SUSY Review

1) Superspace $(N=1, D=4)$: $(x^\mu, \theta^\alpha, \bar{\theta}_{\dot\alpha})$
SUSY transformations: (Grassmann parameters $\xi_\alpha, \bar{\xi}_{\dot\alpha}$)

$$x'^\mu = x^\mu + i(\xi\sigma^\mu\bar{\theta} - \theta\sigma^\mu\bar{\xi})$$
$$\theta'^\alpha = \theta^\alpha + \xi^\alpha$$
$$\bar{\theta}'_{\dot\alpha} = \bar{\theta}_{\dot\alpha} + \bar{\xi}_{\dot\alpha}$$

2) Superfields:

a) Vector superfield (Real)

$$\begin{aligned} V(x,\theta,\bar{\theta}) = {} & C(x) + \theta^\alpha \chi_\alpha(x) + \bar{\theta}_{\dot\alpha}\bar{\chi}^{\dot\alpha}(x) \\ & + \tfrac{1}{2}\theta^2 M(x) + \tfrac{1}{2}\bar{\theta}^2 M^\dagger(x) + \theta\sigma^\mu\bar{\theta}\, V_\mu(x) \\ & + \tfrac{1}{2}\theta^2\bar{\theta}_{\dot\alpha}\bar{\lambda}^{\dot\alpha}(x) + \tfrac{1}{2}\bar{\theta}^2\theta^\alpha\lambda_\alpha(x) \\ & + \tfrac{1}{4}\theta^2\bar{\theta}^2 D(x) \end{aligned}$$

b) Chiral superfield

$$(\bar{D}_{\dot\alpha}\phi = 0) \Rightarrow \quad \phi(x,\theta,\bar{\theta}) = e^{-i\theta\not{\partial}\bar{\theta}}\left[A(x) + \theta^\alpha\psi_\alpha(x) + \theta^2 F(x)\right]$$

Anti-Chiral superfield

$$(D_\alpha\phi^\dagger = 0) \Rightarrow \phi^\dagger(x,\theta,\bar{\theta}) = e^{+i\theta\not{\partial}\bar{\theta}}\left[A^\dagger(x) + \bar{\theta}_{\dot\alpha}\bar{\psi}^{\dot\alpha}(x) + \bar{\theta}^2 F^\dagger(x)\right]$$

3) SUSY Covariant Derivatives

a) Spinor Derivatives: $\left(\frac{\partial}{\partial\theta^\alpha}\theta^\beta = \delta_\alpha{}^\beta \quad \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\bar\theta^{\dot\beta} = \delta_{\dot\alpha}{}^{\dot\beta}\right)$

$$D_\alpha \equiv \frac{\partial}{\partial\theta^\alpha} - i(\not\partial\bar\theta)_\alpha$$

$$\bar D_{\dot\alpha} \equiv -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + i(\theta\not\partial)_{\dot\alpha}$$

$$\{D_\alpha, \bar D_{\dot\alpha}\} = +2i\not\partial_{\alpha\dot\alpha}$$

b) Space-time derivative: ∂_μ

4) Superspace Integration:

$$\int d\theta_\alpha = \frac{\partial}{\partial\theta^\alpha} \qquad \int d^2\theta\,\theta^2 = -4$$

$$\int d\bar\theta_{\dot\alpha} = \frac{\partial}{\partial\bar\theta^{\dot\alpha}} \qquad \int d^2\bar\theta\,\bar\theta^2 = -4$$

a) Vector Measure

$$\int dV \equiv \int d^4x\, d^2\theta\, d^2\bar\theta = \int d^4x\, d\theta^\alpha d\theta_\alpha\, d\bar\theta_{\dot\alpha} d\bar\theta^{\dot\alpha}$$

$$= \int d^4x \frac{\partial}{\partial\theta_\alpha}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\alpha}}$$

(ignore surface terms) $= \int d^4x\, DD\bar D\bar D$

4) b.) Chiral Measure

$$\int dS \equiv \int d^4x\, d^2\theta = \int d^4x \frac{\partial}{\partial\theta_\alpha}\frac{\partial}{\partial\theta^\alpha}$$

$$\left(\text{ignore surface terms}\right) \quad = \int d^4x\, DD$$

Anti-Chiral Measure

$$\int d\bar{S} \equiv \int d^4x\, d^2\bar{\theta} = \int d^4x \frac{\partial}{\partial\bar{\theta}^{\dot\alpha}}\frac{\partial}{\partial\bar{\theta}_{\dot\alpha}}$$

$$\left(\text{ignore surface terms}\right) \quad = \int d^4x\, \bar{D}\bar{D}$$

5) Invariant Action:

$$\int dVV = 16\int d^4x \tfrac{1}{4} D(x)$$

$$= 4\int d^4x\, D(x) \qquad \text{D-term}$$

$$\left.\begin{aligned} \int dS\phi &= -4\int d^4x\, F \\ \int d\bar{S}\bar{\phi} &= -4\int d^4x\, F^\dagger \end{aligned}\right\} \text{F-terms}$$

Susy invariant

$$\delta^Q(\xi,\bar{\xi})\int dVV = 0 = \delta^Q(\xi,\bar{\xi})\int dS\phi$$
$$= \delta^Q(\xi,\bar{\xi})\int d\bar{S}\bar{\phi} .$$

5) a) Vector x Vector = Vector
Chiral x Anti-Chiral = Vector
Vector x (anti-)Chiral = Vector
} Superfields

So $\int dV \, V$ are SUSY invariant

ex. $\int dV \, \bar{\Phi}\Phi$

b) Chiral x chiral = chiral
anti-chiral x anti-chiral = anti-chiral
} superfield

So $\int dS \, \phi$, $\int d\bar{S} \, \bar{\phi}$ are SUSY invariant

ex. $\int dS \, \phi^3$; $\int d\bar{S} \, \bar{\phi}^3$

<u>ex.</u> W-Z model

$$\Gamma = \int dV \, K(\phi, \bar{\phi}) + \int dS \, W(\phi) + \int d\bar{S} \, \bar{W}(\bar{\phi})$$

$K = Z\phi\bar{\phi}$; $W = 4m\phi^2 + g\phi^3$ $\qquad m \to \frac{m}{16}\frac{1}{2}$

$Z = 1/16$ $\qquad \bar{W} = 4m\bar{\phi}^2 + g\bar{\phi}^3$ $\qquad g \to \frac{g}{12}\frac{1}{2}$

$\psi \to \sqrt{2}\,\psi$

$\bar{\psi} \to \sqrt{2}\,\bar{\psi}$

$$\Rightarrow \Gamma = \int d^4x \Big\{ \partial_\mu A \partial^\mu A^\dagger + \frac{i}{2}\psi \overleftrightarrow{\not{\partial}} \bar{\psi} + FF^\dagger$$

$$- m\left[AF + \bar{A}\bar{F} - \frac{1}{2}\psi\psi - \frac{1}{2}\bar{\psi}\bar{\psi} \right]$$

$$- g\left[\tfrac{1}{2}AAF + \tfrac{1}{2}\bar{A}\bar{A}\bar{F} - \frac{1}{2}A\psi^2 - \frac{1}{2}\bar{A}\bar{\psi}^2 \right] \Big\}$$

5) SUSY $\Rightarrow$ No Renormalization Theorem:

$$W(\phi) = W(\phi)^{\text{classical}} \quad (\text{wysiwyg})$$

i.e. $W(\phi)^{\text{quantum corrections}} = 0$

The superpotential is not renormalized
(only wavefunction renormalization
~~no~~ indep. mass or coupling
constant renormalization)

i.e. $$\langle 0 | T\, \tilde{\phi}(p_1,1)\, \tilde{\phi}(p_2,2) \cdots \phi(0,n) |0\rangle^{1PI} \Big|_{\substack{\forall p_i = 0 \\ \text{quantum corrections} \\ (= \text{loops})}} = 0$$