SUSY implies the graviton will have a spin 3/2 partner the gravitino. The spin 3/2 field is known as the Rarita-Schwinger field. Consider the free massive (Majorana) Rarita-Schwinger field $\psi^{\alpha}_{\mu}(x)$, $\bar{\psi}_{\dot{\alpha}\mu}(x)$, it is a vector-spinor field. Such a field obeys the Dirac equation for each value of the vector index

$$\boxed{(i\not{\partial} - m)\Psi_\mu = 0}$$

where
$$\Psi_\mu = \begin{pmatrix} \psi_{\mu\alpha} \\ \bar{\psi}_{\mu}^{\dot{\alpha}} \end{pmatrix} \quad ; \quad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$
as usual in the Weyl representation

along with the constraint

$$\boxed{\gamma^\mu \Psi_\mu = 0 .}$$

From the Dirac equation

$$\gamma^\mu (i\not{\partial} - m)\Psi_\mu = 0$$

$$\Rightarrow \gamma^\mu \gamma^\nu \partial_\nu \Psi_\mu = 0 = \tfrac{1}{2}(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu)\partial_\nu \Psi_\mu$$
$$= \eta^{\mu\nu}\partial_\nu \Psi_\mu = \partial^\mu \Psi_\mu$$

So we also have

$$\boxed{\gamma^\mu \Psi_\mu = 0 .}$$

The plane wave solutions to the spin 3/2 Dirac equation are

$$\Psi_\mu(x) = u_\mu(\vec{p}) e^{-ipx} \quad ; \; p^2 = m^2$$

So $(i\not{\partial} - m)\Psi_\mu = (\not{p} - m) u_\mu(\vec{p}) e^{-ipx} = 0$

$$\Rightarrow \quad (\not{p} - m) u_\mu(\vec{p}) = 0$$

Now let's use the usual Dirac representation for the γ-matrices so that the rest frame field equation

$$(m\gamma^0 - m) u_\mu(\vec{0}) = 0$$

$$\Rightarrow \quad \boxed{\gamma^0 u_\mu(\vec{0}) = u_\mu(\vec{0})}$$

has the usual form it has in the case of spin 1/2 particles: the lower 2 components of $u_\mu(\vec{0})$ vanish for each $\mu = 0, 1, 2, 3$.

That is in the Dirac representation

$$\gamma^0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}; \gamma^1 = \begin{bmatrix} 0 & \sigma^1 \\ -\sigma^1 & 0 \end{bmatrix}; \gamma^2 = \begin{bmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{bmatrix}; \gamma^3 = \begin{bmatrix} 0 & \sigma^3 \\ -\sigma^3 & 0 \end{bmatrix}$$

we have

$$\gamma^0 u_\mu(\vec{0}) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \chi_\mu \\ \phi_\mu \end{bmatrix} = \begin{bmatrix} \chi_\mu \\ -\phi_\mu \end{bmatrix} \equiv +u_\mu(\vec{0})$$

$\Rightarrow \phi_\mu = 0$; χ_μ = arbitrary 2-spinor.

So $u_\mu(\vec{0}) = \begin{bmatrix} \chi_\mu \\ 0 \end{bmatrix}$ 4 2-spinors

The subsidiary condition $\gamma^\mu \Psi_\mu = 0 \Rightarrow$

$$\gamma^\mu u_\mu(\vec{0}) = 0 = \gamma^0 u_0 - \gamma^i u^i$$

but $\gamma^0 u_0 = +u_0 \Rightarrow$ $\boxed{\begin{aligned} u_0(\vec{0}) &= \gamma^i u^i(\vec{0}) \\ &= \vec{\gamma} \cdot \vec{u}(\vec{0}) \end{aligned}}$

but

$$\gamma^i = \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix}; \text{ So } \gamma^i u^i(\vec{0}) = \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix} \begin{bmatrix} \chi^i \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -\sigma^i \chi^i \end{bmatrix}$$

Hence

$$u_0(\vec{0}) = \vec{\gamma}\cdot\vec{u}(\vec{0}) = \begin{bmatrix} 0 \\ -\sigma^i\chi^i \end{bmatrix}$$

$$\| $$

$$\begin{bmatrix} \chi_0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \qquad \chi_0 = 0 \Rightarrow \boxed{u_0(\vec{0}) = 0}$$

$\left(\text{Note: } \partial_\mu \Psi^\mu = 0 \rightarrow m\,u_0(\vec{0}) = 0 \Rightarrow u_0(\vec{0}) = 0\right)$

and

$$\boxed{\sigma^i \chi^i = 0 = \vec{\sigma}\cdot\vec{\chi}}$$

Thus only 4 of the 6 components of $\chi_{1,2,3}$ are independent.

Let $\chi_i = \begin{pmatrix} a_i \\ b_i \end{pmatrix}$ So

$$\vec{\sigma}\cdot\vec{\chi} = \begin{bmatrix} b_1 - ib_2 + a_3 \\ a_1 + ia_2 - b_3 \end{bmatrix} = 0$$

$$\Rightarrow \qquad \boxed{\begin{aligned} a_3 &= -(b_1 - ib_2) \\ b_3 &= +(a_1 + ia_2) \end{aligned}}$$

The spinors $u_\mu(\vec{p})$ are completely fixed by the $u_\mu(\vec{0})$. Thus we only have 4 independent components, just the number of spinors needed to describe a spin 3/2 massive particle with z-components of spin $\pm 3/2, \pm 1/2$ in the particle's rest frame.

Finally the Lagrangian for the massive gravitino is given by

$$\boxed{\mathcal{L} = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\Psi}_\mu \gamma_5 \gamma_\nu \partial_\rho \Psi_\sigma - \frac{1}{4} m \bar{\Psi}_\mu [\gamma^\mu, \gamma^\nu] \Psi_\nu}$$

Recall that Ψ_μ is a Majorana field.

The Dirac equation for Ψ_μ, $(i\not{\partial} - m)\Psi_\mu = 0$, and the constraint $\gamma^\mu \Psi_\mu = 0$ follow from the Euler-Lagrange equation obtained from this Lagrangian.