial A_+^\dagger} = \frac{16}{Z_-}\left[m^2 + \frac{Z_+Z_-}{256Z} gd\right] A_+ + \frac{g^2}{4Z}\left[Z_+A_+A_+^\dagger - Z_-A_-A_-^\dagger\right] Z_+A_+ \qquad (4.0.15)$$

$$= 0$$

and

$$\frac{\partial V}{\partial A_-^\dagger} = \frac{16}{Z_+}\left[m^2 - \frac{Z_+Z_-}{256Z} gd\right] A_- - \frac{g^2}{4Z}\left[Z_+A_+A_+^\dagger - Z_-A_-A_-^\dagger\right] Z_-A_- \qquad (4.0.16)$$

$$= 0 .$$

Hence $\langle A_+\rangle = 0$; $\langle A_-\rangle = v$ with $(m^2 + \frac{Z_+Z_-}{256Z} gd) + \frac{Z_+Z_-}{64Z} g^2 Z_- v^2 = 0$ yielding the minimum. We can now expand the action about this minimum; $A_+ \to A_+$; $A_- \to A_- + v$, and $F_+ \to F_+$; $F_- \to F_- + \frac{mv}{Z_+}$, $D \to D + \frac{1}{64Z}(d - 4Z_-v^2 g)$ to find the mass terms

$$\Gamma_{mass} = \int d^4x \{4Z_- v^2 g^2 V_\mu V^\mu - 4gZ_- v(\psi_-\lambda - \bar{\psi}_-\bar{\lambda}) + 8m(\psi_+\psi_- + \bar{\psi}_+\bar{\psi}_-)\}$$
$$+ \frac{4g}{64Z}(d - 4Z_- v^2 g)[Z_+A_+A_+^\dagger - Z_-A_-A_-^\dagger] - 4Z_- gv(A_- + A_-^\dagger)D\} . \qquad (4.0.17)$$

Hence we see that V_μ becomes massive! The U(1) gauge symmetry is broken.

In addition $V > 0$ so SUSY is broken and the massless Goldstone fermion corresponds to a linear combination of ψ_- and λ while the orthogonal combination of ψ_- and λ is massive along with ψ_+. One scalar field will have mass degenerate with V_μ while one complex scalar will have mass $\sqrt{2m^2}$. Hence we have the SUSY broken when an auxiliary field gets a vacuum value and the gauge symmetry broken when a non-singlet A-field gets a vacuum value.

Finally we would like to consider a model in which SUSY remains unbroken while the gauge symmetry is broken. We can accomplish this by adding to our broken super QED model an additional neutral chiral field ϕ_o. The U(1) invariant action is given by

$$\begin{aligned}\Gamma \;=\; & -2Z\int dVVD\bar{D}\bar{D}DV + \int dV[Z_+\bar{\phi}_+e^{+gV}\phi_+ + Z_-\bar{\phi}_-e^{-gV}\phi_- + Z_o\phi_o\bar{\phi}_o] \\ & + \int dS[4m\phi_+\phi_- + 4\mu\phi_o^2] + \int d\bar{S}[4m\bar{\phi}_+\bar{\phi}_- + 4\mu\bar{\phi}_o^2] \\ & + \int dS[g\phi_o\phi_+\phi_- + \lambda\phi_o^3] + \int d\bar{S}[g^*\bar{\phi}_o\bar{\phi}_+\bar{\phi}_- + \lambda^*\bar{\phi}_o^3] \\ & + \frac{d}{16}\int dVV + f_o\int dS\phi_o + f_o^*\int d\bar{S}\bar{\phi}_o \;.\end{aligned} \tag{4.0.18}$$

The Euler-Lagrange field equations are found by functionally differentiating the action wrt each field

1) $$\frac{\delta\Gamma}{\delta V(x,\theta,\bar{\theta})} = 0 \rightarrow +4ZD\bar{D}\bar{D}DV = +g[Z_+\bar{\phi}_+\phi_+e^{gV} - Z_-\bar{\phi}_-\phi_-e^{-gV}] + \frac{d}{16}$$

2) $$\frac{\delta\Gamma}{\delta\phi_o} = 0 \rightarrow -Z_o\bar{D}\bar{D}\bar{\phi}_o = 8\mu\phi_o + g\phi_+\phi_- + 3\lambda\phi_o^2 + f_o$$

2') $$\frac{\delta\Gamma}{\delta\bar{\phi}_o} = 0 \rightarrow -Z_oDD\phi_o = 8\mu\bar{\phi}_o + g^*\bar{\phi}_+\bar{\phi}_- + 3\lambda^*\bar{\phi}_o^2 + f_o^*$$

$$3)\quad \frac{\delta\Gamma}{\delta\phi_+} = 0 \rightarrow -Z_+\bar{D}\bar{D}(e^{gV}\bar{\phi}_+) = 4m\phi_- + g\phi_o\phi_-$$

$$3')\quad \frac{\delta\Gamma}{\delta\bar{\phi}_+} = 0 \rightarrow -Z_+DD(e^{gV}\phi_+) = 4m\bar{\phi}_- + g^*\bar{\phi}_o\bar{\phi}_-$$

$$4)\quad \frac{\delta\Gamma}{\delta\phi_-} = 0 \rightarrow -Z_-\bar{D}\bar{D}(e^{-gV}\bar{\phi}_-) = 4m\phi_+ + g\phi_o\phi_+ \tag{4.0.19}$$

$$4')\quad \frac{\delta\Gamma}{\delta\bar{\phi}_-} = 0 \rightarrow -Z_-DD(e^{-gV}\phi_-) = 4m\bar{\phi}_+ + g^*\bar{\phi}_o\bar{\phi}_+ \ .$$

The $(\theta,\bar{\theta})$ independent components of the LHS are just the D or F fields while the RHS are just the θ independent A fields. If SUSY is to remain unbroken the $\langle F\rangle$ and $\langle D\rangle$ vacuum values must be zero; this yields the 4 equations with $\langle\phi_\pm\rangle = a_\pm$, $\langle\phi_o\rangle = a_o$

$$1)\quad g[Z_+a_+^*a_+ - Z_-a_-^*a_-] + \frac{d}{16} = 0$$

$$2)\quad 8\mu a_o + ga_+a_- + 3\lambda a_o^2 + f_o = 0 \tag{4.0.20}$$

$$3)\quad 4ma_- + ga_oa_- = 0$$

$$4)\quad 4ma_+ + ga_oa_+ = 0 \ .$$

The two solutions are

$$1)\ \text{if}\ \ d = 0 \qquad a_+ = a_- = 0$$

$$3\lambda a_o^2 + 8\mu a_o + f_o = 0 \tag{4.0.21}$$

since ϕ_o is a U(1) singlet this solution breaks neither SUSY nor gauge invariance,

2) if $d \neq 0$ $\quad a_o = -\frac{4m}{g}$

$$a_+a_- = -\frac{1}{g}\left[f_o - \frac{32m\mu}{g} + \frac{48m^2\lambda}{g^2}\right] \quad . \tag{4.0.22}$$

This satisfies 2-4 but equation 1 can be transformed away since 2-4 is really invariant under the complex extension of U(1)

$a_+ \to e^{\alpha}a_+ = a_+'$, $\quad$ where α is complex, that

$a_- \to e^{-\alpha}a_- = a_-'$ $\quad$ is the potential that determines the vacuum has a higher symmetry than U(1).

So if a_+a_- satisfies 2 so does $a_+'a_-'$, (1) then becomes

$$g[Z_+a_+'^*a_+'e^{-(\alpha+\alpha^*)} - Z_-a_-'^*a_-' e^{+(\alpha+\alpha^*)}] + \frac{d}{16} = 0 \ .$$

Thus $\alpha + \alpha^*$ can be adjusted so that this equation is always satisfied. Hence this D_o term can be transformed away and does not catalyze a SUSY breakdown.

We can expand our action about these minima to find the new mass terms, letting $\phi_\pm \to a_\pm + \phi_\pm$; $\phi_o \to a_o + \phi_o$ we find

$$\begin{aligned}\Gamma = & -2Z\int dVVD\bar{D}\bar{D}DV + \int dV[Z_+\bar{\phi}_+\phi_+e^{gV}+Z_-\bar{\phi}_-\phi_-e^{-gV}+Z_o\phi_o\bar{\phi}_o] \\ & + \int dV[Z_+\bar{\phi}_+(e^{gV}-1)a_++Z_+a_+^*(e^{gV}-1)\phi_++Z_-\bar{\phi}_-(e^{-gV}-1)a_-+Z_-a_-^*(e^{-gV}-1)\phi_- \\ & + Z_+a_+^*a_+(e^{gV}-1-gV)+Z_-a_-^*a_-(e^{-gV}-1+gV)] + \{\int dS[(\overset{0}{\cancel{4m}}+ga_o)\ \phi_+\phi_- \\ & + (4\mu+3\lambda a_o)\phi_o^2+ga_+\phi_o\phi_-+ga_-\phi_o\phi_+] + \int dS[g\phi_o\phi_+\phi_-+\lambda\phi_o^3] + h.c.\} \ .\end{aligned} \tag{4.0.23}$$

So we have mass for V:

$$\frac{g^2}{2}\,(Z_+ a_+^* a_+ + Z_- a_-^* a_-)V^2 \qquad (4.0.24)$$

which cannot be transformed away. The $\phi_{o,+,-}$ mass matrix has a zero eigenvalue corresponding to the Goldstone boson superfield. The scalar field is eaten to give V_μ a mass, while the pseudoscalar and fermion become the new massive degrees of freedom in V. This is the super Higgs mechanism.

For non-abelian gauge groups the situation is similar. The linear chiral terms must be made to vanish, i.e. $\langle F_i \rangle = 0$. When the gauge group does not contain an invariant U(1) the linear V term must also vanish, i.e., $\langle D_i \rangle = 0$; that is

$$a_i^* T_{ij}^a a_j = 0 \,. \qquad (4.0.25)$$

4.1. Super Georgi-Glashow SU(5) Model

We are now ready to turn to the supersymmetric version of the Georgi-Glashow SU(5) model. Recall the left handed matter fields were in the 5^* and 10 representations of SU(5). Hence the SUSY extension of this is to put the matter fields in 5^* and 10 chiral superfields denoted $M_{\bar{5}}, M_{10}$. The bosonic SUSY partners to the quarks and leptons will be called squarks and sleptons (s for SUSY not scalar) or smatter fields. As before we will break the SU(5) down to SU(3) × SU(2) × U(1) by means of an adjoint of Higgs fields--again these will be a chiral superfield denoted ϕ_{24} with the fermi super partners to the bosonic Higgs fields called shiggs fields. Since we have only trilinear pure chiral or pure anti-chiral monomials in the action we now need both a 5 and a $\bar{5}$ of Higgs chiral superfields in order to make the necessary matter field Yukawa terms. We denote these H_5 and $H'_{\bar{5}}$.

Of course we also have the super Yang-Mills field V in the adjoint rep. of SU(5).

The most general SU(5) and SUSY invariant action made from these fields is given by

$$\Gamma^{SSU(5)} = \Gamma_{ym} + \Gamma_K + \Gamma_V \tag{4.1.1}$$

with

$$\Gamma_{ym} = \frac{Z}{g^2} \int dS \; \mathrm{Tr}[W^{\alpha} W_{\alpha}] + \frac{Z}{g^2} \int d\bar{S} \; \mathrm{Tr}[\bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}] \tag{4.1.2}$$

where

$$\begin{aligned} W_{\alpha} &= \bar{D}\bar{D}[e^{-gV^b_a \mathcal{T}^a_b} D_{\alpha} e^{+gV^b_a \mathcal{T}^a_b}] \\ \bar{W}_{\dot{\alpha}} &= DD[e^{+gV^b_a \mathcal{T}^a_b} \bar{D}_{\dot{\alpha}} e^{-gV^b_a \mathcal{T}^a_b}] \end{aligned} \tag{4.1.3}$$

and $\mathcal{T}^a_b$ is the adjoint representation matrix for SU(5)

$$(\mathcal{T}^a_b)^{ce}_{df} = (T^a_b)^c_f \delta^e_d - (T^a_b)^e_d \delta^c_f \tag{4.1.4}$$

with

$$(T^a_b)^c_d = \delta^a_d \delta^c_b - \frac{1}{5} \delta^a_b \delta^c_d \tag{4.1.5}$$

the fundamental representation matrix. The kinetic energy action is

$$\begin{aligned} \Gamma_K = \int dV\{ & Z_{M5} M_{\bar{5}} e^{-gV^b_a T^a_b} \bar{M}_5 + Z_{M10} \bar{M}_{\overline{10}} e^{gV^b_a L^a_b} M_{10} \\ & + Z_{\phi} \bar{\phi}_{24} e^{gV^b_a \mathcal{T}^a_b} \phi_{24} + Z_H \bar{H}_{\bar{5}} e^{gV^b_a T^a_b} H_5 + Z_{H'} H'_{\bar{5}} e^{-gV^b_a T^a_b} \bar{H}'_5 \} \end{aligned} \tag{4.1.6}$$

with the 10 dimensional representation matrix L^a_b given by

$$(L^a_b)^{cd}_{ef} = (T^a_b)^c_e \delta^d_f + (T^a_b)^d_f \delta^c_e \ . \tag{4.1.7}$$

Let's recall the explicit SU(5) gauge transformations to check that this is indeed invariant. For the $\bar{5}$ matter multiplet we have

$$M'_{\bar{5}c} = (e^{-ig\Lambda^b_a T^a_b})^e_c \, M_{\bar{5}e} \tag{4.1.8}$$

and

$$\bar{M}'^d_5 = (e^{+ig\bar{\Lambda}^b_a T^a_b})^d_f \, \bar{M}^f_5 \ . \tag{4.1.9}$$

The vector superfield V^b_a transforms according to

$$(e^{-gV^b_a T^a_b})^e_f = (e^{-ig\Lambda^b_a T^a_b})^e_c (e^{-gV'^b_a T^a_b})^c_d (e^{+ig\bar{\Lambda}^b_a T^a_b})^d_f \tag{4.1.10}$$

so that

$$M'_{\bar{5}c} (e^{-gV'^b_a T^a_b})^c_d \, \bar{M}'^d_5 = M_{\bar{5}} \, e^{-gV^b_a T^a_b} \, \bar{M}_5 \tag{4.1.11}$$

and it is gauge invariant.

(Recall $\bar{\Lambda}^a_b = \Lambda^{*b}_a$, $\bar{\Lambda} = \Lambda^{\dagger}$.) The remaining terms can be checked similarly, e.g.

$$\begin{aligned} \phi'_{24} &= e^{+ig\underline{\underline{\Lambda}}\cdot\underline{\underline{T}}} \phi_{24} \\ \bar{\phi}'_{24} &= e^{+ig\underline{\underline{\bar{\Lambda}}}\cdot\underline{\underline{T}}} \bar{\phi}_{24} \\ e^{g\underline{\underline{V}}\cdot\underline{\underline{T}}} &= e^{-ig\underline{\underline{\bar{\Lambda}}}\cdot\underline{\underline{T}}} e^{g\underline{\underline{V'}}\cdot\underline{\underline{T}}} e^{+ig\underline{\underline{\Lambda}}\cdot\underline{\underline{T}}} \quad \text{etc.} \end{aligned} \tag{4.1.12}$$

Finally we have the pure chiral and anti-chiral action

$$\Gamma_V = \int dSL + \int d\bar{S}\bar{L} \quad \text{with} \quad \bar{L} = L^{\dagger} . \tag{4.1.13}$$

In order to list all the SU(5) invariants made from bilinear and trilinear products of $M_{\bar{5}}$, M_{10}, ϕ_{24}, H_5, $H'_{\bar{5}}$ recall the decomposition of the relevant SU(5) tensor products

$$\begin{aligned}
5 \times \bar{5} &= 1 + 24 \\
5 \times 5 &= 10 + 15 \\
\bar{5} \times \bar{5} &= \overline{10} + \overline{15} \\
\bar{5} \times 10 &= 5 + 45 \\
\bar{5} \times 24 &= \bar{5} + \overline{45} + \overline{70} \\
5 \times 10 &= \overline{10} + \overline{40} \\
5 \times 24 &= 5 + 45 + 70 \\
10 \times 24 &= 10 + 15 + 40 + 175 \\
24 \times 24 &= 1 + 24 + 24 + 75 + 126 + \overline{126} + 200 \\
10 \times 10 &= \bar{5} + \overline{45} + \overline{50} \quad .
\end{aligned} \tag{4.1.14}$$

Thus we must list invariants; first products of two fields; these are just $5 \times \bar{5}$ and 24×24 since only they contain singlets.

$$\begin{aligned}
&M_{\bar{5}}H_5 \\
&H'_{\bar{5}}H_5 \\
&\phi_{24}\phi_{24}
\end{aligned} \tag{4.1.15}$$

Then we have products of three fields

$$\begin{array}{llll} \overline{5} & 5 & 24 & : M_{\overline{5}}H_5\phi_{24}\ ,\ H'_{\overline{5}}H_5\phi_{24} \\ \overline{5} & \overline{5} & 10 & : M_{\overline{5}}M_{\overline{5}}M_{10}\ ,\ M_{\overline{5}}H'_{\overline{5}}M_{10}\ ,\ H'_{\overline{5}}H'_{\overline{5}}M_{10} \\ 5 & 10 & 10 & : M_{10}M_{10}H_5 \\ 24^3 & & & : \phi^3_{24}\ . \end{array} \tag{4.1.16}$$

In order to eliminate terms which mix H_5 and $M_{\overline{5}}$ we also choose Γ to be invariant under the discrete symmetry $M_{\overline{5}} \to -M_{\overline{5}}$; $M_{10} \to -M_{10}$ all other fields being invariant. Thus we find only

$$\begin{array}{l} H'_{\overline{5}}H_5, \phi_{24}\phi_{24} \\ H'_{\overline{5}}\phi_{24}H_5, M_{\overline{5}}M_{10}H'_{\overline{5}}, M_{10}M_{10}H_5, \phi^3_{24}, \end{array} \tag{4.1.17}$$

so

$$L = \frac{m}{2}\phi_{24}{}^a_b\,\phi_{24}{}^b_a + \mu H'_{\overline{5}a}H^a_5 + \lambda_\phi \phi_{24}{}^a_b\,\phi_{24}{}^b_c\,\phi_{24}{}^c_a$$
$$+ \lambda_{\phi H} H'_{\overline{5}a}\phi_{24}{}^a_b\,H^b_5 + \gamma_{mn} M_{\overline{5}ma} M_{10n}{}^{ab} H'_{\overline{5}b} + \Gamma_{mn}\varepsilon_{abcde} M_{10m}{}^{ab} M_{10n}{}^{cd} H^e_5\ . \tag{4.1.18}$$

4.2. Spontaneous Symmetry Breaking of SSU(5)

We now desire a breaking scheme of the sort

$$\text{Super SU(5)} \xrightarrow[M_x \simeq 10^{16}\ \text{GeV}]{\phi_{24}} \text{Super SU(3)} \times \text{SU(2)} \times \text{U(1)}$$

$$\xrightarrow[M_s \simeq 1\ \text{TeV}]{\substack{\text{Explicit but soft} \\ \text{SUSY breaking}}} \text{SU(3)} \times \text{SU(2)} \times \text{U(1)} \xrightarrow[M_W \simeq 100\ \text{GeV}]{H_5, H'_{\overline{5}}} \tag{4.2.1}$$

$$\text{SU(3)} \times U_{em}(1)\ .$$

Since the breaking of SUSY will be soft; that is we will give explicit mass terms to the supersymmetric partners of the matter and gauge fields of the order of $M_s \approx 1$ TeV. This does not violate the higher energy no renormalization theorem of SUSY since the breaking is soft. In addition in order to catalyze the electroweak breaking a non-positive definite mass squared term for the Higgs fields is also added. All of these terms are SU(5) invariant. shiggs masses are not added, in order to keep the SUSY softly broken (sguage fields are acceptable!). Their mass is obtained from the supersymmetric breaking of SU(5) by a natural fine tuning. So we should also add to Γ an explicit SUSY breaking piece Γ^b.

$$\begin{aligned}\Gamma^b = {} & \frac{a}{2} M_s^2 \int dV \theta^2 \bar{\theta}^2 [(\bar{D}^2 D^\alpha V^a_b)(\bar{D}^2 D_\alpha V^b_a) + (D^2 \bar{D}_{\dot{\alpha}} V^a_b)(D^2 \bar{D}^{\dot{\alpha}} V^b_a)] \\ & + b M_s^2 \int dV \theta^2 \bar{\theta}^2 M_{\bar{5}} \bar{M}_5 + c M_s^2 \int dV \theta^2 \bar{\theta}^2 M_{10} \bar{M}_{\overline{10}} \\ & + d M_s^2 \int dV \theta^2 \bar{\theta}^2 H_5 M_H \bar{H}_{\bar{5}} + e M_s^2 \int dV \theta^2 \bar{\theta}^2 \bar{H}'_5 M_{H'} H'_{\bar{5}} \; .\end{aligned} \tag{4.2.2}$$

The first term is just a $\lambda^2 + \bar{\lambda}^2$ mass term while the rest are $AA^\dagger$ masses.

Let's first rewrite Γ in terms of SU(3) × SU(2) × U(1) fields and shift by the large ϕ_{24} vacuum value to check that SUSY is unbroken, but that SU(5) is broken down to SU(3) × SU(2) × U(1).

Recall the SU(3) × SU(2) × U(1) decomposition of the SU(5) 24, 10, 5, $\bar{5}$ from our previous work.

$$\phi_{24b}^{\;\;a} = \begin{bmatrix} \left(H_1^1 - \frac{2}{\sqrt{30}} H_B\right) & H_2^1 & H_3^1 & H_{\bar{x}}^1 & H_{\bar{y}}^1 \\ H_1^2 & \left(H_2^2 - \frac{2}{\sqrt{30}} H_B\right) & H_3^2 & H_{\bar{x}}^2 & H_{\bar{y}}^2 \\ H_1^3 & H_2^3 & \left(H_3^3 - \frac{2}{\sqrt{30}} H_B\right) & H_{\bar{x}}^3 & H_{\bar{y}}^3 \\ H_{x_1} & H_{x_2} & H_{x_3} & \left(\frac{H^0}{\sqrt{2}} + \frac{3H_B}{\sqrt{30}}\right) & H^+ \\ H_{y_1} & H_{y_2} & H_{y_3} & H^- & \left(-\frac{H^0}{\sqrt{2}} + \frac{3H_B}{\sqrt{30}}\right) \end{bmatrix}^{ab} \tag{4.2.3}$$

where H_B is a (1,1,0) and H^0 is the neutral field in a (1,3,0), and these are chiral superfields. So once again we give ϕ_{24} the vacuum values

$$<0|\phi_{24}|0> = \begin{bmatrix} v & & & & \\ & v & & \mathcal{O} & \\ & & v & & \\ & & & \left(-\frac{3}{2} - \frac{1}{2}\varepsilon\right)v & \\ & \mathcal{O} & & & \left(-\frac{3}{2} + \frac{1}{2}\varepsilon\right)v \end{bmatrix} \tag{4.2.4}$$

i.e. $$<0|H_B|0> = -\frac{\sqrt{30}}{2} v$$

$$<0|H^0|0> = -\frac{1}{\sqrt{2}} \varepsilon v \; ; \quad \varepsilon <<< 1 \; . \tag{4,2,5}$$

The 5 of Higgs can be written as

$$H_5^a = \begin{bmatrix} H^1 \\ H^2 \\ H^3 \\ \phi^+ \\ \phi^o \end{bmatrix} \tag{4.2,6}$$

with H^i and ϕ_o^+ chiral superfields and $H^{1,2,3}$ being a $(3,1,-\frac{1}{3})$ and $\begin{pmatrix}\phi^+\\ \phi^o\end{pmatrix}$ a $(1,2,+\frac{1}{2})$. Hence H_5 will have the vacuum value

$$<0|H_5|0> \;=\; \begin{pmatrix}0\\0\\0\\0\\ \frac{v_o}{\sqrt{2}}\end{pmatrix} \qquad \text{i.e.} \quad <0|\phi_o|0> \;=\; \frac{1}{\sqrt{2}}\,v_o \qquad v_o \;\approx\; M_W \quad . \tag{4.2.7}$$

Similarly the $\bar{5}$ of Higgs can be written as

$$H'_{\bar{5}a} \;=\; \begin{pmatrix}H'_1\\H'_2\\H'_3\\ \phi^-\\ \phi'^{\,0}\end{pmatrix} \tag{4.2.8}$$

and again $H'_{1,2,3}$ is a $(\bar{3},1,+\frac{1}{3})$ and $\begin{pmatrix}\phi^-\\ \phi^o\end{pmatrix}$ a $(1,\bar{2},-\frac{1}{2})$. So $H'_{\bar{5}}$ will have the vacuum value

$$<0|H'_{\bar{5}}|0> \;=\; \begin{pmatrix}0\\0\\0\\0\\ \frac{v'_o}{\sqrt{2}}\end{pmatrix} \qquad \text{i.e.} \quad <0|\phi'_o|0> \;=\; \frac{1}{\sqrt{2}}\,v'_o \qquad v'_o \;\approx\; M_W \quad . \tag{4.2.9}$$

For energies large compared to M_s we can neglect the SUSY and electroweak breaking terms and ask if v can be determined so as to keep SUSY good; that is the linear F and D terms should vanish in the shifted action.

For the F terms we need only ask if the linear terms in L vanish. For the D term the linear term in L_K must vanish.

First the F terms, we must have

1) $$\alpha\delta^a_b + \left.\frac{\partial L}{\partial \phi_{24a}^{\ \ b}}\right|_{\substack{<\phi_{24}>=v \\ \text{all other } <>=0}} = 0 = m\phi_{24b}^{\ \ a} + 3\lambda_\phi \phi_{24c}^{\ \ a}\phi_{24b}^{\ \ c} + \lambda_{\phi H} H'_{\bar{5}b} H_5^a + \alpha\delta^a_b$$

2) $$\phi_{24a}^{\ \ a} = 0$$

3) $$\left.\frac{\partial L}{\partial H_5^a}\right|_{\substack{<\phi_{24}>=v \\ \text{all other } <>=0}} = 0 = \mu H'_{\bar{5}a} + \lambda_{\phi H} H'_{\bar{5}b}\phi_{24a}^{\ \ b} + \Gamma_{mn}\varepsilon_{abcde} M_{10m}^{\ \ bc} M_{10n}^{\ \ de} \qquad (4.2.10)$$

4) $$\left.\frac{\partial L}{\partial H'_{\bar{5}a}}\right|_{\substack{<\phi_{24}>=v \\ \text{all other } <>=0}} = 0 = \mu H_5^a + \lambda_{\phi H}\phi_{24b}^{\ \ a} H_5^b + \gamma_{mn} M_{\bar{5}mb} M_{10n}^{\ \ ba}$$

Since $M_{\bar{5}}$ and M_{10} are chosen to have zero vacuum values their linear terms are trivially zero since they appear bilinearly in Γ. Also for high energy ($> M_s$) we are setting $v_o = v'_o = \varepsilon = 0$. So $<H_5> = <H'_{\bar{5}}> = 0$, hence only equation 1 is non-trivial;

$$0 = m<\phi_{24b}^{\ \ a}> + 3\lambda(<\phi_{24}><\phi_{24}>)^a_b + \alpha\delta^a_b \qquad (4.2.11)$$

where the Lagrange multiplier α can be eliminated by taking the trace $\rightarrow$

$$\alpha = \frac{-3}{5}\lambda_\phi \mathrm{Tr}\phi_{24}\phi_{24} \ . \qquad (4.2.12)$$

So letting $\langle\phi_{24b}^{\ a}\rangle \equiv v_b^a$ we have

$$0 = m v_b^a + 3\lambda_\phi [v_c^a v_b^c - \frac{1}{5}\,\delta_b^a\, v_d^c v_c^d] \quad . \tag{4.2.13}$$

For

$$v_b^a = \begin{bmatrix} v & & & & \\ & v & & 0 & \\ & & v & & \\ & 0 & & -\frac{3}{2}v & \\ & & & & -\frac{3}{2}v \end{bmatrix} \tag{4.2.14}$$

this reduces to one equation

$$mv + 3\lambda_\phi [v^2 - \frac{1}{5}(\frac{15}{2}\, v^2)] = 0 \tag{4.2.15}$$

with the simple solution

$$v = \frac{2}{3}\frac{m}{\lambda_\phi} \; . \tag{4.2.16}$$

The linear D term is given by

$$\bar{\phi}_{24}\, \underline{\underline{V}}\cdot\underline{\underline{T}}\, \phi_{24} = v_a^b v_c^d (T_b^a)_{df}^{ce} v_e^f = 0 \; . \tag{4.2.17}$$

Since T_b^a is antisymmetric under a (c,d)(e,f) interchange, this vanishes. Thus we have that at energies > M_s, the supersymmetric SU(5) theory is broken down to a supersymmetric SU(3) × SU(2) × U(1) theory. We can next ask which fields get a large mass as a result of the $v \sim M_x$ breaking. The mass terms in L become

$$L_{mass} = \frac{m}{2}\,Tr\phi_{24}\phi_{24} + \mu H'_{\bar{5}}H_5 + 3\lambda_\phi Tr\phi_{24}v\phi_{24} + \lambda_{\phi H}\, H'_{\bar{5}}vH_5 \ . \qquad (4.2.18)$$

Recall that ϕ_{24} and $H_5, H'_{\bar{5}}$ can be expanded in terms of SU(3) × SU(2) × U(1) fields, so that

$$Tr\phi_{24}\phi_{24} = TrHH + 2H_{\bar{x}}H_x + 2H_{\bar{y}}H_y + H_BH_B + 2H^+H^- + H^oH^o \qquad (4.2.19)$$

and

$$Tr\phi_{24}v\phi_{24} = v\{TrHH - \frac{1}{2}H_{\bar{x}}H_x - \frac{1}{2}H_{\bar{y}}H_y - \frac{1}{2}H_BH_B - 3H^+H^- - \frac{3}{2}H^oH^o\} \ . \qquad (4.2.20)$$

Thus the ϕ_{24} masses become:

$$\begin{aligned}
\left(\frac{m}{2} + 3\lambda_\phi v\right) TrHH &= \frac{5}{2}\,mTrHH \\
\left(\frac{m}{2} - \frac{3}{2}\lambda_\phi v\right)H_BH_B &= -\frac{m}{2}\,H_BH_B \\
\left(m - 9\lambda_\phi v\right) H^+H^- &= -5mH^+H^- \\
\left(\frac{m}{2} - \frac{9}{2}\lambda_o v\right)H^oH^o &= -\frac{5}{2}\,mH^oH^o \\
\left(m - \frac{3}{2}\lambda_\phi v\right)[H_{\bar{x}}H_x + H_{\bar{y}}H_y] &= 0 \ .
\end{aligned} \qquad (4.2.21)$$

As necessary since SU(5) is broken down to SU(3) × SU(2) × U(1) we have 24 - 12 = 12 broken generators (the $X, \bar{X}, Y, \bar{Y}$) and hence 12 zero mass Goldstone bosons $H_x, H_{\bar{x}}, H_y, H_{\bar{y}}$. Since SUSY is good these complete chiral superfields will be eaten by the χ and γ gauge superfields according to the super Higgs mechanism turning these into massive vector superfields (recall the scalar Goldstone bosons are eaten by the ordinary gauge fields $X_\mu, \bar{X}_\mu, Y_\mu, \bar{Y}_\mu$ making them massive while the pseudo-scalar bosons and Weyl spinors in $H_{x,\bar{x},y,\bar{y}}$ become the new massive degrees of freedom in the now massive vector superfields). The remaining 12 Higgs super-mesons are extremely massive ($m \simeq M_x$) as required since H^a_b will lead to proton decay.

The 5 and $\bar{5}$ Higgs mass terms become

$$H'_{\bar{5}}H_5 = H'H + \phi^+\phi^- + \phi^o\phi'^o \tag{4.2.22}$$

and

$$H'_{\bar{5}}vH_5 = vH'H - \frac{3}{2}v\,(\phi^+\phi^- + \phi^o\phi'^o) \ , \tag{4.2.23}$$

so that the masses are given by

$$(\mu+\lambda_{\phi H}v)H'H$$

$$(\mu - \frac{3}{2}\lambda_{\phi H}v)(\phi^+\phi^- + \phi^o\phi'^o) \ . \tag{4.2.24}$$

Since H and H' carry color and couple directly to the matter fields they will be (predominantly) responsible for proton decay hence we have $(\mu+\lambda_{\phi H}v)$ the order of M_x. On the other hand ϕ^o, ϕ'^o will be responsible for electroweak breaking and should have a small mass (0 at this scale of energy > M_s) and so we must "fine tune" the parameters of our model so that $\mu = \frac{3}{2}\lambda_{\phi H}v$. This tuning is technically natural since SUSY is a good symmetry at these energies and the "no-renormalization" theorems imply that there are no radiative corrections to this relation and hence it stays tuned.

As we lower the energy the M_s explicit SUSY breaking mass terms will become important. These will serve to

1) give the s-matter fields a higher mass than the matter fields
2) give the s-gauge fields a mass.
3) give the weak Higgs bosons the correct negative mass squared to catalyze the spontaneous breakdown of the electroweak group; in this sense M_W is determined by M_s; the order of SUSY breaking.

Once SUSY is broken (at energies $< M_s$) the SUSY no-renormalization theorems no longer hold and there are low energy radiative corrections to the effective potential, i.e. masses now receive $O(\alpha M_s)$ corrections. In order not to violate the naturalness of our fine tuning and gauge hierarchy this αM_s should be $O(M_W)$; hence the reason for choosing $M_s \simeq$ 1-100 TeV.

As usual the low energy masses are given in terms of v_o and v_o' as previously (and ε giving the full potential minimum).

Finally let's estimate the proton decay and $\sin^2\theta_W$ and m_b/m_τ ratios predicted in this SUSYGUT.

4.3. Unification Mass, $\sin^2\theta_W$, m_b/m_τ.

Since SUSY puts fermions and bosons on equal footing we will need the RGE β function eq.(1.2.94) with scalar field loops included; it is

$$\beta = -\frac{g^3}{32\pi^2}\left[\frac{11}{3} C_2(G) - \frac{4}{3} T_F - \frac{1}{3} T_s\right] \tag{4.3.1}$$

where recall $C_2(G)$ is the quadratic Casimir operator ($C_2 = N$ for SU(N)).

$$T_F\delta_{ij} = \frac{1}{2}\,\mathrm{Tr}(T^iT^j) \tag{4.3.2}$$

where we sum over all fermi representations (if Dirac fermions sum over both left and right handed representations, while if Weyl fermions only count ψ_α or $\bar{\psi}_{\dot{\alpha}}$ once, not both (these are Majorana fields)). Similarly

$$T_s\delta_{ij} = \mathrm{Tr}(T^iT^j) \tag{4.3.3}$$

where we sum over all scalar field representations. For each super Y-M field we have an ordinary Y-M field in the adjoint rep. as well as a corresponding susy partner Weyl gaugino in the adjoint rep.. These contribute to β $[\frac{11}{3}\,C_2(G) - \frac{4}{3}\frac{1}{2}\,C_2(G)] = 3C_2(G)$ where $T_F = \frac{1}{2}\,C_2(G)$ for the adjoint rep. Weyl fermions. For each chiral superfield we have a Weyl fermion and a complex scalar in the same rep., therefore for each super matter and super Higgs multiplet we find a contribution to β of

$$[-\frac{4}{3}\frac{1}{2}\,T_s - \frac{1}{3}\,T_s] = -T_s = -\frac{1}{d}\,\mathrm{Tr}[T^iT^i]$$

where d = the dimension of the group and the trace is summed over the chiral superfields (i.e. for quarks and leptons this is twice the number of flavors (=4F)). So we have for supersymmetric theories

$$\begin{aligned}\beta &= -\frac{g^3}{32\pi^2}\,[3C_2(G) - \sum_{\substack{\text{chiral}\\\text{super-}\\\text{fields}}}\frac{1}{d}\,\mathrm{Tr}[T^iT^i]\\ &= -\frac{g^3}{32\pi^2}\,[3C_2(G) - \sum_{\substack{\text{chiral}\\\text{super-}\\\text{fields}}} T_s]\quad.\end{aligned} \tag{4.3.4}$$

We would like to apply this formula to find the supersymmetric SU(3) x SU(2) x U(1) running coupling constants $\alpha_i(Q^2)$ and hence obtain $\sin^2\bar{\theta}_W$; M_X and m_b/m_τ. Recall eq.(2.2.11) for the running coupling constants

$$\frac{1}{\alpha_i(Q^2)} - \frac{1}{\alpha_i(M_x^2)} = -8\pi b_i \ln \frac{Q^2}{M_x^2} \tag{4.3.5}$$

where $\beta_i = b_i g_i^3$ for $i = 1,2,3$ and $\alpha_i(Q^2) = \frac{\bar{g}_i^{\,2}(\lambda^2)}{4\pi}$ with $Q^2 = \lambda^2 M_x^2$.

From above we have

$$b_3 = -\frac{1}{32\pi^2}\,[9-2F]$$

$$b_2 = -\frac{1}{32\pi^2}\,[6 - 2F - \tfrac{H}{2}] \quad (> \; 0 \text{ for } F = 3$$

α_2 increases for SUSY theories!)

$$b_1 = +\frac{1}{32\pi^2} \sum_{\substack{\text{chiral}\\\text{super-}\\\text{fields}}} Y^2 = \frac{1}{32\pi^2}\,[\tfrac{10}{3} F + \tfrac{1}{2} H] \tag{4.3.6}$$

where F is the number of families and H the number of Higgs chiral superfield SU(2) doublets. Note that the SUSY slopes b_i are less than the ordinary slopes, b_{o_i},

$$b_{o_3} = -\frac{1}{32\pi^2}\,[11 - \tfrac{4}{3} F]$$

$$b_{o_2} = -\frac{1}{32\pi^2}\,[\tfrac{22}{3} - \tfrac{4}{3} F - \tfrac{1}{6} H]$$

$$b_{o_1} = +\frac{1}{32\pi^2}\,[\tfrac{20}{9} F + \tfrac{1}{6} H] \;, \tag{4.3.7}$$

so the α_i run more slowly in the SUSY case

$$\alpha_i(Q^2) = \frac{\alpha_i(M_x^2)}{1 - 8\pi\alpha_i(M_x^2) b_i \ln \frac{Q^2}{M_x^2}} \quad .$$

This coupled with α_2 not being asymptotically free will yield a larger M_x.

In particular recall we have unification at $Q^2 \geq M_x^2$ with

$$g = g_2 = g_3 = \sqrt{5/3}\, g_1 \quad \text{and}$$

$$\sin^2\theta_w = \frac{g_1^2}{g_1^2 + g_2^2} = \frac{3}{8}$$

$$\frac{\alpha_{QED}}{\alpha_s} = \frac{g_2^2 \sin^2\theta_w}{g_3^2} = \frac{3}{8} \quad . \tag{4.3.8}$$

At lower energies these parameters run according to the general formulae (2.2.12)

$$\sin^2\bar{\theta}_w = \frac{\bar{\alpha}_1}{\bar{\alpha}_1 + \bar{\alpha}_2}$$

$$= \frac{3}{8}[1 + (\frac{g^2}{4\pi})5\pi(\frac{3}{5}\, b_1 - b_2)\ell n\, \frac{Q^2}{M_x^2}] \tag{4.3.9}$$

and (2.2.16)

$$\frac{\bar{\alpha}_{QED}}{\bar{\alpha}_s} = \frac{\bar{\alpha}_2}{\bar{\alpha}_3}\, \sin^2\bar{\theta}_w$$

$$= \frac{3}{8}[1 + (\frac{g^2}{4\pi})\pi(3b_1 + 3b_2 - 8b_3)\ell n\, \frac{Q^2}{M_x^2}] \quad . \tag{4.3.10}$$

These, along with (4.3.5), yield

$$\sin^2\bar{\theta}_w = \frac{1}{[3b_1 + 3b_2 - 8b_3]}\, [3b_2 - 3b_3 + (3b_1 - 5b_2)\, \frac{\bar{\alpha}_{QED}}{\bar{\alpha}_s}]$$

$$\pi\ell n\, \frac{M_x^2}{Q^2} = \frac{1}{[3b_1 + 3b_2 - 8b_3]}\, [\frac{3}{8}\, \frac{1}{\bar{\alpha}_{QED}} - \frac{1}{\bar{\alpha}_s}]$$

$$\frac{g^2}{4\pi} = \frac{1}{[3b_1 + 3b_2 - 8b_3]} \, [(3b_1 + 3b_2)\bar{\alpha}_s - \frac{64}{3} b_3 \, \bar{\alpha}_{QED}] . \qquad (4.3.11)$$

The strong, $\bar{\alpha}_s$, and electromagnetic, $\bar{\alpha}_{QED}$, running fine structure constants are known experimentally at $Q^2 = 10\text{GeV}^2$ as

$$\bar{\alpha}_{QED} \approx \bar{\alpha}_{QED}(0) \approx \frac{1}{137}$$

and

$$\frac{\bar{\alpha}_{QED}}{\bar{\alpha}_s} \simeq .04 \quad .$$

Let's calculate $\sin^2\bar{\theta}_w$, M_x, $\frac{g^2}{4\pi}$ in our ordinary SU(5) and SUSY SU(5) GUTS. Recall we have 3 families in each case, F = 3, but in the ordinary SU(5) GUT we only have one light Higgs doublet, H = 1, which gives the quarks and leptons their masses. However in the SU(5) SUSYGUT we needed two light Higgs doublets, H = 2, in order to make the necessary Yukawa coupling and hence mass terms since we have only tri-linear pure chiral superfield interactions in the supersymmetric Lagrangian; these light doublets were contained in the SUSY SU(5) fields H_5, $H'_{\bar{5}}$.

For the ordinary SU(5) GUT we have

$$\sin^2\bar{\theta}_w = [\frac{1}{6} + \frac{1}{132} H + (\frac{5}{9} - \frac{1}{198} H) \frac{\bar{\alpha}_{QED}}{\bar{\alpha}_s}]$$

$$\ln \frac{M_x^2}{Q^2} = \frac{16\pi}{33} \frac{1}{\bar{\alpha}_{QED}} [\frac{3}{8} - \frac{\bar{\alpha}_{QED}}{\bar{\alpha}_s}]$$

$$\frac{g^2}{4\pi} = \bar{\alpha}_{QED} \, [\frac{64}{3} (\frac{1}{3} - \frac{4}{99} F) + (- \frac{1}{3} + \frac{16}{99} F + \frac{1}{66} H) \frac{\bar{\alpha}_s}{\bar{\alpha}_{QED}}] \ . \qquad (4.3.12)$$

Putting in the numbers we find

$$\sin^2\bar{\theta}_w = 0.20$$
$$M_x = 4 \times 10^{15}\ \text{GeV} \tag{4.3.13}$$
$$\frac{g^2}{4\pi} = \frac{1}{15}$$

for the ordinary Georgi-Glashow SU(5) model.

For the SUSY SU(5) model we have

$$\sin^2\bar{\theta}_w = \frac{1}{6 + \frac{1}{3}H}\left[1 + \frac{1}{6}H + \left(5 - \frac{H}{6}\right)\frac{\bar{\alpha}_{QED}}{\bar{\alpha}_s}\right]$$

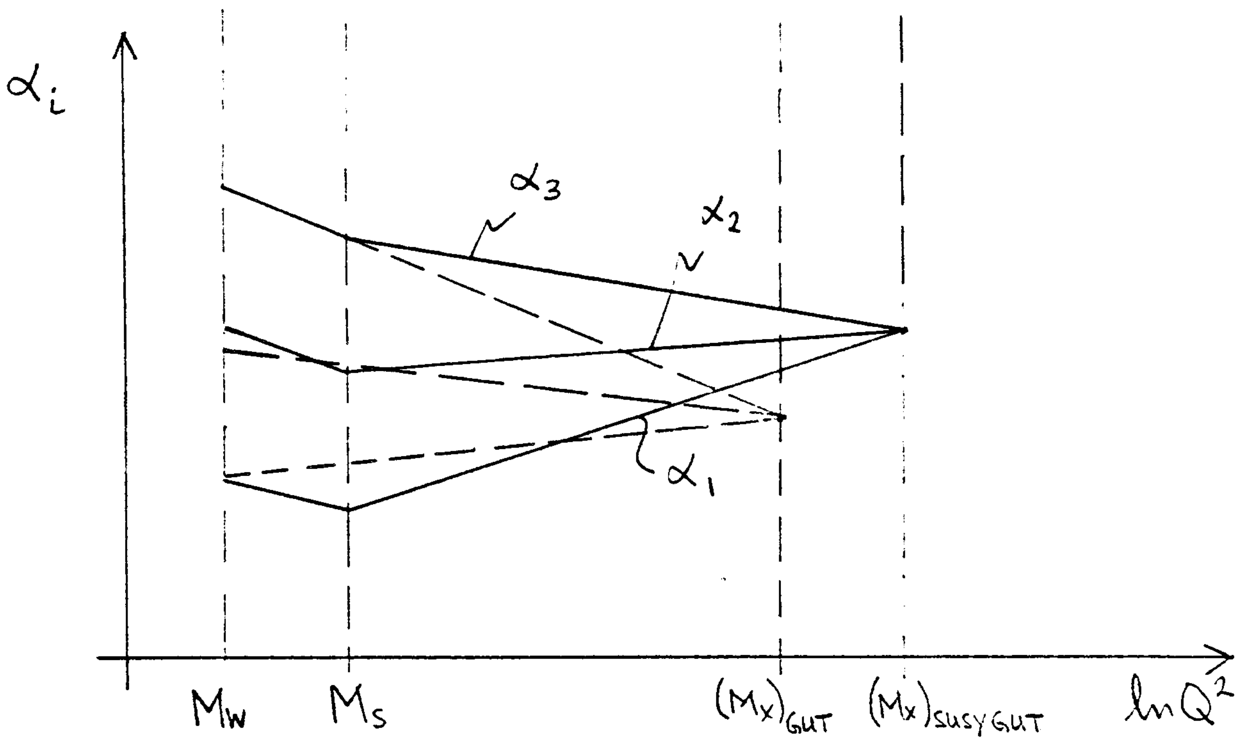
$$\ln\frac{M_x^2}{Q^2} = \frac{16\pi}{27 + \frac{3}{2}H}\,\frac{1}{\bar{\alpha}_{QED}}\left[\frac{3}{8} - \frac{\bar{\alpha}_{QED}}{\bar{\alpha}_s}\right]$$

$$\frac{g^2}{4\pi} = \frac{1}{6 + \frac{1}{3}H}\,\bar{\alpha}_{QED}\left[\frac{64}{3}\left(1 - \frac{2}{9}F\right) + \left(-2 + \frac{16}{9}F + \frac{1}{3}H\right)\frac{\bar{\alpha}_s}{\bar{\alpha}_{QED}}\right] . \tag{4.3.14}$$

Putting in the numbers we find

$$\sin^2\bar{\theta}_w = 0.23$$
$$M_x = 2 \times 10^{17}\ \text{GeV}$$
$$\frac{g^2}{4\pi} = \frac{1}{9} \ . \tag{4.3.15}$$

As expected the SUSY quantities are all a bit larger. Since the proton lifetime τ_p is very sensitive to the value of M_x; $\tau_p \sim M_x^4$, this increase of $M_x \sim 10^{17}$ GeV will result in $\tau_p \gtrsim 10^{38}$ yrs., totally undetectable. However, as we will see, $\tau_p \sim M_x^4$ for direct X,Y boson exchange graphs; for SUSY theories the predominant decay mode will be through Higgs exchange which can result in a $\tau_p \sim M_x^2$ (supression factors). We can pictorially represent the running coupling constants as



The RGE analysis can also be applied to the fermion masses (see e.g. M.B. Einhorn and D.R.T. Jones, Nucl. Phys. B196 (1982) 475) to find for the SUSY GUT case

$$\frac{m_b}{m_\tau} = \left[\frac{\bar{\alpha}_s(2m_b)}{\bar{\alpha}_s(m_t)}\right]^{12/33} \left[\frac{\bar{\alpha}_s(m_t)}{\bar{\alpha}_s(m_w)}\right]^{4/7} \left[\frac{\bar{\alpha}_s(m_w)}{g^2/4\pi}\right]^{8/9}$$

while for the ordinary GUT case we had

$$\frac{m_b}{m_\tau} = \left[\frac{\bar{\alpha}_s(2m_b)}{\bar{\alpha}_s(m_t)}\right]^{12/23} \left[\frac{\bar{\alpha}_s(m_t)}{g^2/4\pi}\right]^{4/7} .$$

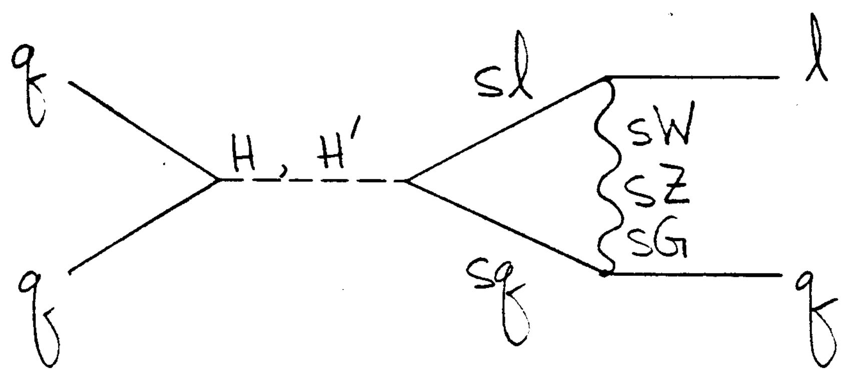
These yield

$$\left(\frac{m_b}{m_\tau}\right)_{SUSYGUT} = 1.09 \left(\frac{m_b}{m_\tau}\right)_{GUT} , \tag{4.3.16}$$

not much change.

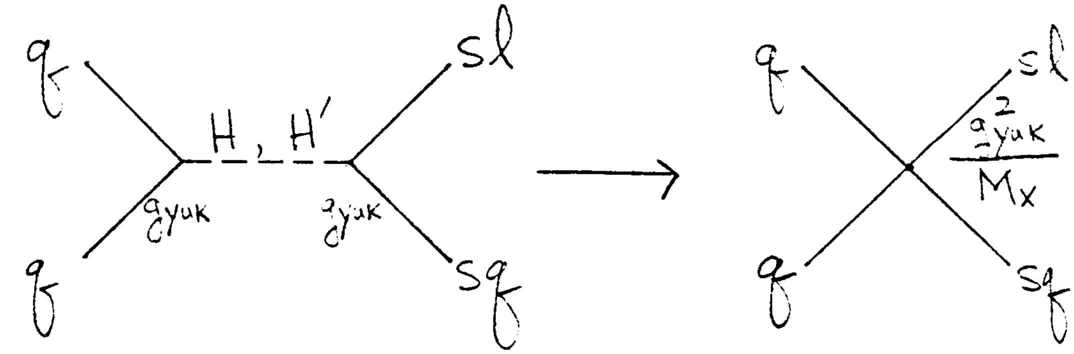
4.4. Proton Decay

Finally we would like to estimate the proton lifetime within the SUSYGUT. SUSY requires the existence of scalar partners for the quarks and leptons and fermion partners for the Yang-Mills fields; we can use these fields to allow the proton to decay through dimension 5 operators. Consider a graph of the form



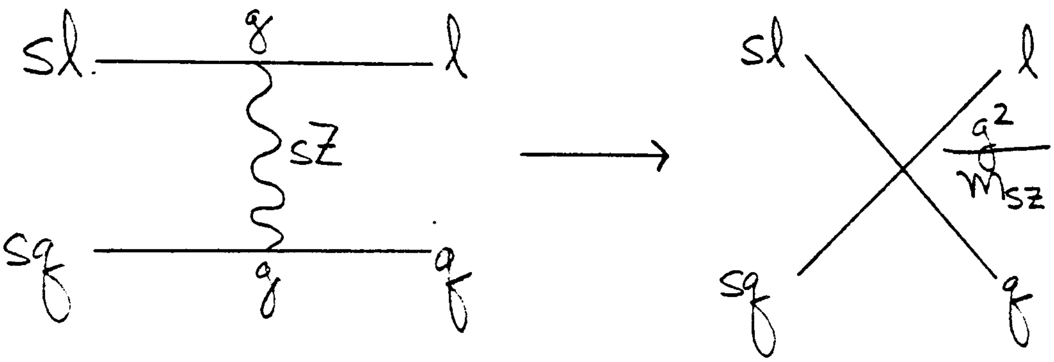
(4.4.1)

The matter superfields $M_{\bar{5}}$, M_{10} have a Yukawa coupling directly to the chiral Higgs superfields H_5, $H'_{\bar{5}}$. The exchange of colorful Higgs fields $H^{1,2,3}$ and $H'_{1,2,3}$ results in baryon number changing processes with effective dimension 5 operators:



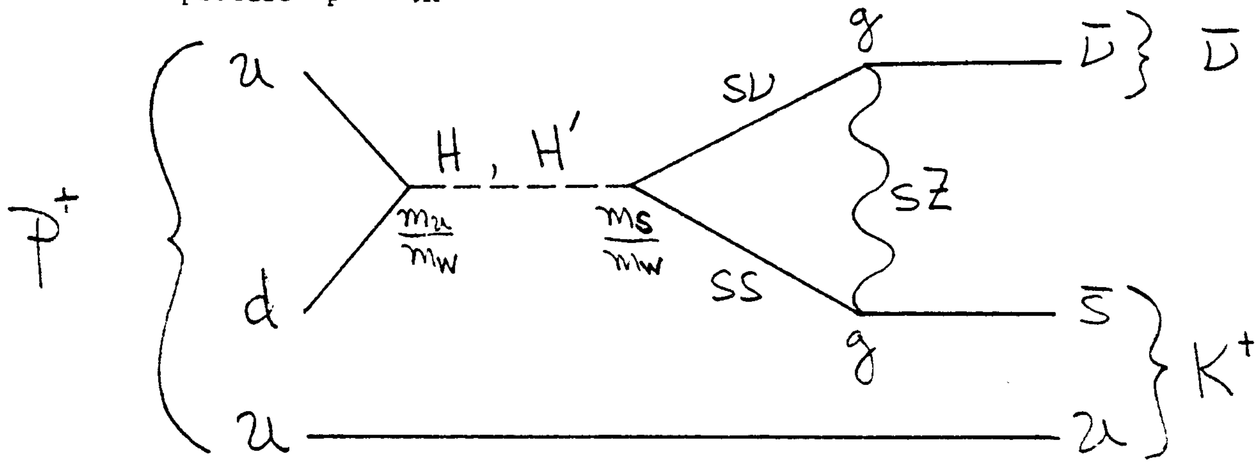
(4.4.2)

where the Yukawa couplings will be related to the masses of the different flavor quarks meeting at the vertex divided by the weak interaction scale, m_W. The scalar quarks, sq, and scalar leptons, sℓ, can then interact via a super gauge Yukawa coupling with the strength of the SUSYGUT coupling constant, g, with their fermionic quark and lepton partners and the fermionic partners to the Yang-Mills fields



(4.4.3)

This gives another effective dimension 5 operator with mass scale set by the mass of the exchanged fermion, m_{sz} in the case above, which goes like the SUSY breaking scale. Putting this together we find, for example, the proton decay process $p^+ \to \bar{\nu}K^+$



(4.4.4)

This amplitude goes like

$$\frac{g^2}{M_x m_{sz}} \left(\frac{m_s m_u}{m_w^2}\right) \quad . \tag{4.4.5}$$

Hence the proton lifetime can be guestimated to be

$$\tau_p^2 \sim \frac{1}{\alpha_{SUSYGUT}^2} \frac{M_{SUSYGUT}^2 \, m_{sz}^2}{m_p^5} \left(\frac{m_w^2}{m_s m_u}\right)^2 . \tag{4.4.6}$$

Comparing this to our ordinary SU(5) GUT proton lifetime we find

$$(\tau_p)_{\substack{SUSY-\\GUT}} \sim \left(\frac{\alpha_{GUT}}{\alpha_{SUSYGUT}}\right)^2 \left(\frac{M_{SUSYGUT} m_{sz}}{M_{GUT}^2}\right)^2 \left(\frac{m_w^2}{m_s m_u}\right)^2 (\tau_p)_{GUT} \quad . \tag{4.4.7}$$

With

$$m_{sz} \sim 1 \text{ TeV} \qquad m_u = 10^{-2}\, m_s ,$$
$$m_w \sim 10^2 \text{ GeV} \qquad m_s = 10^{-1} \text{ GeV}$$

and $M_{SUSYGUT} = 5 \times 10^{17}$ GeV and $M_{GUT} = 3 \times 10^{14}$ GeV this yields

$$(\tau_p)_{\substack{SUSY-\\GUT}} \sim \left(\frac{9}{15}\right)^2 \left(\frac{5 \times 10^{17}\ 10^3}{(3 \times 10^{14})^2}\right)^2 \left(\frac{10^4}{10^{-4}}\right)^2 (\tau_p)_{GUT} \sim (\tau_p)_{GUT} . \tag{4.4.8}$$

The proton lifetime in SUSYGUTS is about the same as that in ordinary GUTS even though baryon number is changed by dimension 5 operators in the former and dimension 6 in the latter. The branching ratios of the various decay modes, however, will be quite different. SUSYGUTS yielding strange meson decay modes as dominant over the pion decay modes, opposite that of the ordinary GUTS. For further details and additional applications of SUSY in GUTS (and beyond) see for example ref. (5a) J. Ellis, CERN Preprint TH-3174.