Finally let's consider the cosmological implications of the MSSM. The imposition of R-parity implied that the LSP is stable. Hence it is possible that the relic LSP's left over from the Big Bang could account for most of the matter of the Universe.

Recall the standard cosmology, 3 ingredients are needed

1) Einstein equations:

Newton's constant

cosmological constant

$$\underbrace{R_{\mu\nu}}_{\text{Ricci tensor}} - \frac{1}{2} g_{\mu\nu} \underbrace{R}_{\text{Ricci scalar}} = + \frac{8\pi G_N}{c^4} \underbrace{T_{\mu\nu}}_{\text{energy-momentum tensor of matter}} + \lambda g_{\mu\nu}$$

obtained from Riemann

2) A metric related to symmetries of physical situation

Homogeneity & isotropy of Universe imply a specific form of the metric

Friedmann-Robertson-Walker Models

$$ds^2 = c^2 dt^2 - \underbrace{a(t)}_{\text{scale factor}}{}^2 \left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right]$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta \, d\varphi^2$$

where $k = -1, 0, +1$ describes the spatial curvature. $k=0$ describes ordinary Euclidean 3-space with "a" giving the overall normalization of physical distances. $(c = 1 = k_B)$

Due to the expansion of the universe $a = a(t)$ is a function of time with $\dot{a}/a > 0$ at the present time. The Einstein equation can be solved yielding an equation for "a" the Friedmann eq.

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G_N}{3} \rho_{total} \qquad \left(\text{ignoring } \Lambda\right)$$

where ρ_{total} = total energy density. Define the

$$\text{Hubble parameter } H(t) \equiv \frac{\dot{a}(t)}{a(t)}$$

The present value of the Hubble (constant) parameter $H_0 = H(t_0)$ is
$\uparrow$ present time

$$H_0 = 73 \pm 3 \ \frac{km}{s \cdot Mpc} .$$

The Universe is flat $(k=0)$ when the energy density equals the critical density ρ_c

$$\rho_c = \frac{3H^2}{8\pi G_N} \left(\simeq 10^{-29} \frac{g}{cm^3} \simeq 3 \times 10^{-47} \, GeV^4\right)$$

presently

Remark: Often we write $H_0 \equiv h \cdot 100 \frac{km}{s \cdot Mpc}$
so that at present $h \approx 0.73 \pm 0.03$

The present numerical value of ρ_c is then

$$\rho_c^0 = \frac{3H_0^2}{8\pi G_N} \approx 1.9 \times 10^{-38} h^2 \frac{kg}{m^3}$$

It is convenient to describe the abundance of a substance in the Universe (matter, radiation or vacuum energy) in units of ρ_c

density parameter

Thus Ω_i quantity of substance "i" with density ρ_i is

$$\Omega_i \equiv \frac{\rho_i}{\rho_c}$$

and

$$\Omega \equiv \sum_i \Omega_i = \sum_i \frac{\rho_i}{\rho_c}$$

so that the Friedmann eq. can be written as

$$\Omega - 1 = \frac{k}{H^2 a^2}$$

So sign of k is determined by Ω

$$\begin{cases} \rho < \rho_c \Leftrightarrow \Omega < 1, & k = -1 \quad \text{open Universe} \\ \rho = \rho_c \Leftrightarrow \Omega = 1, & k = 0 \quad \text{flat} \\ \rho > \rho_c \Leftrightarrow \Omega > 1, & k = +1 \quad \text{closed} \end{cases}$$

Recall expansion of universe is observed in cosmological redshift of light emitted by distant galaxies. Define redshift parameter z

$$1+z = \frac{\lambda_{observed}}{\lambda_{emitted}}$$ for the wavelength of a particular spectral line.

One can derive in the standard FRW model the redshift parameter's relation to the expansion scale factor $a(t)$

$$1+z = \frac{a(t_{observed})}{a(t_{emitted})}.$$

3.) Equation of state specifying the physical properties of matter & energy density.

Suppose the equation of state for species X related the energy density and the pressure according to

$$p_X = \alpha_X \rho_X.$$ Then we

would find Ω_X would depend on time, and hence z according to the scaling factor

$$(1+z)^{3(1+\alpha_X)}.$$

Recall perfect fluid $T_{\mu\nu} = (p+\rho)u_\mu u_\nu - p g_{\mu\nu}$
So energy-momentum conservation $T^{\mu\nu}{}_{;\mu} = 0$ implies the rate of change of the total energy in volume element $V = a^3$, $\frac{d}{dt}(\rho a^3)$, is equal to minus the pressure times the change of volume, $-p\,dV$, that is

$$\frac{d}{dt}(\rho a^3) = -p\frac{d}{dt}a^3 .$$

So if the eq. of state is $\underline{p = \alpha\rho}$ $\Rightarrow$

$$\dot{\rho} a^3 = -3(1+\alpha)\rho a^2 \dot{a}$$

$\Rightarrow$

$$\frac{\dot{\rho}}{\rho} = -3(1+\alpha)\frac{\dot{a}}{a} = -3(1+\alpha)H$$

So $\ln\rho = -3(1+\alpha)\ln a + \text{Const.}$

$$\Rightarrow \boxed{\rho = \text{const.}\, a^{-3(1+\alpha)}}$$

ex. 1) Radiation dominated $\alpha = \frac{1}{3}$; $p = \frac{1}{3}\rho$ (pressure is averaged over 3 spatial dim.)

$$\rho \sim \frac{1}{a^4}$$

Photon energy scales with volume a^3 AND is redshifted a $\Big\}\ a^4$

2) Matter dominated (non-rel.) $\alpha = 0$

$$\rho \sim \frac{1}{a^3}$$

matter density scales with volume a^3

3) Vacuum Energy dominated $\alpha = -1$; $p = -\rho$

$\rho \sim$ const.

Recall 2 Friedmann eq's:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G_N}{3}\rho_{total}$$

$$\frac{2\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = -8\pi G_N p \qquad \left(\begin{array}{c}\text{Usually use}\\ T^{\mu\nu}{}_{;\mu} = 0\\ \text{instead}\end{array}\right)$$

Subtract them $\Rightarrow$

$$\frac{\ddot{a}}{a} = \frac{-4\pi G_N}{3}(\rho + 3p) = -\frac{4\pi G_N}{3}(1+3\alpha)\rho$$

For small t $\quad a \sim t^\beta \Rightarrow \underline{a(t) \sim t^{\frac{2}{3(1+\alpha)}}}$

So 1) Radiation dominated. $a(t) \sim \sqrt{t}$

2) Matter dominated $a(t) \sim t^{2/3}$

3) Vacuum Energy dominated $a(t) \sim e^{Ht}$

Hence we have the density parameters at different times related by their associated redshift

$$\frac{\Omega_X(t)}{\Omega_X(t_0)} = \frac{a(t)^{-3(1+\alpha_X)}}{a(t_0)^{-3(1+\alpha_X)}}$$

$$\Rightarrow \boxed{\Omega_X(t) = (1+z)^{3(1+\alpha_X)}\,\Omega_X(t_0)}$$

Further note for radiation that $\rho \sim \frac{1}{a^4}$ and matter $\rho \sim \frac{1}{a^3}$. In either case for early Universe "a" is small so in the Friedmann eq.

$$H^2(t) + \frac{k}{a^2} = \frac{8\pi G_N}{3}\rho$$

one can ignore the k/a^2 term $\Rightarrow$

$$H^2(t) = \frac{8\pi G_N}{3}\rho$$

So for radiation $H = \frac{\dot{a}}{a} = \frac{1}{2t}$

and the Stefan-Boltzmann Law applies to all relativistic "radiation" ($m \ll T$)

$$\rho(T) = \frac{\pi^2}{30} g_{eff}(T)\, T^4$$

with $g_{eff}(T)$ an effective degeneracy factor counting degrees of freedom for all relativistic particles ($m \ll T$) in thermal equilibrium

For each fermion there is a factor of 7/8 times the number of degrees of freedom due to Fermi-Dirac statistics

For example for the SM at 1 TeV = T all the particles are relativistic there are

quarks:	$\underset{\text{spin}}{2} \cdot \underset{\text{color}}{3} \cdot \underset{\text{up, down}}{2} \times \underset{\text{families}}{3}$	$=$	36
leptons:	$2 \cdot 3 + 1 \cdot 3$	$=$	9
			$45 \times 2 = 90$ (antiparticle)

Gluons	: $\underset{\text{color}}{8} \times \underset{\text{helicity}}{2}$	$=$	16
$W^{\pm}, Z$	: $3 \times \underset{\text{helicity}}{3}$	$=$	9
γ	: 2	$=$	2
Higgs	: 1	$=$	1
			28

So $g_{eff} = 28 + \frac{7}{8} \cdot 90 = 106.75$ at $T = 1\,\text{TeV}$

So we have

$$H = \sqrt{\frac{8\pi G_N}{3}} \frac{\pi}{\sqrt{30}} \sqrt{g_{eff}(t)}\, T^2$$

$$= 1.66 \sqrt{g_{eff}} \frac{T^2}{m_{planck}}$$ and from $H = \frac{1}{2t} \Rightarrow$

$$t = 0.30 \frac{m_{planck}}{\sqrt{g_{eff}}\, T^2}$$

Finally
The energy density can be separated into 3 types

$$\Omega = \Omega_M + \Omega_\Lambda + \Omega_R \text{ with}$$

$$\Omega_M = \Omega_B + \Omega_{DM} \quad \text{where}$$

Ω_Λ is due to the vacuum energy or cosmo. constant
Ω_R is due to radiation energy density
Ω_B is the baryonic matter contribution to the energy density

Ω_{DM} is the non-baryonic, dark matter contribution.

The matter density scales in time as $(1+z)^3$ since a constant comoving number density causes the physical mass density to be diluted with the changing volume.

For large z radiation will scale as $(1+z)^4$ but for present time we measure $\Omega_R \sim 10^{-5}-10^{-4}$, neglect.

The cosmo. constant is constant and so does not change in time i.e. scale $(1+z)^0$.

Thus we have a general expression for the expansion rate of the Universe depends on the equations of state of the different species

(Friedmann eq.)

$$\frac{H(z)^2}{H_0^2} = \left[\Omega_X^0 (1+z)^{3(1+\alpha_X)} + \Omega_K^0 (1+z)^2 + \Omega_M^0 (1+z)^3 + \Omega_R^0 (1+z)^4\right]$$

where

$$\Omega_K^0 \equiv \frac{-k}{a_0^2 H_0^2} \quad \text{and}$$

$$\alpha_\Lambda = -1 \text{ for the cosmo. constant.}$$

Results of WMAP (Wilkinson Microwave Anisotropy Probe)

Age of Universe = 13.7 ± 0.2 Gyrs.

Geometry of Universe is flat : $\Omega_0 = 1$

Dark energy content = 73%

Matter content = 27%

$$\Omega_B^0 h^2 = 0.0224 \pm 0.0009$$

$$\Omega_M^0 h^2 = 0.135^{+0.008}_{-0.009}$$

and a very low hot dark matter density (relic neutrinos). Hence cold dark matter density $\boxed{\Omega_{DM}^0 h^2 = 0.1126^{+0.0161}_{-0.0181} \text{ is}}$ implied.

We will see that the LSP neutralino is a good candidate for the CDM in the Universe.

The Boltzmann equation in the FRW model universe governs the number density $n(t)$ of neutralinos at time t.

$$\frac{dn(t)}{dt} + 3Hn = -\langle \sigma v_{rel.} \rangle [n^2 - n_{eq.}^2]$$

At thermal equilibrium the number of neutralinos in a comoving volume

$N = a^3 n$ is given by the

equilibrium value $N_{eq}(T)$

$$n = g \int \frac{d^3p}{(2\pi)^3} f(\vec{p})$$

where $$f(\vec{p}) = \frac{1}{e^{E/T} \pm 1}$$

where $E = \sqrt{\vec{p}^2 + m^2}$ and $-$ for Bose-Einstein particles and $+$ for Fermi-Dirac particles like the neutralino and g = degeneracy.

Note: $$\rho = g \int \frac{d^3p}{(2\pi)^3} E(\vec{p}) f(\vec{p})$$

$$P = g \int \frac{d^3p}{(2\pi)^3} \frac{|\vec{p}|^2}{3E(\vec{p})} f(\vec{p})$$

Since $E^2 = p^2 + m^2 \Rightarrow p\,dp = E\,dE$

So

$d^3p = 4\pi \sqrt{E^2 - m^2}\, E\,dE$ after integrating over $\int d\Omega = 4\pi$.

Two limits:

1) Non-relativistic: $m \gg T$

$$n_{NR} = g \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$$

$$\rho_{NR} = m\, n_{NR} \quad ; \quad P_{NR} = T \cdot n_{NR} \ll \rho_{NR}$$

and $\langle E \rangle = m + \frac{3}{2}T$ (i.e. $k=1$)

2) Ultra-Relativistic: $T \gg m$

$$\rho_R = \begin{cases} \frac{\pi^2}{30} g T^4 & \text{Bose-Einstein} \\ \frac{7}{8}\left(\frac{\pi^2}{30} g T^4\right) & \text{Fermi-Dirac.} \end{cases}$$

$$n_R = \begin{cases} \frac{\zeta(3)}{\pi^2} g T^3 & \text{Bose-Einstein} \\ \frac{3}{4}\left(\frac{\zeta(3)}{\pi^2} g T^3\right) & \text{Fermi-Dirac} \end{cases}$$

(Recall: $\zeta(3) = 1.20206\ldots$)

$$\langle E \rangle = \rho/n = \begin{cases} 2.70\,T & \text{B-E} \\ 3.15\,T & \text{F-D.} \end{cases}$$

For Photons $m=0$; $g=2$ $\rho_\gamma(T) = \frac{\pi^2}{15} T^4$

The Stefan-Boltzmann law.

This brings us back to the radiation dominated Universe where

$$H^2 = \frac{8\pi G_N}{3}\rho_R = \frac{8\pi G_N}{3}\frac{\pi^2}{30}g_{eff}T^4$$

Or $$H = \sqrt{\frac{8\pi G_N}{3}\frac{\pi^2}{30}}\sqrt{g_{eff}}\,T^2$$

If the number density $n(T)$ is larger than the equilibrium density the particles will annihilate more than they are created. The rate of depletion of neutralinos is

$$\sigma_{N\bar{N}\to f\bar{f}}|\vec{v}|n^2$$

where $\sigma_{N\bar{N}\to f\bar{f}}$ is the annihilation cross-section into $f, \bar{f}$ SM particles, $\vec{v}$ is the neutralino's relative velocity and the rate goes like n_N times $n_{\bar{N}} = n_N$, hence n^2. On the other hand neutralinos are created by the reverse process with a rate proportional to $n_{eq.}^2$. Hence the rate of change of the number density is

$$\frac{dn}{dt} + 3Hn = -\langle\sigma_{N\bar{N}\to f\bar{f}}|\vec{v}|\rangle\left[n^2 - n_{eq}^2\right]$$

The LHS is just $\frac{1}{a^3}\frac{d}{dt}[na^3]$ so the $3Hn$

is just the dilution that comes from the Hubble expansion. $\langle\sigma_{NN\to ff}|\vec{v}|\rangle$ is the thermally averaged cross-section times velocity. The averaging is necessary since the annihilating particles have random thermal velocities & directions. Finally we should sum over all possible annihilation channels to obtain

$$\frac{dn}{dt} + 3Hn = -\langle\sigma_A|\vec{v}|\rangle[n^2 - n_{eq}^2]$$

where σ_A is the total annihilation cross-section.

During the early expansion of the universe the neutralino is in equilibrium with the number density of a non-relativistic massive particle ($g=2$ since it is Majorana)

$$n = T^3\left(\frac{M}{T}\right)^{3/2}\frac{2}{(2\pi)^{3/2}}e^{-M/T}$$

and $$H = \sqrt{\frac{8\pi G_N}{3}\frac{\pi^2}{30}}\sqrt{g_{eff}}\,T^2 .$$

Now in the radiation dominated era we have

$$\rho = \frac{\pi^2}{30} g T^4 \sim a^{-4} \Rightarrow$$

$$a \sim \frac{1}{T}$$

and so $\hat{N} = \frac{n}{T^3}$ $\left(N = a^3 n \sim \frac{n}{T^3} \right)$

is the number of neutralino's per unit comoving volume. And we see that in equilibrium

$$\hat{N} = \frac{n}{T^3} = \frac{2}{(2\pi)^{3/2}} \left(\frac{m}{T}\right)^{3/2} e^{-m/T}$$

As the universe expands and cools,

The $3Hn$ Hubble expansion dominates the now too sparse particle annihilation and we have that

$$\frac{1}{n}\frac{dn}{dt} = -3H = -3\frac{1}{a}\frac{da}{dt}$$

$$\Rightarrow n \sim \frac{1}{a^3} \qquad \left(\text{i.e. } \frac{d}{dt}(a^3 n) = 0\right)$$

The number density of neutralinos reduces only as the Hubble expansion of space — not an exponential decrease with temperature as in thermal equilibrium. The number of neutralinos is larger than it would be in equilibrium. The neutralinos are said to <u>freeze out</u> at this Number

$$\hat{N}_{freeze} = \frac{n(T_{freeze})}{T^3_{freeze}} = \frac{2}{(2\pi)^{3/2}} \left(\frac{m}{T_{Freeze}}\right)^{3/2} e^{-\frac{m}{T_{freeze}}}$$

<u>where $\hat{N}_{Freeze}$ remains constant.</u>

To find the freeze out temperature we can estimate the freeze out to occur when the Hubble expansion is equal to the annihilation rate:

$$\underline{3 H n_{Freeze} = \langle \sigma_A |\vec{v}| \rangle n^2_{Freeze}}$$

$$\Rightarrow 3\sqrt{\frac{8\pi G_N}{3} \frac{\pi^2}{30}} \sqrt{g_{eff}(T_{Freeze})}\, T^2_{Freeze}$$

$$= \langle \sigma_A |\vec{v}| \rangle T^3_{Freeze} \left(\frac{m}{T_{Freeze}}\right)^{3/2} \frac{2}{(2\pi)^{3/2}} e^{-m/T_{Freeze}}.$$

$$\Rightarrow \quad e^{\frac{m}{T_F}} = \frac{\langle \sigma_A |\vec{v}| \rangle T_F \left(\frac{m}{T_F}\right)^{3/2} \frac{2}{(2\pi)^{3/2}}}{3\sqrt{\frac{(2\pi)^3 G_N}{90}}\sqrt{g_{eff}(T_F)}}$$

$$\Rightarrow \quad \boxed{\frac{m}{T_F} = \ln\left[\frac{\langle \sigma_A |\vec{v}| \rangle m \sqrt{40}}{(2\pi)^3 \sqrt{G_N g_{eff}(T_F)}} \sqrt{\frac{m}{T_F}}\right]}$$

Solving this equation iteratively typically leads to values of $m/T_F \approx 20-30$.

Substituting T_F into $\hat{N}_{Freeze}$ we have

$$\boxed{\hat{N}_F = \sqrt{\frac{(2\pi)^3 G_N}{10} g_{eff}(T_F)} \left(\frac{1}{\langle \sigma_A |\vec{v}| \rangle m}\right) \frac{m}{T_F}}$$

Multiplying by T^3 for $T < T_F$ and m for energy density we obtain

$$\boxed{\rho_{\tilde{N}} = \left(\frac{(2\pi)^3 G_N}{10} g_{eff}(T_F)\right)^{1/2} \frac{1}{\langle \sigma_A |\vec{v}| \rangle} \frac{m}{T_F} T^3}$$

Neutralino

Recall $\rho_c^0 = \frac{3H_0^2}{8\pi G_N} \simeq 1.9\times10^{-38} h^2 \frac{kg}{m^3}$

So $$\boxed{\Omega_{\tilde{N}} h^2 = \frac{\rho_N}{\rho_c^0} h^2 = \frac{\rho_{\tilde{N}}}{8.1\times10^{-47}\,GeV^4}}$$

Now $G_N = (1.2\times10^{19}\,GeV)^{-2}$, present $T = 2.7K$
$= 2\times10^{-13}\,GeV$.

Now suppose $T_F \sim \frac{1}{20} m \sim 10\,GeV$ (try m around $200\,GeV$)

Then the # of degrees of freedom are

quarks $u,d,c,s,b = (2\times3)\times5\times2 = 60$
leptons $e,\nu \quad = (2\times3+1\times3)2 = 18$

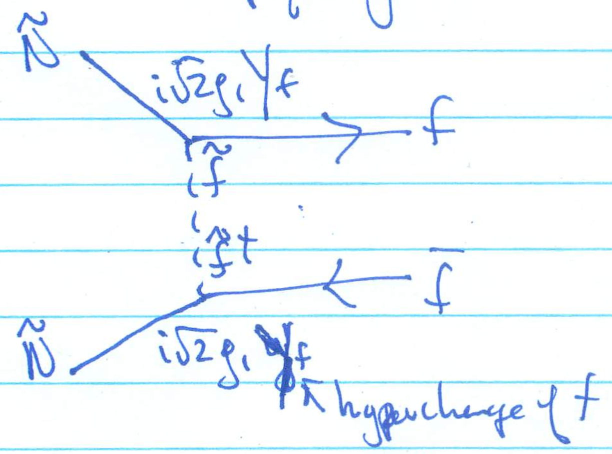
Fermions 78

$\gamma = 2$ Bosons 2

$$g_{eff}(T_F) = 2 + \tfrac{7}{8}78 \simeq 70.3$$

Next consider the annihilation cross-section for neutralino's

The neutralino can be considered mainly a bino $\tilde{N}_1 \approx \tilde{B}$ and so it couples through ~~the~~ generalized Yukawa interactions in $\mathcal{L}_{\tilde{Y}}$ (p.-128-) to yield ~~a~~ annihilation into the leptons & quarks (other than top, if light)



which gives an amplitude that goes like

$$\mathcal{M} \sim \frac{2 g_1^2 Y_f^2}{M_{susy}^2}$$

; hence an annihilation cross-section summed over all fermions coupled to hypercharge

$$\sigma_A \sim 8\pi\alpha^2 \frac{m^2 N_A}{M_{susy}^4} |\vec{v}|$$

where we ignored $Y_f^2 \sim 1$ and let the sfermion masses in the propagator all be $\sim \frac{1}{M_{susy}^2}$ and α is the em fine structure constant. N_A is the sum over all "light" fermion decay channels

$$N_A = \overbrace{u,d,c,s,b}^{3\times5} + \overbrace{e,\mu,\tau}^{3} + \overbrace{\nu_e,\nu_\mu,\nu_\tau}^{3} = 21$$

and the $|\vec{v}|$ helicity (p-wave) supression appears since the neutralino is a Majorana fermion (H. Goldberg, Phys. Rev. Lett. 50 (1983) p. 1419).

The thermal average of the non-relativistic neutralino $\vec{v}^2$ yields

$$\left\langle \tfrac{1}{2} m \vec{v}^2 \right\rangle = \tfrac{3}{2} T_{\text{Freezeout}}$$

So

$$\boxed{\langle \sigma_A |\vec{v}| \rangle = 24\pi \alpha^2 N_A \frac{m}{M_{SUSY}^4} T_F}$$

Hence

$$\boxed{\rho_{\tilde{N}} = \sqrt{\frac{(2\pi)^3 G_N g_{eff}(T_F)}{10}} \; \frac{1}{24\pi\alpha^2 N_A} \frac{M_{SUSY}^4}{m^2} \left(\frac{m}{T_F}\right)^2 T^3}$$

Dividing by $\frac{\rho_c^0}{h^2}$ and using $m/T_F \simeq 20$ we find

$$\Omega_{\tilde{N}} h^2 = \frac{\rho_{\tilde{N}}}{8.1\times10^{-47}\,\mathrm{GeV}^4}$$

$$\boxed{\Omega_{\tilde{N}} h^2 = 1.65\times10^{-6}\left[\frac{M^4_{SUSY}}{m^2}\right]\frac{1}{\mathrm{GeV}^2}}$$

$\Rightarrow$ Since WMAP places the bound

$$\Omega_{DM} h^2 < 0.129$$

we have

$$\frac{M^4_{SUSY}}{m^2} < 0.129\,(600\times10^3)\,\mathrm{GeV}^2$$

$$\Rightarrow \boxed{m > \frac{M^2_{SUSY}}{280}\ (\mathrm{GeV})}$$

So we see that this is in the right range of masses for the neutralinos of around $m \gtrsim 200$ GeV to be a dark matter candidate!!

Just as a check consider

$$\frac{m}{T_F} = \ln\left[\frac{\langle\sigma_A|\vec{v}|\rangle m\sqrt{40}}{(2\pi)^3\sqrt{G_N g_{eff}(T_F)}}\cdot\sqrt{\frac{m}{T_F}}\right]$$

$$= -\frac{1}{2}\ln\left(\frac{m}{T_F}\right) + \ln\left[\frac{24\pi\alpha^2 N_A\sqrt{40}\, m^3}{(2\pi)^3\sqrt{G_N g_{eff}(T_F)}\, M_{susy}^4}\right]$$

Try $\frac{m^3}{M_{susy}^4} \approx \frac{1}{200\,\text{GeV}}$ $\Rightarrow$

$$\frac{m}{T_F} = -\frac{1}{2}\ln\frac{m}{T_F} + \underbrace{\ln\left[1.521\times10^{13}\right]}_{\approx 30.4}$$

$\Rightarrow$

$$\frac{m}{T_F} \approx 30 \checkmark$$