Physics 660 Quantum Mechanics

I. Schrödinger Wave Mechanics

1.1. Introduction

During the early part of this century, physicists were confronted with many new phenomena on the atomic scale (\(10^{-8}\) cm) that could not be satisfactorily accounted for by classical mechanics. From the properties of electromagnetic processes, Planck and Einstein deduced that electromagnetic radiation consisted of fundamental particles, photons. Under certain circumstances, the photons exhibited wave-like behavior, such as diffraction and interference, while at other times the photons exhibited particle-like behavior, such as when colliding with electrons in Compton scattering experiments or as in the photo-electric effect. Equally astonishing, under certain situations, electrons were shown to exhibit wave-like behavior such as in diffraction and interference experiments. This wave-particle duality is inexplicable within the framework of classical
Physics. As well, many other phenomena arose for which classical physics had no explanation. These included the appearance of atomic spectral lines at fixed frequencies, the low temperature behavior of the specific heat of matter and the stability of matter, and many others. Quantum theory emerged as a new theoretical framework which is able to account for all these observations. In addition as the decades have passed, quantum theory has provided the basis to account for numerous additional phenomena such as superconductivity, superfluidity, nuclear reactions, as well as a framework for the field theories of the presently observable elementary particles and their fundamental gauge theory interactions.

To begin our study of quantum mechanics, we shall consider some of the original ideas leading to this new theory. Planck and Einstein associated an electromagnetic wave of angular frequency \( \omega \) with a collection of photons each of energy \( E \) via the relation
E = \frac{1}{2}m \dot{v}^2, with \( \hbar = \frac{\hbar}{2\pi} \) and \( \hbar \) being Planck's constant. de Broglie then suggested that not only photons but all free particles have an associated wave of frequency \( \nu = \frac{E}{\hbar} \) (Planck-Einstein relation), and a wavelength \( \lambda = \frac{2\pi}{\nu} = \frac{2\pi m}{\hbar} \) (de Broglie wavelength). Here the wave number \( k \) is given by \( k = \frac{\nu}{c} \). In general, the energy and momentum of a non-relativistic particle of mass \( m \) are related by \( E = \frac{p^2}{2m} \). Combining this with the Planck-Einstein and de Broglie relations so that \( \nu = \frac{\hbar k^2}{2m} \), the group velocity of the matter wave is given as

\[ v = \frac{\partial \nu}{\partial k} = \frac{\hbar k}{m} = \frac{p}{m}, \]

which is the magnitude of the velocity of the non-relativistic particle.

We should continue with this historical approach and introduce the quantization of Bohr orbits (the circumference of the orbit, \( 2\pi r \), contains an integer number of wavelengths, \( n = \frac{2\pi r}{\lambda} = kr \); hence the action is quantized, \( pr = n\hbar \)) and so on.
However, as crucial as these intermediate steps might have been to the development of the quantum theory, or as useful as they may be as approximations, they are perhaps incorrect. Hence we shall proceed directly to the correct theory.