Neutrino Masses

Let's recall the different types of fermion mass terms:

Recall our various representations for spinors

Dirac: $\Psi_D = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}$ in Weyl Rep. $\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$

Charge Conjugation $\Psi^c \equiv C \bar{\Psi}_D^T = \begin{bmatrix} \chi \\ \bar{\psi} \end{bmatrix}$

$\bar{\Psi}^c \equiv \Psi^{c\dagger}\gamma^0 = -\Psi_D^T C^{-1} = \boxed{\psi \;\; \bar{\chi}}$

$\gamma_5 = +i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{bmatrix} -1 & 0 \\ 0 & +1 \end{bmatrix}$; $\gamma_5^2 = 1$

where $C^{-1}\gamma_\mu C = -\gamma_\mu^T$

$$C = -C^{-1} = -C^\dagger = -C^T = i\gamma^2\gamma^0 = \begin{bmatrix} i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{bmatrix}$$

Helicity Projections:

$$\Psi_L = \tfrac{1}{2}(1-\gamma_5)\Psi_D = \begin{bmatrix} \psi \\ 0 \end{bmatrix} \; ; \; \bar{\Psi}_L \equiv \Psi_L^\dagger\gamma^0 = \boxed{0 \;\; \bar{\psi}}$$

$$\Psi_R = \tfrac{1}{2}(1+\gamma_5)\Psi_D = \begin{bmatrix} 0 \\ \bar{\chi} \end{bmatrix} \; ; \; \bar{\Psi}_R \equiv \Psi_R^\dagger\gamma^0 = \boxed{\chi \;\; 0}$$

$$\Psi^c_L \equiv \tfrac{1}{2}(1-\gamma_5)\Psi^c = \begin{bmatrix} \chi \\ 0 \end{bmatrix} \quad ; \quad \Psi^c_R = \tfrac{1}{2}(1+\gamma_5)\Psi^c = \begin{bmatrix} 0 \\ \bar{\psi} \end{bmatrix}$$

$$\overline{\Psi^c_L} \equiv \left((\Psi^c)_L\right)^\dagger \gamma^0 = \begin{bmatrix} 0 & \bar{\chi} \end{bmatrix} \quad ; \quad \overline{\Psi^c_R} = \begin{bmatrix} \psi & 0 \end{bmatrix}$$

Hence $\Psi^c_{\substack{L \\ R}} = C\,\overline{\Psi}^T_{\substack{R \\ L}}$; $\overline{\Psi^c_{\substack{L \\ R}}} = -\Psi^T_{\substack{R \\ L}} C^{-1}$

&

$$\Psi^c_{\substack{L \\ R}} = C\left(\overline{\Psi^c_{\substack{L \\ R}}}\right)^T ; \quad \overline{\Psi}_{\substack{R \\ L}} = \Psi^{c\,T}_{\substack{L \\ R}} C$$

Majorana Fermion

$$\begin{array}{ccccc} \Psi_M & \equiv & \Psi^c_M & = & C\,\overline{\Psi}^T_M \\ \| & & & & \| \\ \begin{bmatrix} \psi \\ \bar{\chi} \end{bmatrix} & & & & \begin{bmatrix} \chi \\ \bar{\psi} \end{bmatrix} \end{array}$$

$$\Rightarrow \psi = \chi \; ; \; \bar{\psi} = \bar{\chi}$$

So $\Psi_M = \begin{bmatrix} \psi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{bmatrix}$

$$\overline{\Psi}_M = \Psi_M^\dagger \gamma^0 = \begin{bmatrix} \psi^\alpha & \bar{\psi}_{\dot{\alpha}} \end{bmatrix}$$

Lorentz Invariant Mass Terms

1) Dirac Mass Terms: Connect L-R components of same field.

$$\mathcal{L}_D = m_D \overline{\Psi}_D \Psi_D = m_D \left[\overline{\Psi}_L \Psi_R + \overline{\Psi}_R \Psi_L \right]$$

The mass eigenfield is $\Psi_D = \Psi_L + \Psi_R$.

Equivalently

$$\mathcal{L}_D = m_D \left[\psi^\alpha \chi_\alpha + \overline{\psi}_{\dot\alpha} \overline{\chi}^{\dot\alpha} \right]$$

$$= m_D \left[\overline{\Psi^c_R} \, \Psi^c_L + \overline{\Psi^c_L} \, \Psi^c_R \right]$$

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2) Majorana Mass Terms: Connect L-R of charge conjugate fields

$$\mathcal{L}_{M-} = \tfrac{1}{2} m_- \left(\psi^\alpha \psi_\alpha + \overline{\psi}_{\dot\alpha} \overline{\psi}^{\dot\alpha} \right)$$

$$\mathcal{L}_{M+} = \tfrac{1}{2} m_+ \left(\chi^\alpha \chi_\alpha + \overline{\chi}_{\dot\alpha} \overline{\chi}^{\dot\alpha} \right)$$

The 2 mass eigenfields are ψ and χ.

Equivalently:

$$\mathcal{L}_{M-} = \frac{1}{2} m_- \overline{\Psi}_M \Psi_M \quad \text{with } \Psi_M = \begin{bmatrix} \psi_\alpha \\ \bar{\psi}^{\dot\alpha} \end{bmatrix}$$

$$\mathcal{L}_{M+} = \frac{1}{2} m_+ \overline{X}_M X_M \quad \text{with } X_M = \begin{bmatrix} \chi_\alpha \\ \bar{\chi}^{\dot\alpha} \end{bmatrix}.$$

Ψ_M & X_M are the mass eigenfields.

Note: $\Psi_M = \Psi_L + \Psi_R^c \quad ; \quad \Psi_M^c = \Psi_M$

$X_M = \Psi_L^c + \Psi_R \quad ; \quad X_M^c = X_M$

(i.e. $(\Psi_R^c)^c = \Psi_L$, etc.)

Inverting

$$\Psi_L = \tfrac{1}{2}(1-\gamma_5)\Psi_M$$

$$\Psi_R = \tfrac{1}{2}(1+\gamma_5) X_M$$

$$\Psi_R^c = \tfrac{1}{2}(1+\gamma_5)\Psi_M$$

$$\Psi_L^c = \tfrac{1}{2}(1-\gamma_5) X_M$$

Note $\Psi_D = \Psi_L + \Psi_R = \frac{1}{2}(1-\gamma_5)\Psi_M + \frac{1}{2}(1+\gamma_5) X_M$

So $\overline{\Psi}_D = \overline{\Psi}_M \gamma_+ + \overline{X}_M \gamma_-$

Hence we also have

$$\mathcal{L}_{M-} = \frac{1}{2} m_- \left[\overline{\Psi^c_R}\, \Psi_L + \overline{\Psi_L}\, \Psi^c_R \right]$$

$$\mathcal{L}_{M+} = \frac{1}{2} m_+ \left[\overline{\Psi_R}\, \Psi^c_L + \overline{\Psi^c_L}\, \Psi_R \right]$$

Note for Dirac masses we need 2-Majorana-Weyl fields. A single left-handed field can have a Majorana mass.

$$\gamma_5 \tfrac{1}{2}(1 \pm \gamma_5) = \pm \tfrac{1}{2}(1 \pm \gamma_5) \quad \text{i.e. } \gamma_5 \gamma_\pm = \pm \gamma_\pm$$

Finally

$$\Psi_D' \equiv \gamma_5 \Psi_D = \gamma_5 (\Psi_L + \Psi_R) = (-\Psi_L + \Psi_R)$$

$$\Psi_M' \equiv \gamma_5 \Psi_M = -\Psi_L + \Psi^c_R$$

$$X_M' \equiv \gamma_5 X_M = -\Psi^c_L + \Psi_R$$

Hence

$$\mathcal{L}_D = m_D \overline{\Psi_D} \Psi_D = -m_D \overline{\Psi_D'}\, \Psi_D'$$

$$\mathcal{L}_{M-} = \tfrac{1}{2} m_- \overline{\Psi_M} \Psi_M = -\tfrac{1}{2} m_- \overline{\Psi_M'}\, \Psi_M'$$

$$\mathcal{L}_{M+} = \tfrac{1}{2} m_+ \overline{X_M} X_M = -\tfrac{1}{2} m_+ \overline{X_M'}\, X_M'$$

The Ψ_D', Ψ_M', Σ_M' fields are interpreted as the correct mass eigenfields for the minus values of fermion masses.

When both Dirac & Majorana mass terms are present we have

$$\mathcal{L}_{DM} = m_D \bar{\Psi}_D \Psi_D + \tfrac{1}{2} m_- \left[\bar{\Psi}^c_R \Psi_L + \bar{\Psi}_L \Psi^c_R \right]$$
$$+ \tfrac{1}{2} m_+ \left[\bar{\Psi}_R \Psi^c_L + \bar{\Psi}^c_L \Psi_R \right]$$

$$= \tfrac{1}{2} m_D \left[\bar{\Psi}_M \Sigma_M + \bar{\Sigma}_M \Psi_M \right]$$
$$+ \tfrac{1}{2} m_- \bar{\Psi}_M \Psi_M + \tfrac{1}{2} m_+ \bar{\Sigma}_M \Sigma_M$$

$$= \tfrac{1}{2} \left[\bar{\Psi}_M \ \bar{\Sigma}_M \right] \begin{bmatrix} m_- & m_D \\ m_D & m_+ \end{bmatrix} \begin{bmatrix} \Psi_M \\ \Sigma_M \end{bmatrix}$$

The mass matrix $M = \begin{bmatrix} m_- & m_D \\ m_D & m_+ \end{bmatrix}$

Where we used $\overline{\Psi}_L = \overline{\Psi}_M \gamma_+$, etc.

$$\overline{\Psi}_D \Psi_D = \frac{1}{2}\left[\overline{\Psi}_L \Psi_R + \overline{\Psi}_R \Psi_L\right]$$

$$+ \frac{1}{2}\left[\overline{\Psi}^c_R \Psi^c_L + \overline{\Psi}^c_L \Psi^c_R\right]$$

$$= \frac{1}{2}\left[\overline{\Psi}_M \overline{X}_M + \overline{X}_M \Psi_M\right]$$

Check (p.-240-)

$$= \frac{1}{2}\left[\overline{\Psi}_M \gamma_+ \gamma_+ X_M + \overline{X}_M \gamma_- \gamma_- \Psi_M\right]$$

$$+ \frac{1}{2}\left[\overline{\Psi}_M \gamma_- \gamma_- X_M + \overline{X}_M \gamma_+ \gamma_+ \Psi_M\right]$$

$$\checkmark = \frac{1}{2}\left[\overline{\Psi}_M X_M + \overline{X}_M \Psi_M\right] .$$

has 2 mass eigenvalues:

$$\boxed{M_{\pm} = \frac{1}{2}\left[(m_+ + m_-) \pm \sqrt{(m_+ - m_-)^2 + 4m_D^2}\right]}$$

The mass eigenfields are given by

$$\begin{pmatrix} N_- \\ N_+ \end{pmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \Psi_M \\ X_M \end{bmatrix}$$

with

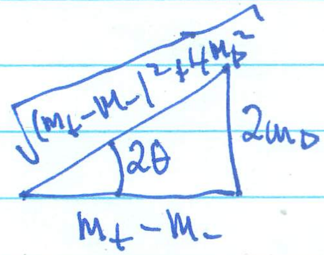
$$\tan 2\theta = \frac{2m_D}{m_+ - m_-}$$

Inverting the above formulae we have

$$M_+ + M_- = m_+ + m_- \; ; \; M_+ - M_- = \sqrt{(m_+ - m_-)^2 + 4m_D^2}$$

$$M_+ M_- = m_+ m_- - m_D^2$$

&



$$\sin 2\theta = \frac{2m_D}{\sqrt{(m_+ - m_-)^2 + 4m_D^2}}$$

$$\Rightarrow \boxed{m_D = \frac{1}{2}(M_+ - M_-)\sin 2\theta}$$

$$m_+ - m_- = 2m_D \cot 2\theta = (M_+ - M_-)\cos 2\theta$$
$$m_+ + m_- = M_+ + M_-$$

$$\Rightarrow \quad m_+ = \tfrac{1}{2}\left[M_+(1+\cos 2\theta) + M_-(1-\cos 2\theta)\right]$$

$$\boxed{m_+ = M_+\cos^2\theta + M_-\sin^2\theta}$$

$$\boxed{m_- = M_+\sin^2\theta + M_-\cos^2\theta}$$

Hence the most general Dirac & Majorana mass term for a 4 component spinor (Dirac) describes 2 Majorana particles with distinct masses.

A Dirac fermion corresponds to the degenerate mass limit $m_+ = 0 = m_-$ of the general 2 Majorana field case. Also if $m_+, m_- \neq 0$ the mass term violates any additive quantum number symmetry that the field has ex. electric charge since ex. $\overline{\Psi^c_R}\Psi_L$ is not invariant since Ψ^c has the opposite charge as Ψ, then $\overline{\Psi^c}$ has the same charge!

In the case of neutrino's, Majorana masses violate lepton number.

So the neutrino of the SM is a left-handed field only — as part of the lepton SU(2) doublet

$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \text{for each generation.}$$

This is a $(2, -\frac{1}{2})$ under $SU(2) \times U(1)$. The right handed fields are all singlets:

$$e_R \quad (1, -1) \quad \text{or equivalently}$$

e^c_L is a $(1, +1)$.

The SM Higgs field $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ is a $(2, +\frac{1}{2})$ while $\tilde{\phi} = i\sigma^2 \phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$ is a $(2, -\frac{1}{2})$.

$$\left(\phi^\dagger \sim (\bar{2}, -\tfrac{1}{2}) \right)$$

The only potential bilinears we can make are

$$\bar{l}_L e_R \sim (2, +\tfrac{1}{2}) \times (1, -1) = (2, -\tfrac{1}{2})$$

$$\overline{l^c_R} l_L \sim (2, -\tfrac{1}{2}) \times (2, -\tfrac{1}{2}) = (1, -1) + (3, -1)$$

$$\overline{e^c_L} e_R \sim (1, -1) \times (1, -1) = (1, -2)$$

Given the Higgs $\phi \sim (2, +\frac{1}{2})$ only $\bar{l}_L \phi e_R + h.c.$ is present in the SM, as we have seen, which leaves the neutrino massless.

There are 3 ways to give the neutrino mass

1) Enlarge the Higgs sector
2) Enlarge the lepton sector
3) Both.

1) If we add to the Higgs sector another Higgs so that we can make invariant Yukawa couplings, neutrino masses will arise.

Possible extensions

a) $H \sim (3, +1)$
b) $h^+ \sim (1, +1)$
c) $k^{++} \sim (1, +2)$

Consider the triplet Higgs case:

$$H \equiv \sigma^i H^i = \begin{pmatrix} H^+ & \sqrt{2}\, H^{++} \\ \sqrt{2}\, H^0 & -H^+ \end{pmatrix}$$

And additional Yukawa is possible

$$\frac{1}{2} y \, \overline{l^c_R} \epsilon \sigma^i l_L H^i + h.c.$$

and a potential term

$$k \phi H^* \phi + h.c.$$

plus self-couplings which are such that

H can develop a vev $\langle \sigma \cdot H \rangle = \begin{pmatrix} 0 & 0 \\ v_H & 0 \end{pmatrix}$. This leads to a Majorana mass for the neutrino

$$\frac{1}{2}\left[(y v_H)\, \overline{\nu_R^c}\, \nu_L + (y v_H)\, \overline{\nu}_L\, \nu_R^c \right]$$

$$= \frac{1}{2}(y v_H)\, \overline{\nu}_M \nu_M$$

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2) Add an additional singlet right-handed neutrino for each generation
$\nu_R \sim (1,0)$.

The additional Lagrangian terms are possible

$$\mathcal{L} = f\, \overline{l}_L \tilde{\phi}\, \nu_R + \frac{1}{2} M_+ \overline{\nu_L^c}\, \nu_R + h.c.$$

$$\longrightarrow m_D \overline{\nu}_L \nu_R + \frac{1}{2} M_+ \overline{\nu_L^c}\, \nu_R + h.c.$$

Thus we are lead to mass terms of the form

$$\mathcal{L}_\nu = m_D \left[\overline{\nu}_L \nu_R + \overline{\nu}_R \nu_L \right] + \frac{1}{2} M_+ \left[\overline{\nu_L^c}\, \nu_R + \overline{\nu}_R \nu_L^c \right]$$

$$= \frac{1}{2} \left[\overline{\nu}_M \quad \overline{N}_M \right] \begin{bmatrix} 0 & m_D \\ m_D & M_+ \end{bmatrix} \begin{bmatrix} \nu_M \\ N_M \end{bmatrix}$$

where we have defined

$$\nu_M \equiv \nu_L + \nu_R^c \qquad (= {}^2 I_m)$$

$$N_m \equiv \nu_R + \nu_L^c \qquad (= \Sigma_m)$$

So

$$\boxed{M_\nu = \begin{bmatrix} 0 & m_D \\ m_D & m_+ \end{bmatrix}}$$

The mass eigenvalues are given by

$$M_- = \frac{1}{2}\left[(m_+) - \sqrt{m_+^2 + 4m_D^2}\right]$$

$$M_+ = \frac{1}{2}\left[m_+ + \sqrt{m_+^2 + 4m_D^2}\right]$$

Now if $m_D \ll m_+$ we can expand the square roots to find

$$\boxed{\begin{aligned} M_- &= -2\frac{m_D^2}{m_+} \ll M_+ . \\ M_+ &\simeq m_+ \end{aligned}}$$

See-Saw Mechanism

The mixing angle $\tan 2\theta = \frac{2m_D}{m_+} \ll 1$

$$\Rightarrow \theta \simeq \frac{m_D}{m_+} \ll 1$$

So the mass eigenstates are

$$N_- = \nu_M - \frac{m_D}{m_+} N_M$$

$$N_+ = N_M + \frac{m_D}{m_+} \nu_M .$$

So we have one light Majorana field —

$$N_- = \nu_M - \frac{m_D}{m_+} N_M \,, \quad M_- = -2\frac{m_D^2}{m_+}$$

which is mainly $\nu_M = \nu_L + \nu_R^c$ (Recall the left-handed neutrino of the SM. $(\nu_R^c)^c = \nu_L$)

and one heavy Majorana field

$$N_+ = N_M + \frac{m_D}{m_+} \nu_M \,, \quad M_+ = m_+$$

and $N_M = \nu_R + \nu_L^c$ (recall $(\nu_L^c)^c = \nu_R$) is the right handed neutrino.

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3) Some short distance lepton number violating process leads to a low energy SM effective action term of the form

$$\mathcal{L}_1 = \frac{x}{\Lambda} \overline{l_R^c} \cdot \phi \; l_L \cdot \phi + h.c.$$

$$\longrightarrow \frac{1}{2} x \frac{v^2}{\Lambda} \left[\overline{\nu_R^c} \nu_L + \overline{\nu}_L \nu_R^c \right]$$

again this leads to a Majorana mass term for the neutrino: $m_- = x\frac{v^2}{\Lambda}$.

For $m_\nu \simeq 1\,\mathrm{eV}$ and $x \sim O(1) \Rightarrow \Lambda > 10^{10}\,\mathrm{TeV}$.

Neutrino Masses in the MSSM

Consider the SUSY extension of the see-saw mechanism. We add to the MSSM fields an additional gauge singlet chiral superfield for each generation

$$N^c_F = e^{-i\theta\not\partial\bar\theta}\left[\tilde\nu^c_F + \sqrt{2}\,\theta^\alpha \nu^c_{F\alpha} + \theta^2 F_{\nu^c_F}\right]$$

($F = 1,2,3 = e, \mu, \tau$ Family)

and the corresponding anti-chiral superfield

$$\bar{N}^c_F = e^{+i\theta\not\partial\bar\theta}\left[\tilde\nu^{c\dagger}_F + \sqrt{2}\,\bar\theta_{\dot\alpha}\bar\nu^{c\dot\alpha}_F + \bar\theta^2 F^\dagger_{\nu^c_F}\right]$$

These are complete gauge singlets: $(1,1,0)$ under $SU(3) \times SU(2) \times U(1)$.

Recall a "Dirac" neutrino field is made up of ν & ν^c

$$\nu_{Dirac} = \begin{pmatrix} \nu \\ \bar\nu^c \end{pmatrix}$$

So $\nu_L = \begin{pmatrix}\nu\\ 0\end{pmatrix}$ $\quad \nu_R = \begin{pmatrix}0\\ \overline{\nu^c}\end{pmatrix}$

$\overline{\nu_L} = \begin{pmatrix}0 & \bar{\nu}\end{pmatrix}$ $\quad \overline{\nu_R} = \begin{pmatrix}\nu^c & 0\end{pmatrix}$

for the chiral or helicity projections.

In 4-component notation

$$\nu^c = \begin{pmatrix}\nu^c\\ \bar{\nu}\end{pmatrix}, \quad \overline{\nu^c} = \begin{pmatrix}\nu & \overline{\nu^c}\end{pmatrix}$$

Weyl

So $\nu^c_L = \begin{bmatrix}\nu^c\\ 0\end{bmatrix}$ $\quad \nu^c_R = \begin{bmatrix}0\\ \bar{\nu}\end{bmatrix}$

$\overline{\nu^c_L} = \begin{pmatrix}0 & \overline{\nu^c}\end{pmatrix}$ $\quad \overline{\nu^c_R} = \begin{pmatrix}\nu & 0\end{pmatrix}$

Majorana fields corresponding to the neutrino fields are

$$\nu_M \equiv \begin{pmatrix}\nu\\ \bar{\nu}\end{pmatrix} = \nu_L + \nu_R^c$$

$$N_M \equiv \begin{pmatrix}\nu^c\\ \overline{\nu^c}\end{pmatrix} = \nu^c_L + \nu_R$$

Two new superpotential terms along with the Kähler potential term that are 3-2-1 invariant can be made:

$$\boxed{\begin{aligned}\Gamma_\nu &= \int dV \underbrace{\tfrac{1}{16}\overline{N^c_F} N^c_F}_{=K_\nu} + \int dS\, W_\nu + \int d\bar{S}\, \overline{W}_\nu \\ &\text{with superpotential terms} \\ W_\nu &= \tfrac{1}{2} M_{N_F} N^c_F N^c_F - L_F \overset{\epsilon}{\cdot} H_2 N^c_G\, y_{N\,FG} \\ &= \tfrac{1}{2} M_{N_F} N^c_F N^c_F + H_2 \cdot L\, y_N N^c\end{aligned}}$$

where M_{N_F} are chosen diagonal — basis for RH neutrino's wlog. y_N is a 3x3 family matrix.

As usual we must also add soft SUSY breaking terms. As usual we can introduce them as spurion fields for M_N & y_N

$$M_N \longrightarrow M_N(1 + \tfrac{1}{4} B_N \theta^2) = M_N(\theta)$$

$$y_N \longrightarrow y_N + \tfrac{1}{4} A_N \theta^2 = y_N(\theta)$$

& for the Kähler potential $Z \to Z(1 - \frac{1}{16} m^2_{\tilde{N}} \theta^2 \bar{\theta}^2)$
$= Z(\theta, \bar{\theta})$

This yields the soft Susy breaking terms

$$\boxed{\begin{aligned}\mathcal{L}_{N\not{S}} = &-\tilde{\nu}^{c\dagger} m_{\tilde{N}}^2 \tilde{\nu}^c - \tfrac{1}{2} M_{NF} \tilde{\nu}^c_F B_{NFG} \tilde{\nu}^c_G \\ &- \tfrac{1}{2} \bar{M}_{NF} \tilde{\nu}^{c\dagger}_F \bar{B}_{NFG} \tilde{\nu}^{c\dagger}_G \\ &- H_u \cdot \tilde{l} A_N \tilde{\nu}^c \underbrace{- H_u^\dagger \cdot \tilde{l}^{\dagger} \bar{A}_N \tilde{\nu}^{c\dagger}}\end{aligned}}$$

$$= -\tilde{\nu}^{c\dagger} A_N^\dagger H_u^\dagger \cdot \tilde{l}^{\dagger}$$

As in the (s)quark sector we now have lepton family mixing along with lepton-flavor violating processes due to the soft-Susy breaking parameters ex $\mu \to e\gamma$, $\tau \to \mu\gamma$, $\mu \to e^+e^-e^-$. Thus once again the Susy parameter space will be restricted by these (lack of) observations.

Also M_{NF} is a Majorana mass for the neutrinos and in general will be considered large, O(GUT) scale while the soft-Susy breaking parameters are of the order of $M_{Susy} \sim$ weak scale. So we expect
3 light neutrinos and
3 heavy neutrinos.

Also we expect 3 heavy sneutrinos and

3 sneutrinos with M_{SUSY} masses. In general we must solve the 6x6 (3 families and 2 types of (s)neutrino) mixing matrices to find the diagonal mass eigenstates. But the non-mixing approximation gives us an idea of the mass pattern.

There is much more MSSM phenomenology to study. It is left to the student to continue. Let's move on to the possible unification of the 3-2-1 gauge symmetries.
