The gauge coupling constants are not the only running constants — in fact all of the parameters run. The heavy third generation Yukawa couplings run (the 1st & 2nd generation Yukawa's are small and usually their running is neglected)

Yukawa Coupling Running for The third generation

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left[-\sum_{i=1}^{3} c_i^t g_i^2 + 6\cdot 16\, y_t^2 + 16\, y_b^2\right]$$

$$\frac{dy_b}{dt} = \frac{y_b}{16\pi^2}\left[-\sum_{i=1}^{3} c_i^b g_i^2 + 16\, y_t^2 + 6\cdot 16\, y_b^2 + 16\, y_\tau^2\right]$$

$$\frac{dy_\tau}{dt} = \frac{y_\tau}{16\pi^2}\left[-\sum_{i=1}^{3} c_i^\tau g_i^2 + 3\cdot 16\, y_b^2 + 4\cdot 16\, y_\tau^2\right]$$

with $c^t = \begin{pmatrix} 13/15 \\ 3 \\ 16/3 \end{pmatrix}$; $c^b = \begin{pmatrix} 7/15 \\ 3 \\ 16/3 \end{pmatrix}$; $c^\tau = \begin{pmatrix} 9/5 \\ 3 \\ 0 \end{pmatrix}$

Likewise all the soft-susy breaking parameters run

Gaugino Masses:

$$\frac{dM_i}{dt} = \frac{g_i^2}{16\pi^2} M_i \Big[-6 C_2(G_i) + 2 \overset{\sum_i' \text{ chiral superfields}}{T_S} \Big]$$

$$= 2 \frac{\alpha_i}{4\pi} M_i \Big[-3 C_2(G_i) + \underset{\sum_i' \text{ chiral superfields}}{} T_S \Big]$$

Notice: RHS is same β_i as gauge coupling

$$\frac{dM_i}{dt} = 2 \frac{M_i}{g_i} \left[\frac{g_i^3}{16\pi^2} \left(-3 C_2(G_i) + \sum_S{}' T_S \right) \right]$$

$$= 2 \frac{M_i}{g_i} \beta_i \qquad \text{i.e. } \frac{dg_i}{dt} = \beta_i$$

Hence $\frac{d}{dt}\left(\frac{M_i}{g_i^2}\right) = 0$ and $\frac{M_i}{g_i^2}$ is scale independent

So if $\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2}$ in a particular model

they stay equal at all scales. So if this is true in a SUSY GUT the gaugino masses must unify when the coupling constants unify at M_{GUT} (called GUT relation for gaugino masses).

The 1-loop RGEs for the soft SUSY breaking parameters are given in Falck, Z. Physik C30 (1986) 247. We will neglect intergeneration mixing and write the tri-linear breaking as $4A_i y_i$: Soft-SUSY breaking parameters:

$$\frac{dA_t}{dt} = \frac{2}{16\pi^2}\left[-\sum_i c_i^t g_i^2 M_i + 6\cdot 16 y_t^2 A_t + 16 y_b^2 A_b\right]$$

$$\frac{dA_b}{dt} = \frac{2}{16\pi^2}\left[-\sum_i c_i^b g_i^2 M_i + 6\cdot 16 y_b^2 A_b + 16 y_t^2 A_t + 16 y_\tau^2 A_\tau\right]$$

$$\frac{dA_\tau}{dt} = \frac{2}{16\pi^2}\left[-\sum_i c_i^\tau g_i^2 M_i + 3\cdot 16 y_b^2 A_b + 4\cdot 16 y_\tau^2 A_\tau\right]$$

$$\frac{dB}{dt} = \frac{2}{16\pi^2}\left[-\frac{3}{5} g_1^2 M_1 - 3 g_2^2 M_2 + 3\cdot 16 y_b A_b + 3\cdot 16 y_t^2 A_t + 16 y_\tau^2 A_\tau\right]$$

$$\frac{d\mu}{dt} = \frac{\mu}{16\pi^2}\left[-\frac{3}{5} g_1^2 - 3 g_2^2 + 3\cdot 16 y_t^2 + 3\cdot 16\cdot y_b^2 + 16 y_\tau^2\right]$$

$$\frac{dm_{q_t}^2}{dt} = \frac{2}{16\pi^2}\left[-\frac{1}{15} g_1^2 M_1^2 - 3 g_2^2 M_2^2 - \frac{16}{3} g_3^2 M_3^2 + \frac{1}{10} g_1^2 S + 16 y_t^2 X_t + 16 y_b^2 X_b\right]$$

$$\frac{dm^2_{u^c t}}{dt} = \frac{2}{16\pi^2}\left[-\frac{16}{15}g_1^2 M_1^2 - \frac{16}{3}g_3^2 M_3^2 - \frac{2}{5}g_1^2 S + 2\cdot 16 y_t^2 X_t\right]$$

$$\frac{dm^2_{d^c b}}{dt} = \frac{2}{16\pi^2}\left[-\frac{4}{15}g_1^2 M_1^2 - \frac{16}{3}g_3^2 M_3^2 + \frac{1}{5}g_1^2 S + 2\cdot 16 y_b^2 X_b\right]$$

$$\frac{dm^2_{l\tau}}{dt} = \frac{2}{16\pi^2}\left[-\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 - \frac{3}{10}g_1^2 S + 16 y_\tau^2 X_\tau\right]$$

$$\frac{dm^2_{e^c\tau}}{dt} = \frac{2}{16\pi^2}\left[-\frac{12}{5}g_1^2 M_1^2 + \frac{3}{5}g_1^2 S + 2\cdot 16 y_\tau^2 X_\tau\right]$$

$$\frac{dm^2_{H_d}}{dt} = \frac{2}{16\pi^2}\left[-\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 - \frac{3}{10}g_1^2 S + 3\cdot 16 y_b^2 X_b + 16 y_\tau^2 X_\tau\right]$$

$$\frac{dm^2_{H_u}}{dt} = \frac{2}{16\pi^2}\left[-\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 + \frac{3}{10}g_1^2 S + 3\cdot 16 y_t^2 X_t\right]$$

where

$$X_t = m^2_{q_t} + m^2_{u^c t} + m^2_{H_u} + A_t^2$$

$$X_b = m^2_{q_t} + m^2_{d^c b} + m^2_{H_d} + A_b^2$$

$$X_\tau = m^2_{l\tau} + m^2_{e^c\tau} + m^2_{H_d} + A_\tau^2$$

sum over families

and

$$S = m_{H_u}^2 - m_{H_d}^2 + \mathrm{Tr}\left[m_q^2 - m_l^2 - 2m_{u^c}^2 + m_{d^c}^2 + m_{e^c}^2 \right]$$

The RGE's for the first two families can be obtained by replacing the Yukawa coupling constants appropriately as well as the $X_{t,b,\tau}$ parameters.

These RG equations can be "run" by choosing different model dependent initial conditions at a high momentum scale — then integrate the equations down to a lower scale.

For example the "universality" initial conditions at the $M_{GUT} = 2\times10^{16}$ GeV scale can be chosen:

GUT normalized U(1)

MSUGRA (minimal supergravity model — simplest supergravity GUT model)

$$g_1 = g_2 = g_3 \equiv g_{GUT}$$

$$M_1 = M_2 = M_3 \equiv m_{1/2}$$

$$m_{q_i}^2 = m_{u_i^c}^2 = m_{d_i^c}^2 = m_{l_i}^2 = m_{e_i^c}^2 = m_{H_u}^2 = m_{H_d}^2 \equiv m_0^2$$

$$A_t = A_b = A_\tau = A_0 \text{ with}$$

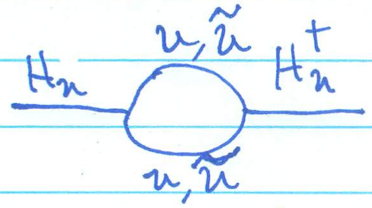
all off diagonal soft SUSY breaking scalar masses and tri-linear A parameters are set to zero. Family mixing occurs only via the Yukawa couplings.

For example: $m_0 = 100$ GeV, $m_{1/2} = 200$ GeV, $A_0 = 0$ and $\tan\beta = 4$, Baer & Tata have shown the running of the soft SUSY breaking parameters in their text. Note at low energy $M_1 : M_2 : M_3 \sim 1 : 2 : 7$ so that the gluino should be much heavier than the lighter chargino or the 2 lighter neutralinos. This is in agreement with the $\frac{\alpha_1}{m_1} = \frac{\alpha_2}{m_2} = \frac{\alpha_3}{m_3}$ ratio of gauge couplings at low energy.

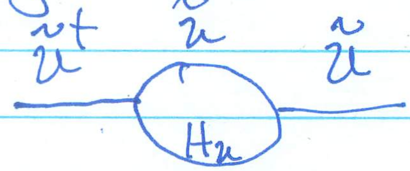
Especially interesting is the evolution of the $M^2_{H_u}$ mass squared parameter which at high energy is positive, but due to the large top quark Yukawa coupling interactions its radiative corrections drive it negative. Hence normalizing the mass squared as positive at a high scale $\mu \gg M_Z$ leads to large radiative corrections $\ln(\mu/M_Z)$ which can be summed to yield the 1-loop (or higher) effective potential which exhibits a truer picture of the potential than at tree level.

The electroweak symmetry is broken and we refer to this as <u>radiative EWSB</u>.

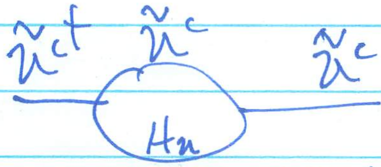
A simpler discussion of this effect is seen by just considering just the large top Yukawa coupling contribution to the soft SUSY breaking scalar masses. Consider the diagrams for the soft SUSY



$(m^2_{H_u} H_u^\dagger H_u)$



$(m_q^2 \tilde{q}^\dagger \tilde{q})$



$(m^2_{u^c} \tilde{u}^{c\dagger} \tilde{u}^c)$

breaking masses that come from the superpotential Yukawa couplings to the top quark. The RGE mix $m^2_{H_u}, m^2_{q_t}, m^2_{u^c_t}$ (from above) $\leftarrow$ (the 33 entry)

$$\frac{d}{dt}\begin{bmatrix} m^2_{H_u} \\ m^2_{u^c} \\ m^2_q \end{bmatrix} = \frac{(16 y_t^2)}{8\pi^2}\begin{bmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} m^2_{H_u} \\ m^2_{u^c} \\ m^2_q \end{bmatrix}$$

$$+\frac{(16y_t^2)}{8\pi^2}A_t^2\begin{pmatrix}3\\2\\1\end{pmatrix}.$$

The factors of 3 come from the 3 colors of (s)quarks running around the loop, likewise the factors of 2 come from the SU(2) doublets running around the loops. For simplicity we will ignore the A_t term. also we will ignore the running of the y_t it is just the large fixed value that gives the top mass!

So

$$\frac{d}{dt}\begin{bmatrix}m_{H_u}^2\\m_{u^c}^2\\m_q^2\end{bmatrix}=\frac{(16y_t^2)}{8\pi^2}\begin{bmatrix}3&3&3\\2&2&2\\1&1&1\end{bmatrix}\begin{bmatrix}m_{H_u}^2\\m_{u^c}^2\\m_q^2\end{bmatrix}$$

can be solved by transforming to the eigenbasis for the matrix

$$\begin{bmatrix}3&3&3\\2&2&2\\1&1&1\end{bmatrix}.$$

The eigenvalues are (0, 0, 6) with corresponding eigenvectors

$$\begin{pmatrix}0\\-1\\1\end{pmatrix},\begin{pmatrix}-1\\1\\0\end{pmatrix},\begin{pmatrix}3\\2\\1\end{pmatrix}$$

Now the eigenvectors run according to their eigenvalue $e^{-\lambda t}$ (since we are running "down" to t from the Planck scale now) So the eigenvector with eigenvalue 6 is damped out of the running to the 0-0 eigenvalue plane. Now the msugra universal initial condition at M_{planck} is

$$\left.\begin{bmatrix} m_{H_u}^2 \\ m_{u^c}^2 \\ m_q^2 \end{bmatrix}\right|_{\substack{t=0 \\ \text{at } M_{planck}}} = m_0^2 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} .$$

The vector $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ can be expanded in terms of eigenvectors as

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{2}\left[\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}\right]$$

The last eigenvector is the eigenvalue 6 and rapidly scales to zero as we run to low energy. So at M_Z we find

$$\left.\begin{bmatrix} m_{H_u}^2 \\ m_{u^c}^2 \\ m_q^2 \end{bmatrix}\right|_{M_W} \simeq \frac{1}{2} m_0^2 \left[\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}\right] = \frac{1}{2} m_0^2 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

since the zero eigenvalues eigenvectors do not run. Thus we see that the radiative corrections have changed the sign of $m^2_{H_u}$. The large top Yukawa coupling has destabilized the vacuum and the effective potential breaks $SU(2) \times U(1) \longrightarrow U(1)$ of electromagnetism. This is known as REWSB.

(Of course we still must check that the 2 conditions for stable D-flat directions and the maximum at the origin are satisfied to conclude electroweak symmetry is indeed broken.)