So we have mass for V:

$$\frac{g^2}{2}\,(Z_+a_+^*a_+ + Z_-a_-^*a_-)V^2 \qquad (4.0.24)$$

which cannot be transformed away. The $\phi_{o,+,-}$ mass matrix has a zero eigenvalue corresponding to the Goldstone boson superfield. The scalar field is eaten to give V_μ a mass, while the pseudoscalar and fermion become the new massive degrees of freedom in V. This is the super Higgs mechanism.

For non-abelian gauge groups the situation is similar. The linear chiral terms must be made to vanish, i.e. $\langle F_i\rangle = 0$. When the gauge group does not contain an invariant U(1) the linear V term must also vanish, i.e., $\langle D_i\rangle = 0$; that is

$$a_i^* T_{ij}^a a_j = 0 \; . \qquad (4.0.25)$$

4.1. Super Georgi-Glashow SU(5) Model

We are now ready to turn to the supersymmetric version of the Georgi-Glashow SU(5) model. Recall the left handed matter fields were in the 5^* and 10 representations of SU(5). Hence the SUSY extension of this is to put the matter fields in 5^* and 10 chiral superfields denoted $M_{\bar{5}}, M_{10}$. The bosonic SUSY partners to the quarks and leptons will be called squarks and sleptons (s for SUSY not scalar) or smatter fields. As before we will break the SU(5) down to SU(3) × SU(2) × U(1) by means of an adjoint of Higgs fields--again these will be a chiral superfield denoted ϕ_{24} with the fermi super partners to the bosonic Higgs fields called shiggs fields. Since we have only trilinear pure chiral or pure anti-chiral monomials in the action we now need both a 5 and a $\bar{5}$ of Higgs chiral superfields in order to make the necessary matter field Yukawa terms. We denote these H_5 and $H'_{\bar{5}}$.

Of course we also have the super Yang-Mills field V in the adjoint rep. of SU(5).

The most general SU(5) and SUSY invariant action made from these fields is given by

$$\Gamma^{SSU(5)} = \Gamma_{ym} + \Gamma_K + \Gamma_V \tag{4.1.1}$$

with

$$\Gamma_{ym} = \frac{Z}{g^2} \int dS \, Tr[W^\alpha W_\alpha] + \frac{Z}{g^2} \int d\bar{S} \, Tr[\bar{W}_{\dot\alpha} \bar{W}^{\dot\alpha}] \tag{4.1.2}$$

where

$$\begin{aligned} W_\alpha &= \bar{D}\bar{D}[e^{-gV^b_a T^a_b} D_\alpha e^{+gV^b_a T^a_b}] \\ \bar{W}_{\dot\alpha} &= DD[e^{+gV^b_a T^a_b} \bar{D}_{\dot\alpha} e^{-gV^b_a T^a_b}] \end{aligned} \tag{4.1.3}$$

and T^a_b is the adjoint representation matrix for SU(5)

$$(T^a_b)^{ce}_{df} = (T^a_b)^c_f \delta^e_d - (T^a_b)^e_d \delta^c_f \tag{4.1.4}$$

with

$$(T^a_b)^c_d = \delta^a_d \delta^c_b - \frac{1}{5} \delta^a_b \delta^c_d \tag{4.1.5}$$

the fundamental representation matrix. The kinetic energy action is

$$\begin{aligned} \Gamma_K = \int dV \{ & Z_{M5} M_{\bar{5}} e^{-gV^b_a T^a_b} \bar{M}_5 + Z_{M10} \bar{M}_{\overline{10}} e^{gV^b_a L^a_b} M_{10} \\ & + Z_\phi \bar{\phi}_{24} e^{gV^b_a T^a_b} \phi_{24} + Z_H \bar{H}_{\bar{5}} e^{gV^b_a T^a_b} H_5 + Z_{H'} H'_{\bar{5}} e^{-gV^b_a T^a_b} \bar{H}'_5 \} \end{aligned} \tag{4.1.6}$$

with the 10 dimensional representation matrix L^a_b given by

$$(L^a_b)^{cd}_{ef} = (T^a_b)^c_e \delta^d_f + (T^a_b)^d_f \delta^c_e \ . \qquad (4.1.7)$$

Let's recall the explicit SU(5) gauge transformations to check that this is indeed invariant. For the $\bar{5}$ matter multiplet we have

$$M'_{\bar{5}c} = (e^{-ig\Lambda^b_a T^a_b})^e_c \, M_{\bar{5}e} \qquad (4.1.8)$$

and

$$\bar{M}'^{d}_{5} = (e^{+ig\bar{\Lambda}^b_a T^a_b})^d_f \, \bar{M}^f_5 \ . \qquad (4.1.9)$$

The vector superfield V^b_a transforms according to

$$(e^{-gV^b_a T^a_b})^e_f = (e^{-ig\Lambda^b_a T^a_b})^e_c (e^{-gV'^b_a T^a_b})^c_d (e^{+ig\bar{\Lambda}^b_a T^a_b})^d_f \qquad (4.1.10)$$

so that

$$M'_{\bar{5}c} (e^{-gV'^b_a T^a_b})^c_d \, \bar{M}'^d_5 = M_{\bar{5}} \, e^{-gV^b_a T^a_b} \, \bar{M}_5 \qquad (4.1.11)$$

and it is gauge invariant.

(Recall $\bar{\Lambda}^a_b = \Lambda^{*b}_a$, $\bar{\Lambda} = \Lambda^\dagger$.) The remaining terms can be checked similarly, e.g.

$$\begin{aligned} \phi'_{24} &= e^{+ig\underline{\underline{\Lambda}}\cdot\underline{\underline{T}}} \phi_{24} \\ \bar{\phi}'_{24} &= e^{+ig\underline{\underline{\bar{\Lambda}}}\cdot\underline{\underline{T}}} \bar{\phi}_{24} \\ e^{g\underline{\underline{V}}\cdot\underline{\underline{T}}} &= e^{-ig\underline{\underline{\bar{\Lambda}}}\cdot\underline{\underline{T}}} e^{g\underline{\underline{V'}}\cdot\underline{\underline{T}}} e^{+ig\underline{\underline{\Lambda}}\cdot\underline{\underline{T}}} \quad \text{etc.} \end{aligned} \qquad (4.1.12)$$

Finally we have the pure chiral and anti-chiral action

$$\Gamma_V = \int dSL + \int d\bar{S}\bar{L} \quad \text{with} \quad \bar{L} = L^{\dagger} . \tag{4.1.13}$$

In order to list all the SU(5) invariants made from bilinear and trilinear products of $M_{\bar{5}}$, M_{10}, ϕ_{24}, H_5, $H'_{\bar{5}}$ recall the decomposition of the relevant SU(5) tensor products

$$\begin{aligned}
5 \times \bar{5} &= 1 + 24 \\
5 \times 5 &= 10 + 15 \\
\bar{5} \times \bar{5} &= \overline{10} + \overline{15} \\
\bar{5} \times 10 &= 5 + 45 \\
\bar{5} \times 24 &= \bar{5} + \overline{45} + \overline{70} \\
5 \times 10 &= \overline{10} + \overline{40} \\
5 \times 24 &= 5 + 45 + 70 \\
10 \times 24 &= 10 + 15 + 40 + 175 \\
24 \times 24 &= 1 + 24 + 24 + 75 + 126 + \overline{126} + 200 \\
10 \times 10 &= \bar{5} + \overline{45} + \overline{50} .
\end{aligned} \tag{4.1.14}$$

Thus we must list invariants; first products of two fields; these are just $5 \times \bar{5}$ and 24×24 since only they contain singlets.

$$\begin{aligned}
&M_{\bar{5}} H_5 \\
&H'_{\bar{5}} H_5 \\
&\phi_{24} \phi_{24}
\end{aligned} \tag{4.1.15}$$

Then we have products of three fields

$$\begin{array}{llll} \bar{5} & 5 & 24 & : \ M_{\bar{5}}H_5\phi_{24}\ ,\ H'_{\bar{5}}H_5\phi_{24} \\ \bar{5} & \bar{5} & 10 & : \ M_{\bar{5}}M_{\bar{5}}M_{10}\ ,\ M_{\bar{5}}H'_{\bar{5}}M_{10}\ ,\ H'_{\bar{5}}H'_{\bar{5}}M_{10} \\ 5 & 10 & 10 & : \ M_{10}M_{10}H_5 \\ 24^3 & & & : \ \phi_{24}^3 \ . \end{array} \tag{4.1.16}$$

In order to eliminate terms which mix H_5 and $M_{\bar{5}}$ we also choose Γ to be invariant under the discrete symmetry $M_{\bar{5}} \to -M_{\bar{5}}$; $M_{10} \to -M_{10}$ all other fields being invariant. Thus we find only

$$\begin{array}{l} H'_{\bar{5}}H_5, \phi_{24}\phi_{24} \\ H'_{\bar{5}}\phi_{24}H_5, M_{\bar{5}}M_{10}H'_{\bar{5}}, M_{10}M_{10}H_5, \phi_{24}^3, \end{array} \tag{4.1.17}$$

so

$$L = \frac{m}{2}\phi_{24}{}^a_b\,\phi_{24}{}^b_a + \mu H'_{\bar{5}a}H_5^a + \lambda_\phi \phi_{24}{}^a_b\,\phi_{24}{}^b_c\,\phi_{24}{}^c_a + \lambda_{\phi H}H'_{\bar{5}a}\phi_{24}{}^a_b\,H_5^b + \gamma_{mn}M_{\bar{5}ma}M_{10n}{}^{ab}H'_{\bar{5}b} + \Gamma_{mn}\epsilon_{abcde}M_{10m}{}^{ab}M_{10n}{}^{cd}H_5^e \ . \tag{4.1.18}$$

4.2. Spontaneous Symmetry Breaking of SSU(5)

We now desire a breaking scheme of the sort

$$\text{Super SU(5)} \xrightarrow[M_x \simeq 10^{16}\ \text{GeV}]{\phi_{24}} \text{Super SU(3)} \times \text{SU(2)} \times \text{U(1)}$$

$$\xrightarrow[M_s \simeq 1\ \text{TeV}]{\text{Explicit but soft SUSY breaking}} \text{SU(3)} \times \text{SU(2)} \times \text{U(1)} \xrightarrow[M_W \simeq 100\ \text{GeV}]{H_5, H'_{\bar{5}}} \tag{4.2.1}$$

$$\text{SU(3)} \times \text{U}_{em}(1) \ .$$

Since the breaking of SUSY will be soft; that is we will give explicit mass terms to the supersymmetric partners of the matter and gauge fields of the order of $M_s \approx 1$ TeV. This does not violate the higher energy no renormalization theorem of SUSY since the breaking is soft. In addition in order to catalyze the electroweak breaking a non-positive definite mass squared term for the Higgs fields is also added. All of these terms are SU(5) invariant. shiggs masses are not added, in order to keep the SUSY softly broken (sguage fields are acceptable!). Their mass is obtained from the supersymmetric breaking of SU(5) by a natural fine tuning. So we should also add to Γ an explicit SUSY breaking piece Γ^b.

$$\begin{aligned}\Gamma^b = & \frac{a}{2} M_s^2 \int dV \theta^2 \bar{\theta}^2 [(\bar{D}^2 D^\alpha V_b^a)(\bar{D}^2 D_\alpha V_a^b) + (D^2 \bar{D}_{\dot{\alpha}} V_b^a)(D^2 \bar{D}^{\dot{\alpha}} V_a^b)] \\ & + bM_s^2 \int dV \theta^2 \bar{\theta}^2 M_{\bar{5}} \bar{M}_5 + cM_s^2 \int dV \theta^2 \bar{\theta}^2 M_{10} \bar{M}_{\overline{10}} \\ & + dM_s^2 \int dV \theta^2 \bar{\theta}^2 H_5 M_H \bar{H}_{\bar{5}} + eM_s^2 \int dV \theta^2 \bar{\theta}^2 \bar{H}'_5 M_{H'} H'_{\bar{5}} \ .\end{aligned} \tag{4.2.2}$$

The first term is just a $\lambda^2 + \bar{\lambda}^2$ mass term while the rest are $AA^\dagger$ masses.

Let's first rewrite Γ in terms of SU(3) × SU(2) × U(1) fields and shift by the large ϕ_{24} vacuum value to check that SUSY is unbroken, but that SU(5) is broken down to SU(3) × SU(2) × U(1).

Recall the SU(3) × SU(2) × U(1) decomposition of the SU(5) 24, 10, 5, $\bar{5}$ from our previous work.

$$\phi_{24b}^{\ a} = \begin{bmatrix} \left(H_1^1 - \frac{2}{\sqrt{30}} H_B\right) & H_2^1 & H_3^1 & H_{\bar{x}}^1 & H_{\bar{y}}^1 \\ H_1^2 & \left(H_2^2 - \frac{2}{\sqrt{30}} H_B\right) & H_3^2 & H_{\bar{x}}^2 & H_{\bar{y}}^2 \\ H_1^3 & H_2^3 & \left(H_3^3 - \frac{2}{\sqrt{30}} H_B\right) & H_{\bar{x}}^3 & H_{\bar{y}}^3 \\ H_{x_1} & H_{x_2} & H_{x_3} & \left(\frac{H^0}{\sqrt{2}} + \frac{3H_B}{\sqrt{30}}\right) & H^+ \\ H_{y_1} & H_{y_2} & H_{y_3} & H^- & \left(-\frac{H^0}{\sqrt{2}} + \frac{3H_B}{\sqrt{30}}\right) \end{bmatrix}^{ab} \tag{4.2.3}$$

where H_B is a (1,1,0) and H^0 is the neutral field in a (1,3,0), and these are chiral superfields. So once again we give ϕ_{24} the vacuum values

$$<0|\phi_{24}|0> = \begin{bmatrix} v & & & & \\ & v & & 0 & \\ & & v & & \\ & 0 & & \left(-\frac{3}{2} - \frac{1}{2}\varepsilon\right)v & \\ & & & & \left(-\frac{3}{2} + \frac{1}{2}\varepsilon\right)v \end{bmatrix} \tag{4.2.4}$$

i.e.

$$<0|H_B|0> = -\frac{\sqrt{30}}{2} v$$

$$<0|H^0|0> = -\frac{1}{\sqrt{2}} \varepsilon v \ ; \quad \varepsilon <<< 1 \ . \tag{4.2.5}$$

The 5 of Higgs can be written as

$$H_5^a = \begin{bmatrix} H^1 \\ H^2 \\ H^3 \\ \phi^+ \\ \phi^o \end{bmatrix} \tag{4.2.6}$$

with H^i and ϕ_o^+ chiral superfields and $H^{1,2,3}$ being a $(3,1,-\frac{1}{3})$ and $\begin{pmatrix}\phi^+ \\ \phi^o\end{pmatrix}$ a $(1,2,+\frac{1}{2})$. Hence H_5 will have the vacuum value

$$\langle 0|H_5|0\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{v_o}{\sqrt{2}} \end{pmatrix} \qquad \text{i.e.} \quad \langle 0|\phi_o|0\rangle = \frac{1}{\sqrt{2}} v_o \qquad v_o \simeq M_W \quad . \tag{4.2.7}$$

Similarly the $\bar{5}$ of Higgs can be written as

$$H'_{\bar{5}a} = \begin{pmatrix} H'_1 \\ H'_2 \\ H'_3 \\ \phi^- \\ \phi'^{\,0} \end{pmatrix} \tag{4.2.8}$$

and again $H'_{1,2,3}$ is a $(\bar{3},1,+\frac{1}{3})$ and $\begin{pmatrix}\phi^- \\ \phi^o\end{pmatrix}$ a $(1,\bar{2},-\frac{1}{2})$. So $H'_{\bar{5}}$ will have the vacuum value

$$\langle 0|H'_{\bar{5}}|0\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{v'_o}{\sqrt{2}} \end{pmatrix} \qquad \text{i.e.} \quad \langle 0|\phi'_o|0\rangle = \frac{1}{\sqrt{2}} v'_o \qquad v'_o \simeq M_W \quad . \tag{4.2.9}$$

For energies large compared to M_s we can neglect the SUSY and electroweak breaking terms and ask if v can be determined so as to keep SUSY good; that is the linear F and D terms should vanish in the shifted action.

For the F terms we need only ask if the linear terms in L vanish. For the D term the linear term in L_K must vanish.

First the F terms, we must have

$$1)\qquad \alpha\delta^a_b + \left.\frac{\partial L}{\partial \phi_{24a}{}^b}\right|_{\substack{\langle\phi_{24}\rangle = v \\ \text{all other } \langle\rangle = 0}} = 0 = m\phi_{24b}{}^a + 3\lambda_\phi \phi_{24c}{}^a \phi_{24b}{}^c + \lambda_{\phi H} H'_{\bar{5}b} H^a_5 + \alpha\delta^a_b$$

$$2)\qquad \phi_{24a}{}^a = 0$$

$$3)\qquad \left.\frac{\partial L}{\partial H^a_5}\right|_{\substack{\langle\phi_{24}\rangle = v \\ \text{all other } \langle\rangle = 0}} = 0 = \mu H'_{\bar{5}a} + \lambda_{\phi H} H'_{\bar{5}b} \phi_{24a}{}^b + \Gamma_{mn} \varepsilon_{abcde} M_{10m}^{bc} M_{10n}^{de} \qquad (4.2.10)$$

$$4)\qquad \left.\frac{\partial L}{\partial H'_{\bar{5}a}}\right|_{\substack{\langle\phi_{24}\rangle = v \\ \text{all other } \langle\rangle = 0}} = 0 = \mu H^a_5 + \lambda_{\phi H} \phi_{24b}{}^a H^b_5 + \gamma_{mn} M_{\bar{5}mb} M_{10n}^{ba}$$

Since $M_{\bar{5}}$ and M_{10} are chosen to have zero vacuum values their linear terms are trivially zero since they appear bilinearly in Γ. Also for high energy ($> M_s$) we are setting $v_o = v'_o = \varepsilon = 0$. So $\langle H_5\rangle = \langle H'_{\bar{5}}\rangle = 0$, hence only equation 1 is non-trivial;

$$0 = m\langle\phi_{24b}{}^a\rangle + 3\lambda(\langle\phi_{24}\rangle\langle\phi_{24}\rangle)^a_b + \alpha\delta^a_b \qquad (4.2.11)$$

where the Lagrange multiplier α can be eliminated by taking the trace $\rightarrow$

$$\alpha = \frac{-3}{5}\lambda_\phi \mathrm{Tr}\,\phi_{24}\phi_{24}\ . \qquad (4.2.12)$$

So letting $\langle\phi_{24b}^{\ a}\rangle \equiv v_b^a$ we have

$$0 = mv_b^a + 3\lambda_\phi[v_c^a v_b^c - \frac{1}{5}\delta_b^a v_d^c v_c^d] \quad . \qquad (4.2.13)$$

For

$$v_b^a = \begin{bmatrix} v & & & & \\ & v & & 0 & \\ & & v & & \\ & 0 & & -\frac{3}{2}v & \\ & & & & -\frac{3}{2}v \end{bmatrix} \qquad (4.2.14)$$

this reduces to one equation

$$mv + 3\lambda_\phi[v^2 - \frac{1}{5}(\frac{15}{2}v^2)] = 0 \qquad (4.2.15)$$

with the simple solution

$$v = \frac{2}{3}\frac{m}{\lambda_\phi} \ . \qquad (4.2.16)$$

The linear D term is given by

$$\bar{\phi}_{24}\ \underline{\underline{V}}\cdot\underline{\underline{T}}\ \phi_{24} = v_a^b v_c^d (T_b^a)_{df}^{ce} v_e^f = 0 \ . \qquad (4.2.17)$$

Since T_b^a is antisymmetric under a (c,d)(e,f) interchange, this vanishes. Thus we have that at energies > M_s, the supersymmetric SU(5) theory is broken down to a supersymmetric SU(3) × SU(2) × U(1) theory. We can next ask which fields get a large mass as a result of the $v \sim M_x$ breaking. The mass terms in L become

$$L_{mass} = \frac{m}{2} \mathrm{Tr}\phi_{24}\phi_{24} + \mu H'_{\bar{5}}H_5 + 3\lambda_\phi \mathrm{Tr}\phi_{24}v\phi_{24} + \lambda_{\phi H} H'_{\bar{5}}vH_5 \ . \tag{4.2.18}$$

Recall that ϕ_{24} and $H_5, H'_{\bar{5}}$ can be expanded in terms of SU(3) × SU(2) × U(1) fields, so that

$$\mathrm{Tr}\phi_{24}\phi_{24} = \mathrm{Tr}HH + 2H_{\bar{x}}H_x + 2H_{\bar{y}}H_y + H_BH_B + 2H^+H^- + H^oH^o \tag{4.2.19}$$

and

$$\mathrm{Tr}\phi_{24}v\phi_{24} = v\{\mathrm{Tr}HH - \frac{1}{2} H_{\bar{x}}H_x - \frac{1}{2} H_{\bar{y}}H_y - \frac{1}{2} H_BH_B - 3H^+H^- - \frac{3}{2} H^oH^o\} \ . \tag{4.2.20}$$

Thus the ϕ_{24} masses become:

$$\begin{aligned}
\left(\frac{m}{2} + 3\lambda_\phi v\right) \mathrm{Tr}HH &= \frac{5}{2} m\mathrm{Tr}HH \\
\left(\frac{m}{2} - \frac{3}{2}\lambda_\phi v\right) H_BH_B &= -\frac{m}{2} H_BH_B \\
\left(m - 9\lambda_\phi v\right) H^+H^- &= -5mH^+H^- \\
\left(\frac{m}{2} - \frac{9}{2}\lambda_o v\right) H^oH^o &= -\frac{5}{2} mH^oH^o \\
\left(m - \frac{3}{2}\lambda_\phi v\right)[H_{\bar{x}}H_x + H_{\bar{y}}H_y] &= 0 \ .
\end{aligned} \tag{4.2.21}$$

As necessary since SU(5) is broken down to SU(3) × SU(2) × U(1) we have 24 - 12 = 12 broken generators (the $X, \bar{X}, Y, \bar{Y}$) and hence 12 zero mass Goldstone bosons $H_x, H_{\bar{x}}, H_y, H_{\bar{y}}$. Since SUSY is good these complete chiral superfields will be eaten by the X and Y gauge superfields according to the super Higgs mechanism turning these into massive vector superfields (recall the scalar Goldstone bosons are eaten by the ordinary gauge fields $X_\mu, \bar{X}_\mu, Y_\mu, \bar{Y}_\mu$ making them massive while the pseudo-scalar bosons and Weyl spinors in $H_{x,\bar{x},y,\bar{y}}$ become the new massive degrees of freedom in the now massive vector superfields). The remaining 12 Higgs super-mesons are extremely massive ($m \simeq M_x$) as required since H^a_b will lead to proton decay.

The 5 and $\bar{5}$ Higgs mass terms become

$$H'_{\bar{5}}H_5 \quad = \quad H'H + \phi^+\phi^- + \phi^o\phi'^o \tag{4.2.22}$$

and

$$H'_{\bar{5}}vH_5 \quad = \quad vH'H - \frac{3}{2}v\ (\phi^+\phi^- + \phi^o\phi'^o)\quad , \tag{4.2.23}$$

so that the masses are given by

$$(\mu+\lambda_{\phi H}v)H'H$$

$$(\mu - \frac{3}{2}\lambda_{\phi H}v)(\phi^+\phi^- + \phi^o\phi'^o)\ . \tag{4.2.24}$$

Since H and H' carry color and couple directly to the matter fields they will be (predominantly) responsible for proton decay hence we have $(\mu+\lambda_{\phi H}v)$ the order of M_x. On the other hand ϕ^o, ϕ'^o will be responsible for electro-weak breaking and should have a small mass (0 at this scale of energy > M_s) and so we must "fine tune" the parameters of our model so that $\mu = \frac{3}{2}\lambda_{\phi H}v$. This tuning is technically natural since SUSY is a good symmetry at these energies and the "no-renormalization" theorems imply that there are no radiative corrections to this relation and hence it stays tuned.

As we lower the energy the M_s explicit SUSY breaking mass terms will become important. These will serve to

1) give the s-matter fields a higher mass than the matter fields
2) give the s-gauge fields a mass.
3) give the weak Higgs bosons the correct negative mass squared to catalyze the spontaneous breakdown of the electroweak group; in this sense M_W is determined by M_s; the order of SUSY breaking.

Once SUSY is broken (at energies $< M_s$) the SUSY no-renormalization theorems no longer hold and there are low energy radiative corrections to the effective potential, i.e. masses now receive $O(\alpha M_s)$ corrections. In order not to violate the naturalness of our fine tuning and gauge hierarchy this αM_s should be $O(M_W)$; hence the reason for choosing $M_s \simeq$ 1-100 TeV.

As usual the low energy masses are given in terms of v_o and v_o' as previously (and ε giving the full potential minimum).

Finally let's estimate the proton decay and $\sin^2\theta_W$ and m_b/m_τ ratios predicted in this SUSYGUT.

4.3. Unification Mass, $\sin^2\theta_W$, m_b/m_τ.

Since SUSY puts fermions and bosons on equal footing we will need the RGE β function eq.(1.2.94) with scalar field loops included; it is

$$\beta = -\frac{g^3}{32\pi^2}\left[\frac{11}{3} C_2(G) - \frac{4}{3} T_F - \frac{1}{3} T_s\right] \qquad (4.3.1)$$

where recall $C_2(G)$ is the quadratic Casimir operator ($C_2 = N$ for SU(N)).

$$T_F \delta_{ij} = \frac{1}{2} \mathrm{Tr}(T^i T^j) \qquad (4.3.2)$$

where we sum over all fermi representations (if Dirac fermions sum over both left and right handed representations, while if Weyl fermions only count ψ_α or $\bar{\psi}_{\dot{\alpha}}$ once, not both (these are Majorana fields)). Similarly

$$T_s \delta_{ij} = Tr(T^i T^j) \tag{4.3.3}$$

where we sum over all scalar field representations. For each super Y-M field we have an ordinary Y-M field in the adjoint rep. as well as a corresponding susy partner Weyl gaugino in the adjoint rep.. These contribute to β $[\frac{11}{3} C_2(G) - \frac{4}{3}\frac{1}{2} C_2(G)] = 3C_2(G)$ where $T_F = \frac{1}{2} C_2(G)$ for the adjoint rep. Weyl fermions. For each chiral superfield we have a Weyl fermion and a complex scalar in the same rep., therefore for each super matter and super Higgs multiplet we find a contribution to β of

$$[-\frac{4}{3}\frac{1}{2} T_s - \frac{1}{3} T_s] = -T_s = -\frac{1}{d} Tr[T^i T^i]$$

where d = the dimension of the group and the trace is summed over the chiral superfields (i.e. for quarks and leptons this is twice the number of flavors (=4F)). So we have for supersymmetric theories

$$\beta = -\frac{g^3}{32\pi^2}[3C_2(G) - \sum_{\substack{\text{chiral}\\ \text{super-}\\ \text{fields}}} \frac{1}{d} Tr[T^i T^i]$$

$$= -\frac{g^3}{32\pi^2}[3C_2(G) - \sum_{\substack{\text{chiral}\\ \text{super-}\\ \text{fields}}} T_s] \quad . \tag{4.3.4}$$

We would like to apply this formula to find the supersymmetric SU(3) x SU(2) x U(1) running coupling constants $\alpha_i(Q^2)$ and hence obtain $\sin^2\bar{\theta}_W$; M_X and m_b/m_τ. Recall eq.(2.2.11) for the running coupling constants

$$\frac{1}{\alpha_i(Q^2)} - \frac{1}{\alpha_i(M_x^2)} = -8\pi b_i \ln \frac{Q^2}{M_x^2} \tag{4.3.5}$$

where $\beta_i = b_i g_i^3$ for $i = 1,2,3$ and $\alpha_i(Q^2) = \frac{\bar{g}_i^{\,2}(\lambda^2)}{4\pi}$ with $Q^2 = \lambda^2 M_x^2$.

From above we have

$$b_3 = -\frac{1}{32\pi^2}[9-2F]$$

$$b_2 = -\frac{1}{32\pi^2}\left[6 - 2F - \frac{H}{2}\right]$$ (> 0 for F = 3 α_2 increases for SUSY theories!)

$$b_1 = +\frac{1}{32\pi^2}\sum_{\substack{\text{chiral}\\ \text{super-}\\ \text{fields}}} Y^2 = \frac{1}{32\pi^2}\left[\frac{10}{3}F + \frac{1}{2}H\right] \tag{4.3.6}$$

where F is the number of families and H the number of Higgs chiral superfield SU(2) doublets. Note that the SUSY slopes b_i are less than the ordinary slopes, b_{o_i},

$$b_{o_3} = -\frac{1}{32\pi^2}\left[11 - \frac{4}{3}F\right]$$

$$b_{o_2} = -\frac{1}{32\pi^2}\left[\frac{22}{3} - \frac{4}{3}F - \frac{1}{6}H\right]$$

$$b_{o_1} = +\frac{1}{32\pi^2}\left[\frac{20}{9}F + \frac{1}{6}H\right] , \tag{4.3.7}$$

so the α_i run more slowly in the SUSY case

$$\alpha_i(Q^2) = \frac{\alpha_i(M_x^2)}{1 - 8\pi\alpha_i(M_x^2)b_i \ln \frac{Q^2}{M_x^2}} .$$

This coupled with α_2 not being asymptotically free will yield a larger M_x.

In particular recall we have unification at $Q^2 \geq M_x^2$ with

$$g = g_2 = g_3 = \sqrt{5/3}\, g_1 \quad \text{and}$$

$$\sin^2\theta_w = \frac{g_1^2}{g_1^2 + g_2^2} = \frac{3}{8}$$

$$\frac{\alpha_{QED}}{\alpha_s} = \frac{g_2^2 \sin^2\theta_w}{g_3^2} = \frac{3}{8} \quad . \tag{4.3.8}$$

At lower energies these parameters run according to the general formulae (2.2.12)

$$\sin^2\bar{\theta}_w = \frac{\bar{\alpha}_1}{\bar{\alpha}_1 + \bar{\alpha}_2}$$

$$= \frac{3}{8}[1 + (\frac{g^2}{4\pi})5\pi(\frac{3}{5}\, b_1 - b_2)\ln \frac{Q^2}{M_x^2}] \tag{4.3.9}$$

and (2.2.16)

$$\frac{\bar{\alpha}_{QED}}{\bar{\alpha}_s} = \frac{\bar{\alpha}_2}{\bar{\alpha}_3} \sin^2\bar{\theta}_w$$

$$= \frac{3}{8}[1 + (\frac{g^2}{4\pi})\pi(3b_1 + 3b_2 - 8b_3)\ln \frac{Q^2}{M_x^2}] \quad . \tag{4.3.10}$$

These, along with (4.3.5), yield

$$\sin^2\bar{\theta}_w = \frac{1}{[3b_1 + 3b_2 - 8b_3]}\, [3b_2 - 3b_3 + (3b_1 - 5b_2)\, \frac{\bar{\alpha}_{QED}}{\bar{\alpha}_s}]$$

$$\pi \ln \frac{M_x^2}{Q^2} = \frac{1}{[3b_1 + 3b_2 - 8b_3]}\, [\frac{3}{8}\, \frac{1}{\bar{\alpha}_{QED}} - \frac{1}{\bar{\alpha}_s}]$$

$$\frac{g^2}{4\pi} = \frac{1}{[3b_1 + 3b_2 - 8b_3]} [(3b_1 + 3b_2)\bar{\alpha}_s - \frac{64}{3} b_3 \bar{\alpha}_{QED}]. \qquad (4.3.11)$$

The strong, $\bar{\alpha}_s$, and electromagnetic, $\bar{\alpha}_{QED}$, running fine structure constants are known experimentally at $Q^2 = 10\text{GeV}^2$ as

$$\bar{\alpha}_{QED} \approx \bar{\alpha}_{QED}(0) \simeq \frac{1}{137}$$

and

$$\frac{\bar{\alpha}_{QED}}{\bar{\alpha}_s} \simeq .04 \quad .$$

Let's calculate $\sin^2\bar{\theta}_W$, M_X, $\frac{g^2}{4\pi}$ in our ordinary SU(5) and SUSY SU(5) GUTS. Recall we have 3 families in each case, F = 3, but in the ordinary SU(5) GUT we only have one light Higgs doublet, H = 1, which gives the quarks and leptons their masses. However in the SU(5) SUSYGUT we needed two light Higgs doublets, H = 2, in order to make the necessary Yukawa coupling and hence mass terms since we have only tri-linear pure chiral superfield interactions in the supersymmetric Lagrangian; these light doublets were contained in the SUSY SU(5) fields H_5, $H'_{\bar{5}}$.

For the ordinary SU(5) GUT we have

$$\sin^2\bar{\theta}_W = [\frac{1}{6} + \frac{1}{132} H + (\frac{5}{9} - \frac{1}{198} H) \frac{\bar{\alpha}_{QED}}{\bar{\alpha}_s}]$$

$$\ln \frac{M_X^2}{Q^2} = \frac{16\pi}{33} \frac{1}{\bar{\alpha}_{QED}} [\frac{3}{8} - \frac{\bar{\alpha}_{QED}}{\bar{\alpha}_s}]$$

$$\frac{g^2}{4\pi} = \bar{\alpha}_{QED} [\frac{64}{3} (\frac{1}{3} - \frac{4}{99} F) + (- \frac{1}{3} + \frac{16}{99} F + \frac{1}{66} H) \frac{\bar{\alpha}_s}{\bar{\alpha}_{QED}}] . \qquad (4.3.12)$$

Putting in the numbers we find

$$\sin^2\bar{\theta}_w = 0.20$$
$$M_x = 4 \times 10^{15}\ \text{GeV} \qquad (4.3.13)$$
$$\frac{g^2}{4\pi} = \frac{1}{15}$$

for the ordinary Georgi-Glashow SU(5) model.

For the SUSY SU(5) model we have

$$\sin^2\bar{\theta}_w = \frac{1}{6 + \frac{1}{3} H} \left[1 + \frac{1}{6} H + \left(5 - \frac{H}{6}\right) \frac{\bar{\alpha}_{QED}}{\bar{\alpha}_s}\right]$$

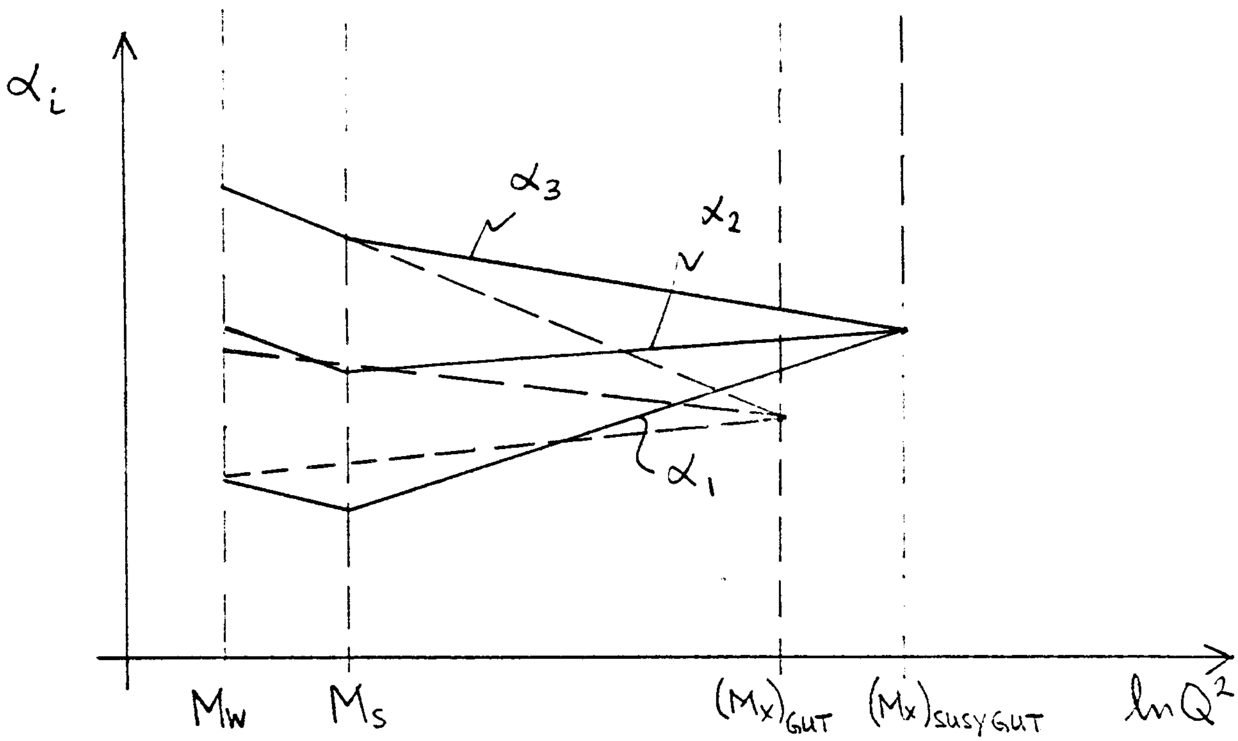
$$\ell n \frac{M_x^2}{Q^2} = \frac{16\pi}{27 + \frac{3}{2} H} \frac{1}{\bar{\alpha}_{QED}} \left[\frac{3}{8} - \frac{\bar{\alpha}_{QED}}{\bar{\alpha}_s}\right]$$

$$\frac{g^2}{4\pi} = \frac{1}{6 + \frac{1}{3} H} \bar{\alpha}_{QED} \left[\frac{64}{3}\left(1 - \frac{2}{9} F\right) + \left(-2 + \frac{16}{9} F + \frac{1}{3} H\right) \frac{\bar{\alpha}_s}{\bar{\alpha}_{QED}}\right] . \qquad (4.3.14)$$

Putting in the numbers we find

$$\sin^2\bar{\theta}_w = 0.23$$
$$M_x = 2 \times 10^{17}\ \text{GeV}$$
$$\frac{g^2}{4\pi} = \frac{1}{9} . \qquad (4.3.15)$$

As expected the SUSY quantities are all a bit larger. Since the proton lifetime τ_p is very sensitive to the value of M_x; $\tau_p \sim M_x^4$, this increase of $M_x \sim 10^{17}$ GeV will result in $\tau_p \gtrsim 10^{38}$ yrs., totally undetectable. However, as we will see, $\tau_p \sim M_x^4$ for direct X,Y boson exchange graphs; for SUSY theories the predominant decay mode will be through Higgs exchange which can result in a $\tau_p \sim M_x^2$ (supression factors). We can pictorially represent the running coupling constants as



The RGE analysis can also be applied to the fermion masses (see e.g. M.B. Einhorn and D.R.T. Jones, Nucl. Phys. B196 (1982) 475) to find for the SUSY GUT case

$$\frac{m_b}{m_\tau} = \left[\frac{\bar{\alpha}_s(2m_b)}{\bar{\alpha}_s(m_t)}\right]^{12/33} \left[\frac{\bar{\alpha}_s(m_t)}{\bar{\alpha}_s(m_w)}\right]^{4/7} \left[\frac{\bar{\alpha}_s(m_w)}{g^2/4\pi}\right]^{8/9}$$

while for the ordinary GUT case we had

$$\frac{m_b}{m_\tau} = \left[\frac{\bar{\alpha}_s(2m_b)}{\bar{\alpha}_s(m_t)}\right]^{12/23} \left[\frac{\bar{\alpha}_s(m_t)}{g^2/4\pi}\right]^{4/7} .$$

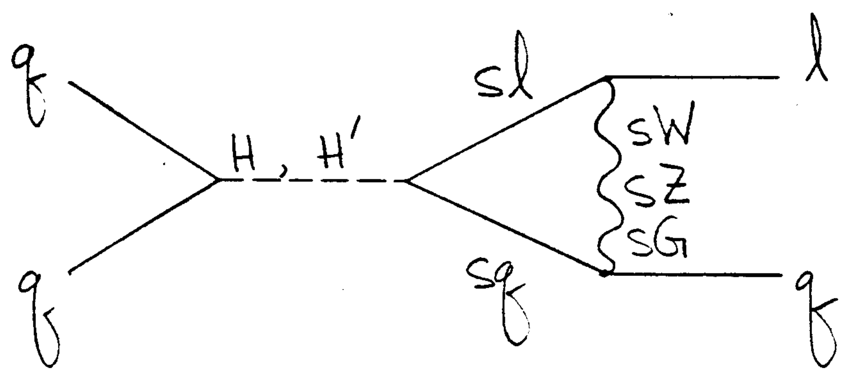
These yield

$$\left(\frac{m_b}{m_\tau}\right)_{SUSYGUT} = 1.09 \left(\frac{m_b}{m_\tau}\right)_{GUT} , \qquad (4.3.16)$$

not much change.

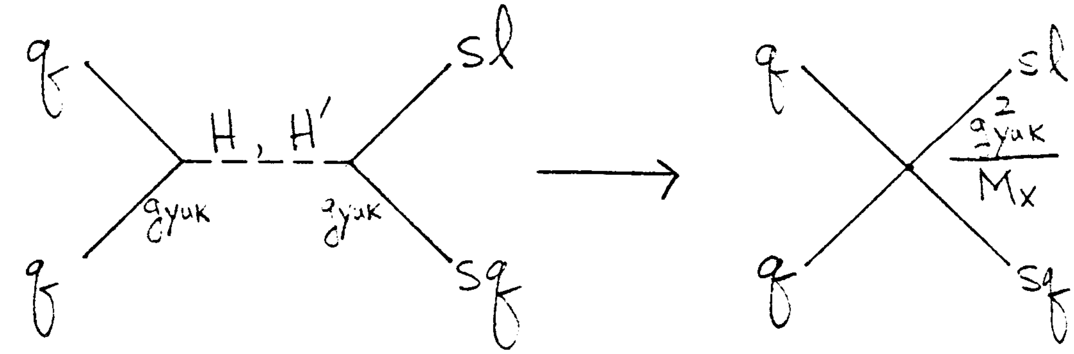
4.4. Proton Decay

Finally we would like to estimate the proton lifetime within the SUSYGUT. SUSY requires the existence of scalar partners for the quarks and leptons and fermion partners for the Yang-Mills fields; we can use these fields to allow the proton to decay through dimension 5 operators. Consider a graph of the form



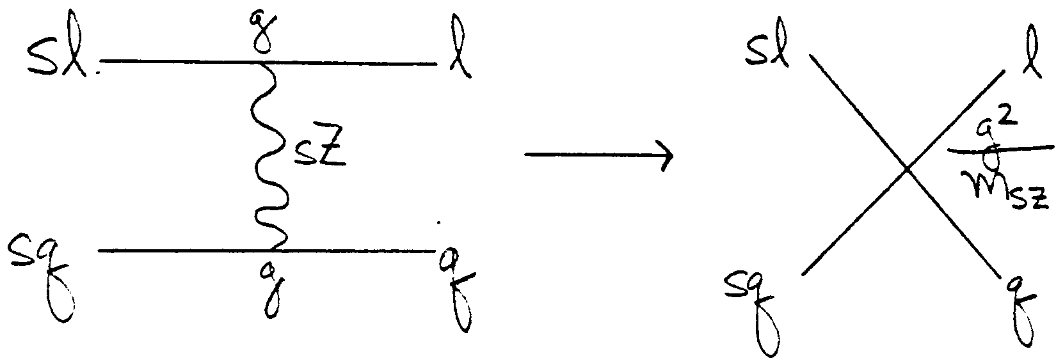
(4.4.1)

The matter superfields $M_{\bar{5}}$, M_{10} have a Yukawa coupling directly to the chiral Higgs superfields H_5, $H'_{\bar{5}}$. The exchange of colorful Higgs fields $H^{1,2,3}$ and $H'_{1,2,3}$ results in baryon number changing processes with effective dimension 5 operators:



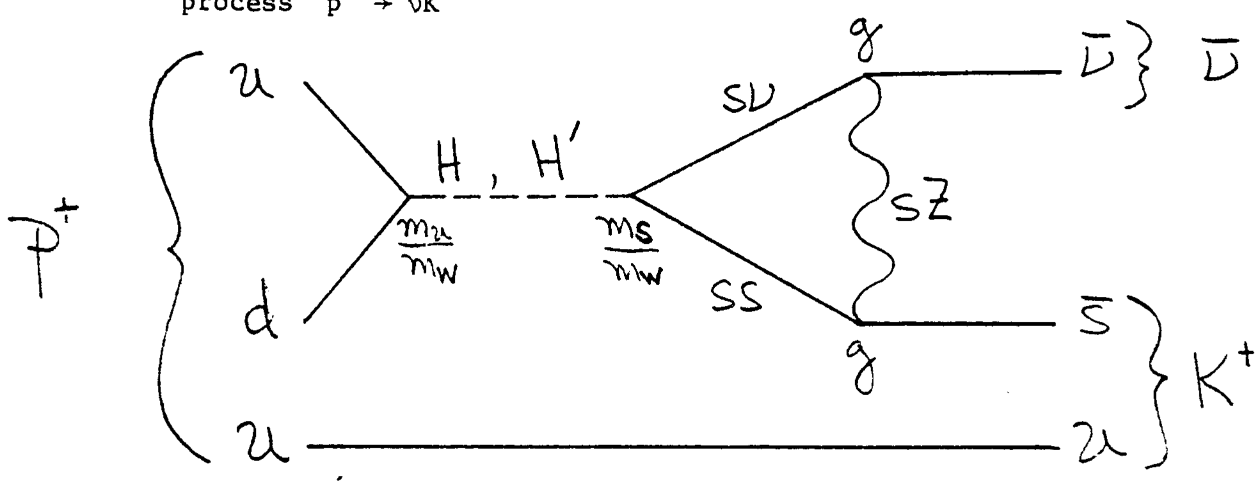
(4.4.2)

where the Yukawa couplings will be related to the masses of the different flavor quarks meeting at the vertex divided by the weak interaction scale, m_W. The scalar quarks, sq, and scalar leptons, sℓ, can then interact via a super gauge Yukawa coupling with the strength of the SUSYGUT coupling constant, g, with their fermionic quark and lepton partners and the fermionic partners to the Yang-Mills fields



(4.4.3)

This gives another effective dimension 5 operator with mass scale set by the mass of the exchanged fermion, m_{sz} in the case above, which goes like the SUSY breaking scale. Putting this together we find, for example, the proton decay process $p^+ \to \bar{\nu}K^+$



(4.4.4)

This amplitude goes like

$$\frac{g^2}{M_x m_{sz}} \left(\frac{m_s m_u}{m_w^2}\right) \quad . \tag{4.4.5}$$

Hence the proton lifetime can be guestimated to be

$$\tau_p^2 \sim \frac{1}{\alpha^2_{SUSYGUT}} \frac{M^2_{SUSYGUT}\, m^2_{sz}}{m_p^5} \left(\frac{m_w^2}{m_s m_u}\right)^2 . \tag{4.4.6}$$

Comparing this to our ordinary SU(5) GUT proton lifetime we find

$$(\tau_p)_{\substack{SUSY-\\GUT}} \sim \left(\frac{\alpha_{GUT}}{\alpha_{SUSYGUT}}\right)^2 \left(\frac{M_{SUSYGUT} m_{sz}}{M^2_{GUT}}\right)^2 \left(\frac{m_w^2}{m_s m_u}\right)^2 (\tau_p)_{GUT} \quad . \tag{4.4.7}$$

With

$$m_{sz} \sim 1 \text{ TeV} \qquad m_u = 10^{-2}\, m_s ,$$
$$m_w \sim 10^2 \text{ GeV} \qquad m_s = 10^{-1} \text{ GeV}$$

and $M_{SUSYGUT} = 5 \times 10^{17}$ GeV and $M_{GUT} = 3 \times 10^{14}$ GeV this yields

$$(\tau_p)_{\substack{SUSY-\\GUT}} \sim \left(\frac{9}{15}\right)^2 \left(\frac{5 \times 10^{17}\ 10^3}{(3 \times 10^{14})^2}\right)^2 \left(\frac{10^4}{10^{-4}}\right)^2 (\tau_p)_{GUT} \sim (\tau_p)_{GUT} . \tag{4.4.8}$$

The proton lifetime in SUSYGUTS is about the same as that in ordinary GUTS even though baryon number is changed by dimension 5 operators in the former and dimension 6 in the latter. The branching ratios of the various decay modes, however, will be quite different. SUSYGUTS yielding strange meson decay modes as dominant over the pion decay modes, opposite that of the ordinary GUTS. For further details and additional applications of SUSY in GUTS (and beyond) see for example ref. (5a) J. Ellis, CERN Preprint TH-3174.