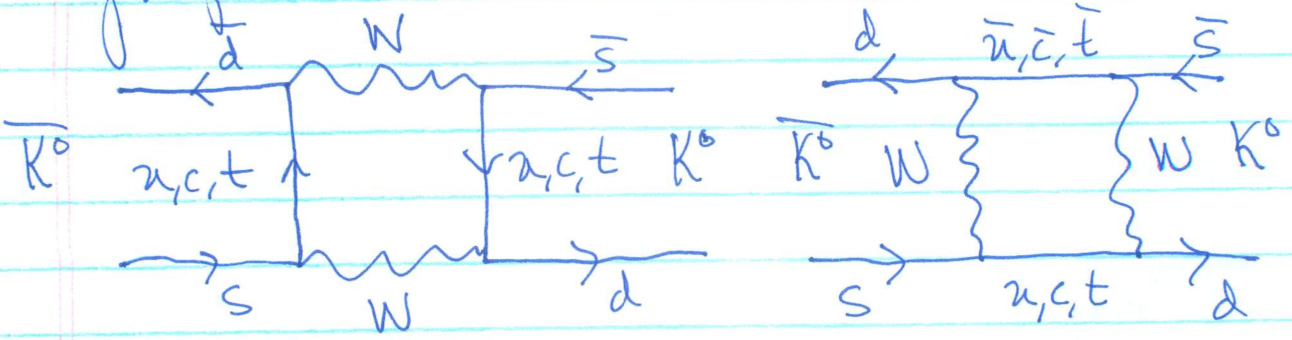
III.C.) The SUSY Flavor Problem & SUSY CP Problem

There are FCNC problems in the quark sector. In the SM these occur at the one-loop level at the rate that generally agrees with experiment. In the Kaon system there is $K_L - K_S$ mass difference that occurs in 1-loop from box graphs like



The mixing between the third generation and the first two is small, so the top contribution is small.

First note that the coupling to $W^\pm_\mu$ was given by the charged weak current

$$J^\mu_W = \bar{e}\gamma^\mu(1-\gamma_5)\nu + \bar{d}\gamma^\mu(1-\gamma_5)A_{CKM}\, u$$

with the CKM matrix

$$A^\dagger_{CKM} = V \underset{d}{=} \left(A^{d\dagger}_L A^u_L\right)^\dagger$$

$$= \begin{array}{c} \\ u \\ c \\ t \end{array} \begin{array}{c} \begin{array}{ccc} d & s & b \end{array} \\ \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \end{array}$$

So we have the CKM matrix at each vertex. Glashow, Iliopoulos and Maiani found that if all the quark masses were the same this would vanish since then we just have the unitarity of the CKM matrix

$$\text{box} \propto V^{\dagger}_{di} V_{is} V^{\dagger}_{si} V_{id} = \delta_{ds}\delta_{sd} = 0$$

with $(V^{\dagger}V)_{ij} = \delta_{ij}$.

But the box will then go as the m^2_{quark} for each type of quark and not vanish but is surpressed — This is call the GIM mechanism and GIM surpression $\left(\sim \frac{M^2_{quark}}{M_W^4}\right)$
Now the charm quark is the major contribution and we find an effective operator for the mixing of

RG QCD correction

$$\mathcal{H}_{K^0\bar{K}^0} = \frac{G_F^2}{16\pi^2}\left[(V^*_{cd}V_{cs})^2 m_c^2 \, H\!\left(\frac{M_c^2}{M_W^2}\right)\right] \times (0.85)$$

$$\times \bar{d}\gamma_\mu(1-\gamma_5)s \, \bar{d}\gamma^\mu(1-\gamma_5)s$$

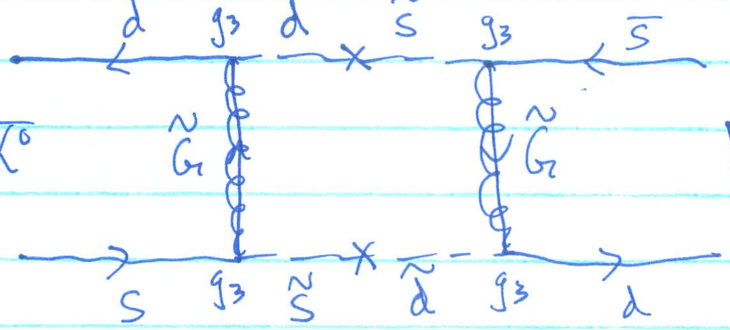
with

$$H(x) = \left[\frac{1}{4} + \frac{9}{4}\frac{1}{1-x} - \frac{3}{2}\frac{1}{(1-x)^2}\right] - \frac{3}{2}\frac{x^2}{(1-x)^3}\ln x .$$

This leads to an amplitude for $K^0-\bar{K}^0$ mixing

$$\mathcal{M}^{SM}_{K^0\bar{K}^0} \simeq \frac{2}{\pi^2} G_F^2 m_c^2 \sin^2\theta_c \cos^2\theta_c \qquad (\text{i.e. } V_{ud} = \cos\theta_c)$$

Now in the MSSM we have additional graphs involving the squarks and gluinos



(also chargino & neutralino loops contribute)

The $\times$ vertex corresponds to an off-diagonal squark mass matrix contribution soft SUSY breaking term

$$\mathcal{L}_{\not{S}K} \supset -\tilde{d}^{\dagger}(m_{\tilde{q}}^2)_{12}\,\tilde{s} \qquad \text{for example.}$$

$\uparrow$ family 1, family 2 } off-diagonal

This leads to an amplitude in addition to the SM one

$$\mathcal{M}^{MSSM}_{K^0\bar{K}^0} \simeq 4\alpha_3^2 \left[\frac{M^2_{\tilde{q}\,12}}{M^2_{SUSY}}\right]^2 \frac{1}{M^2_{SUSY}}$$

where M_{SUSY} corresponds to the superpartner masses which are all around the SUSY breaking scale.

Since the SM result roughly accounts for the $K_L - K_S$ mass difference we require

$$\mathcal{M}^{SM}_{K^0\bar{K}^0} \gg \mathcal{M}^{MSSM}_{K^0\bar{K}^0}$$

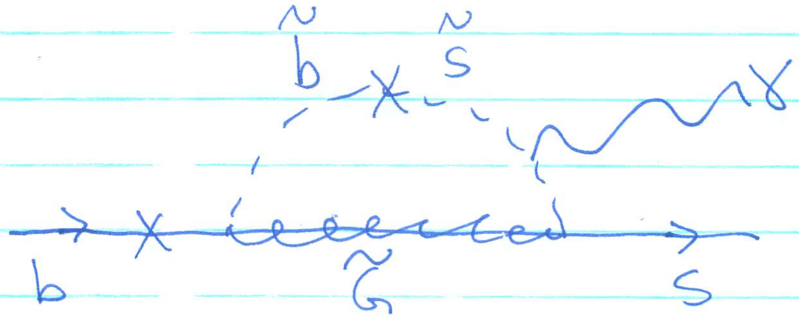
$$\Rightarrow \quad 4\alpha_3^2 \left[\frac{m^2_{\tilde{q}12}}{M^2_{susy}}\right]^2 \left(\frac{1}{M^2_{susy}}\right) \ll \frac{2}{\pi^2} G_F^2 m_c^2 \sin^2\theta_c \cos^2\theta_c$$

$$\Rightarrow \quad \left(\frac{m^2_{\tilde{q}}}{M^2_{susy}}\right) < 4\times10^{-3}\,\frac{M_{susy}}{500\,\text{GeV}}$$

Hence we have severe bounds on the squark mass matrix ruling out large sectors of parameter space.

This is an example of the SUSY flavor problem.

Kaons are not the only example - large off-diagonal $(m^2_{\tilde{q}})_{23}$ or $(m^2_{\tilde{d}^c})_{23}$ or $(A_d)_{23}$ results in large flavor violating gluino vertices and unacceptable $b \to s\gamma$ decays from diagrams involving squark and gluino loops ex.



In general the diagrams involve the unitary matrices U that diagonalise the quark mass matrix and $\tilde{U}$ the unitary matrix that diagonalizes the quark mass matrix. The $\tilde{G}-\tilde{q}-q$ interaction in the mass diagonal basis will be $\propto U\tilde{U}^{\dagger}$. Hence the box diagrams will be proportional to

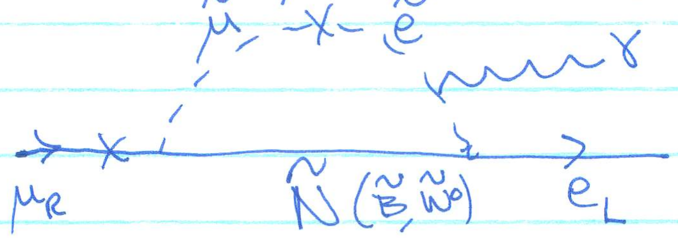
$$\sum_{\substack{i,j \\ =\tilde{d},\tilde{s},\tilde{b}}} [U\tilde{U}^{\dagger}]_{ai} [U\tilde{U}^{\dagger}]^{*}_{bi} [U\tilde{U}^{\dagger}]_{aj} [U\tilde{U}^{\dagger}]^{*}_{bj} F(m_i^2, m_j^2)$$

So a necessary condition for FC processes is that there are large off-diagonal entries in the $U\tilde{U}^{\dagger}$ matrix.

Addition constraints on squark masses and mixing matrices come from $B-\bar{B}$ & $D-\bar{D}$ mass differences.

Likewise the lepton sector can also have FCNC processes like $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$

ex.



lepton number violation

$\mu \to e\gamma$

Again this is non-zero since the slepton mass matrices m_ℓ^2, $m_{e^c}^2$ are not diagonal in the same basis as the lepton mass matrix.

Further the trilinear breaking terms A_u, A_d, A_ℓ can give rise to unwanted FCNC processes by off-diagonal elements and large radiative mass corrections from diagonal matrix elements —

ex.

$$\mathcal{L}_{SB} \supset -H_d^0 (A_d)_{12} \tilde{d}_1 \tilde{d}^c_2 \longrightarrow -\frac{v_d}{\sqrt{2}} (A_d)_{12} \tilde{d}\, \tilde{s}^c$$

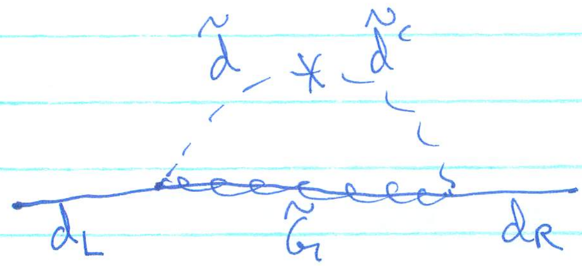
family index

off diagonal K^0-$\bar{K}^0$ restricted

and

$$\mathcal{L}_{SB} \supset -H_d^0 (A_d)_{11} \tilde{d}_1 \tilde{d}^c_1 \longrightarrow -\frac{v_d}{\sqrt{2}} (A_d)_{11} \underbrace{\tilde{d}\, \tilde{d}^c}_{\text{diagonal}}$$

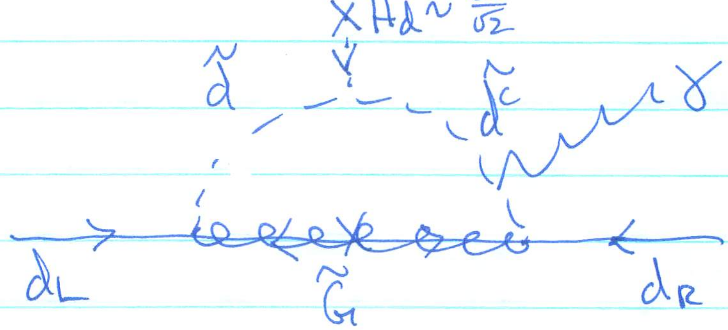
ex.



loop mass restricted to down quark $< M_d$ restricts $(A_d)_{11}$

Also CP violating effects will suppress a large part of parameter space. Again the imaginary parts of squark mass^2 matrix elements enter into K^0-$\bar{K}^0$ system to give CP violation which yields tight bounds on the phases of squark mixing matrix to be small.

Also the A-terms introduce electric dipole moment contributions for the d-quark and hence the neutron. A non-trivial complex phase is needed for this in the soft-SUSY breaking parameters. The effective EDM operator is $d_R^\dagger \sigma^{\mu\nu} d_L F_{\mu\nu}$ arises from a similar graph to the d-loop mass correction



This leads to a very small phase for $(A_d)_{11}$.

The smallness of so many CP violating phases is called the SUSY CP problem

To avoid these issues there is a way to identify "safe neighborhoods" of the 105 dimensional parameter space. Three safe neighborhoods have been identified:

1) "Soft Breaking Universality" requires 3 conditions to be satisfied

a) Squark & slepton soft SUSY breaking masses are proportional to the identity in the same basis where quark & lepton mass matrices are diagonal.

b) Tri-linear A-term matrices are proportional to Yukawa matrices

c) No new nontrivial phases beyond the SM.

Universality & reality of soft SUSY breaking masses

a) $(m_q^2) = m_q^2 \mathbb{1}$; $(m_{u^c}^2) = m_{u^c}^2 \mathbb{1}$; $(m_{d^c}^2) = m_{d^c}^2 \mathbb{1}$

$(m_\ell^2) = m_\ell^2 \mathbb{1}$; $(m_{e^c}^2) = m_{e^c}^2 \mathbb{1}$

c) and all mass^2 are real.

b) $(A_u) = A_u (y_u)$; $(A_d) = A_d (y_d)$; $(A_\ell) = A_\ell (y_e)$

c) and no phases, all real $A_{u,d,\ell}$. This is a "super GIM mechanism".

2) "<u>More Minimal Supersymmetric Model</u>": Only require leading quadratic divergences in the Higgs mass from top, gauge boson and Higgs loops to cancel. This requires that the superpartners $\tilde{t}$, $\tilde{t}^c$, $\tilde{b}$, $\tilde{H}_u$, $\tilde{H}_d$, $\tilde{B}$, $\tilde{W}$ have masses below 1 TeV while 1st and 2nd generation sparticles should be as heavy as 40 TeV. The heaviness of 1st & 2nd generation squarks and sleptons <u>surpresses</u> FCNC processes. This is also known as the <u>decoupling solution</u>. Caution as 2-loop running below the heavy squark threshold may drive the top squark mass squared negative.

3) "<u>Alignment</u>": Squark & quark mass matrices are diagonalized by the same unitary transformation $U\tilde{U}^\dagger \approx 1$. The squarks can be non-degenerate in mass. The quark & squark mass matrices are said to be "aligned". This can be arranged in models which have horizontal symmetries linking the various generations.

Alignment: $$m_q^2 = y_u^* y_u^T + y_d^* y_d^T$$

$$m_{u^c}^2 = y_u^\dagger y_u$$

$$m_{d^c}^2 = y_d^\dagger y_d .$$

Of course we eventually will build (or try to) models which predict suppression of FCNC & CP violating processes.

III.D.) Renormalized Perturbation Theory and Running coupling constants and masses

The soft SUSY breaking in the MSSM maintains the stability of the dimensionful parameters as they only receive logarithmic divergent radiative corrections. If the scale μ is the renormalization scale of the theory, then quantum corrections to the various parameters of the MSSM will receive contributions like $\lambda^2 \ln\left(\frac{\mu}{m}\right)$, m the SUSY parameters in the calculation and λ^2 some coupling or mass. Although potentially large if μ is the scale of unification M_{GUT} these log's can be summed by RG techniques to give perturbation theory in terms of running couplings and masses that depend on the energy scale μ.

Recall the running of the gauge coupling constants in the SM.