Physics 660 Quantum Mechanics

I. Schrödinger Wave Mechanics

1.1. Introduction

During the early part of this century physicists were confronted with many new phenomena on the atomic scale ($\sim 10^{-8}$ cm) that could not be satisfactorily accounted for by classical mechanics. From the properties of electromagnetic processes Planck and Einstein deduced that electromagnetic radiation consisted of fundamental particles, photons. Under various circumstances the photons exhibited wave-like behavior, such as diffraction and interference, while at other times the photons exhibited particle-like behavior, such as when colliding with electrons in Compton scattering experiments or as in the photo-electric effect. Equally astonishing particles like electrons where shown under certain situations to exhibit wave-like behavior as in diffraction and interference experiments. This wave-particle duality is inexplicable within the framework of classical physics. As well, many other phenomena arose for which classical physics had no explanation. These included the appearance of atomic spectral lines at fixed frequencies, the low temperature behavior of the specific heat of matter and the stability of matter, and many others. Quantum theory emerged as a new theoretical framework which is able to account for all these observations. In addition, as the decades have passed, quantum theory has provided the basis to account for numerous additional phenomena such as superconductivity, superfluidity, nuclear reactions, as well as a framework for the field theories of the presently observed elementary particles and their fundamental gauge theory interaction forces.

To begin our study of quantum mechanics we shall consider some of the original ideas leading to this new theory. Planck and Einstein associated an electromagnetic wave of angular frequency $\omega (= 2\pi\nu)$ with a collection of photons each of energy E via the relation

$E = \hbar\omega$, with $\hbar = h/2\pi$ and h being Planck's constant. deBroglie then suggested that not only photons but ~~that~~ all free particles have an associated wave of frequency $\omega = E/\hbar$ (Planck-Einstein relation) and wavelength $\lambda \equiv \frac{2\pi}{k} = \frac{2\pi\hbar}{p}$ (deBroglie wavelength). Here the wave number k is given by $k = p/\hbar$. In general, the energy and momentum of a non-relativistic particle of mass m are related by $E = \frac{p^2}{2m}$. Combining this with the Planck-Einstein and deBroglie relations so that $\omega = \frac{\hbar k^2}{2m}$, the group velocity of the matter wave is given as

$$u = \frac{d\omega}{dk} = \frac{\hbar k}{m} = \frac{p}{m},$$

which is the magnitude of the velocity of the non-relativistic particle.

We could continue with this historical approach and introduce the quantization of Bohr orbits (the circumference of the orbit, $2\pi r$, contains an integral number of wavelengths, $n = \frac{2\pi r}{\lambda} = kr$, hence the action is quantized, $pr = n\hbar$) and so on.

However, as crucial as these intermediate steps might have been to the development of the quantum theory, or as useful as they may be as approximations, they are, per se, incorrect. Hence we shall proceed directly to the correct theory.