III.B.) (s)Top Corrections to the Higgs mass.

Recall in the tree approximation we found that the lightest scalar Higgs mass was

$$m_h^2 = \frac{1}{2}\left[(m_A^2 + M_Z^2) - \sqrt{(m_A^2+M_Z^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta}\right]$$

which yielded the bound

$$m_h \leq M_Z |\cos 2\beta| \leq M_Z$$

indeed if $\tan\beta = 1 \Rightarrow m_h = 0$!! These values for m_h are already ruled out by LEP-II experiments!

However the radiative corrections will be large for h and will allow m_h to be increased above M_Z. Current calculations (including 2 loops) allow for m_h to be as high as 130 GeV, which will be probed at the LHC.

Let's calculate the one-loop top, stop quark corrections.

Now

$$\left[(m_A^2+M_Z^2)^2-4m_A^2M_Z^2\cos^2 2\beta\right]$$

$$=\left(m_A^2+M_Z^2-2M_Z^2\cos^2 2\beta\right)^2-4M_Z^4\cos^4 2\beta$$
$$+4M_Z^4\cos^2 2\beta$$

$$=\left(m_A^2+M_Z^2-2M_Z^2\cos^2 2\beta\right)^2+4M_Z^4\cos^2 2\beta\underbrace{\left(1-\cos^2 2\beta\right)}_{=\sin^2 2\beta}$$

$$=\left(m_A^2+M_Z^2-2M_Z^2\cos^2 2\beta\right)^2+\underbrace{M_Z^4\sin^2 4\beta}_{\geq 0}$$

$\Rightarrow$

$$\geq\left(m_A^2+M_Z^2-2M_Z^2\cos^2 2\beta\right)^2$$

$\Rightarrow$

$$\boxed{-\sqrt{(m_A^2+M_Z^2)^2-4m_A^2M_Z^2\cos^2 2\beta}\leq-\sqrt{\left(m_A^2+M_Z^2-2M_Z^2\cos^2 2\beta\right)^2}}$$

$\Rightarrow$

$$m_h^2\leq\frac{1}{2}\left[\left(\cancel{m_A^2}+\cancel{M_Z^2}\right)-\left(\cancel{m_A^2}+\cancel{M_Z^2}-2M_Z^2\cos^2 2\beta\right)\right]$$

$$\leq M_Z^2\cos^2 2\beta$$

$$\Rightarrow\quad\boxed{m_h\leq M_Z|\cos 2\beta|}$$

We could use the radiative correction to the h-h-propagator — but we will also need the H-H & H-h mixing propagators — then re-diagonalize the mass matrix. So we might as well use the complex $H_u^0, H_u^{0\dagger}$ propagators — then re-diagonalize the tree ~~plus~~ one-loop mass matrix. So we need the top & stop interactions with the $H_u^0, H_u^{0\dagger}$ fields — with the top quarks and squarks in the mass eigenbasis. The leading contribution comes from the Yukawa coupling since the go like m_t^2/v^2 — we ignore the M_W^2/v^2, $\frac{\mu}{m_t}\sin\alpha$, A_t couplings as smaller and/or off-diagonal.

So the H_u^0 to top couplings are given by the $\mathcal{L}_Y$ & $\mathcal{L}_F$ terms

$$\mathcal{L}^{H_u, t, \tilde{t}} = -4 H_u^0\, u\, y_u\, u^c - 4 H_u^{0\dagger}\, \bar{u}^c\, y_u^\dagger\, \bar{u}$$

$$-16 H_u^{0\dagger} H_u \left[\tilde{u}\, y_u y_u^\dagger \tilde{u}^\dagger + \tilde{u}^{c\dagger} y_u^\dagger y_u \tilde{u}^c\right]$$

$$\text{Recall } -4 y_u^* = \Gamma^u = -\frac{\sqrt{2}}{v \sin\beta} M^u$$

and since we are interested in the stop we have

$$\boxed{\mathcal{L}^{t,\tilde{t}}_{H_u^0} = -\frac{\sqrt{2}\, m_t}{v \sin\beta}\left[H_u^0 \overbrace{t\, t^c}^{=\bar{t}_R t_L} + H_u^{0\dagger} \overbrace{\bar{t}^c \bar{t}}^{=\bar{t}_L t_R}\right] - \frac{2 m_t^2}{v^2 \sin^2\beta}\left(H_u^{0\dagger} H_u^0\right)\left[\underbrace{\tilde{t}^{c\dagger} \tilde{t}^c + \tilde{t}\, \tilde{t}^\dagger}_{=\tilde{t}_1^\dagger \tilde{t}_1 + \tilde{t}_2^\dagger \tilde{t}_2}\right]}$$

$$\boxed{\mathcal{L}^{t,\tilde{t}}_{H_u^0} = \frac{-\sqrt{2}\, m_t}{v \sin\beta}\left[H_u^0\, \bar{t}\gamma_- t + H_u^{0\dagger}\, \bar{t}\gamma_+ t\right] - \left(\frac{\sqrt{2}\, m_t}{v\sin\beta}\right)^2 \left[\left(\frac{v_u}{\sqrt{2}} + H_u^{0\dagger}\right)\left(\frac{v_u}{\sqrt{2}} + H_u^0\right)\right] \times \left[\tilde{t}_1^\dagger \tilde{t}_1 + \tilde{t}_2^\dagger \tilde{t}_2\right]}$$

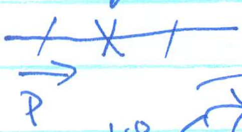
where we have shifted the H_u^0 by its vev.

Now the radiative corrections to the $H_u^0 - H_u^{0\dagger}$ & $H_u^0 - H_u^0$, $H_u^{0\dagger} - H_u^{0\dagger}$ 1PI functions (self-energies) will give the perturbative corrections to the mass matrix. (Although the physical mass is at the pole of the propagator – we will calculate the mass at the zero momentum value). The self-energy 1PI graphs are

F.T.

$$\langle 0|T\,\tilde{H}^0_u(p)\,H^{0\dagger}_u(0)|0\rangle^{1PI} =$$

tree value $= +\Gamma^{tree}_{H^0_u H^{0\dagger}_u}$



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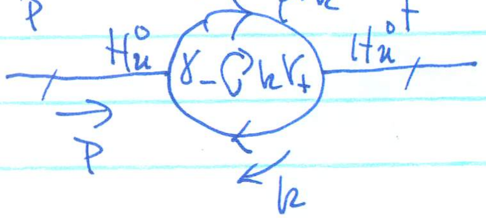

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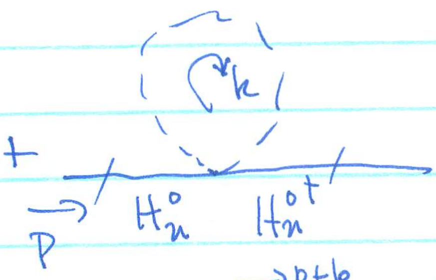

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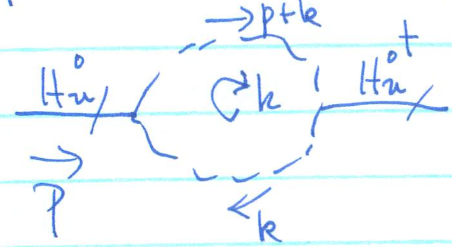

$$\langle 0|T\,\tilde{H}^0_u(p)\,H^0_u(0)|0\rangle^{1PI} =$$

tree $= +\Gamma^{tree}_{H^0_u H^0_u}$

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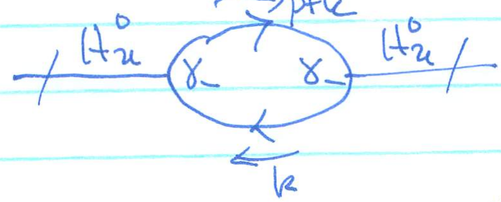

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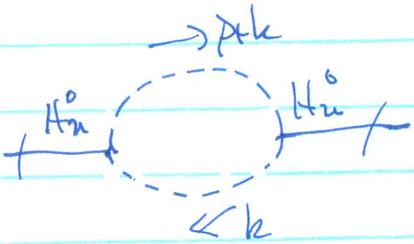
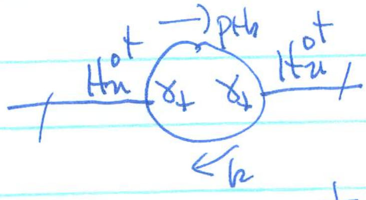


$$\langle 0|T\,\tilde{H}^{0\dagger}_u(p)\,H^{0\dagger}_u(0)|0\rangle^{1PI} =$$

tree $= +\Gamma^{tree}_{H^{0\dagger}_u H^{0\dagger}_u}$

$+$



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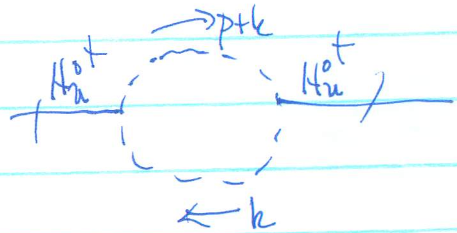
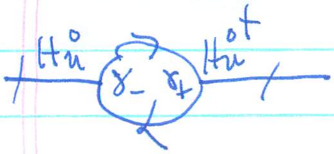
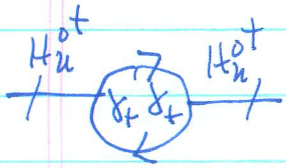


So we have

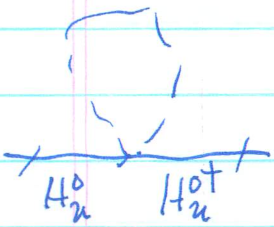


$$= -3\int\frac{d^4k}{(2\pi)^4}\left[\frac{-i\sqrt{2}\,m_t}{v\sin\beta}\right]^2 \mathrm{Tr}\left[\frac{\gamma_-\, i(\not k+m_t)\,\gamma_+\, i(\not p+\not k+m_t)}{[(p+k)^2-m_t^2][k^2-m_t^2]}\right]$$

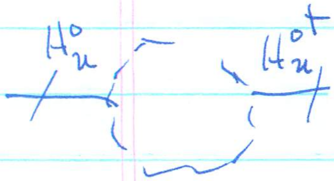
$$= -3\int\frac{d^4k}{(2\pi)^4}\left[\frac{-i\sqrt{2}\,m_t}{v\sin\beta}\right]^2 \mathrm{Tr}\left[\frac{\gamma_-\, i(\not k+m_t)\,\gamma_-\, i(\not p+\not k+m_t)}{[(p+k)^2-m_t^2][k^2-m_t^2]}\right]$$



$$= -3\int\frac{d^4k}{(2\pi)^4}\left[\frac{-i\sqrt{2}\,m_t}{v\sin\beta}\right]^2 \mathrm{Tr}\left[\frac{\gamma_+\, i(\not k+m_t)\,\gamma_+\, i(\not p+\not k+m_t)}{[k^2-m_t^2][(p+k)^2-m_t^2]}\right]$$

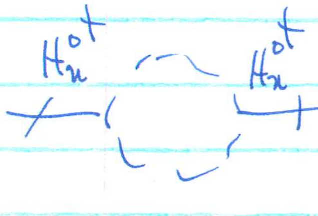


$$= \left[i\frac{2m_t^2}{v^2\sin^2\beta}\right]\int\frac{d^4k}{(2\pi)^4}\left[\frac{i}{k^2-m_{t_1}^2}+\frac{i}{k^2-m_{t_2}^2}\right]\cdot 3$$



$$= 3\left[\frac{v_u}{\sqrt{2}}\,\frac{i\,2m_t^2}{v^2\sin^2\beta}\right]^2\int\frac{d^4k}{(2\pi)^4}\left[\frac{i}{[(p+k)^2-m_{t_1}^2]}\,\frac{i}{[k^2-m_{t_1}^2]} + \frac{i}{[(p+k)^2-m_{t_2}^2]}\,\frac{i}{[k^2-m_{t_2}^2]}\right]$$

$$= 3\cdot\left[i\frac{v_u}{\sqrt{2}}\,\frac{2m_t^2}{v^2\sin^2\beta}\right]^2\int\frac{d^4k}{(2\pi)^4}\left[\frac{i}{[(p+k)^2-m_{t_1}^2]}\,\frac{i}{[k^2-m_{t_1}^2]} + \frac{i}{[(p+k)^2-m_{t_2}^2]}\,\frac{i}{[k^2-m_{t_2}^2]}\right]$$



$$= 3\cdot\left[i\,\frac{v_u}{\sqrt{2}}\,\frac{2m_t^2}{v^2\sin^2\beta}\right]^2 \int \frac{d^4k}{(2\pi)^4}\left[\frac{i}{\left[(p+k)^2-m_{\tilde t_1}^2\right]}\,\frac{i}{\left[k^2-m_{\tilde t_1}^2\right]} + \frac{i}{\left[(p+k)^2-m_{\tilde t_2}^2\right]}\,\frac{i}{\left[k^2-m_{\tilde t_2}^2\right]}\right]$$

Now recall

$$\left[\frac{v_u}{\sqrt{2}}\,\frac{2m_t^2}{v^2\sin^2\beta}\right]^2 = 2m_t^4\,\underbrace{\frac{v_u^2}{v^2}}_{=\sin^2\beta}\,\frac{1}{\sin^2\beta}\,\frac{1}{v^2\sin^2\beta}$$

$$= \frac{2m_t^4}{v^2\sin^2\beta}$$

Also

$$\mathrm{Tr}\left[\gamma_-(\not k+m_t)\gamma_+(\not p+\not k+m_t)\right] = \mathrm{Tr}\left[\gamma_-\not k(\not p+\not k)\right]$$

$$= 2\left[p\cdot k + k^2\right]$$

$$\mathrm{Tr}\left[\gamma_\mp(\not k+m_t)\gamma_\mp(\not p+\not k+m_t)\right] = 2m_t^2$$

Evaluating the self energies & hence mass matrix at $p=0$ yields

$$\langle 0|T\,\tilde{H}^0_u(0)\,H^{0\dagger}_u(0)|0\rangle^{1PI} = \Gamma^{tree}_{H^0_u H^{0\dagger}_u}(0)$$

$$+3\left[\frac{2m_t^2}{v^2\sin^2\beta}\right]\int\frac{d^4k}{(2\pi)^4}\left[\frac{-2k^2}{[k^2-m_t^2]^2}+\frac{1}{[k^2-m_{t_1}^2]}+\frac{1}{[k^2-m_{t_2}^2]}\right.$$

$$\left.+\frac{m_t^2}{[k^2-m_{t_1}^2]^2}+\frac{m_t^2}{[k^2-m_{t_2}^2]^2}\right]$$

$$\langle 0|T\,\tilde{H}^0_u(0)\,H^0_u(0)|0\rangle^{1PI} = \Gamma_{H^0_u H^0_u}(0)$$

$$+3\left[\frac{2m_t^2}{v^2\sin^2\beta}\right]\int\frac{d^4k}{(2\pi)^4}\left[\frac{-2m_t^2}{[k^2-m_t^2]^2}\right.$$

$$\left.+\frac{m_t^2}{[k^2-m_{t_1}^2]^2}+\frac{m_t^2}{[k^2-m_{t_2}^2]^2}\right]$$

$$\langle 0|T\,\tilde{H}^{0\dagger}_u(0)\,H^{0\dagger}_u(0)|0\rangle^{1PI} = \Gamma^{tree}_{H^{0\dagger}_u H^{0\dagger}_u}(0)$$

$$+3\left[\frac{2m_t^2}{v^2\sin^2\beta}\right]\int\frac{d^4k}{(2\pi)^4}\left[\frac{-2m_t^2}{[k^2-m_t^2]^2}\right.$$

$$\left.+\frac{m_t^2}{[k^2-m_{t_1}^2]^2}+\frac{m_t^2}{[k^2-m_{t_2}^2]^2}\right]$$

Notice if SUSY is good $m_{\tilde t_1} = m_{\tilde t_2} = m_t$ the last loop integrals vanish — so will the first 1PI function since we can add & subtract a mass^2. In general then

$$\langle 0|T\,\tilde H_u^0(0)\,H_u^{0\dagger}(0)|0\rangle^{1PI} = \Gamma^{tree}_{H_u^0 H_u^{0\dagger}}(0)$$

$$+3\cdot\left[\frac{2m_t^2}{v^2\sin^2\beta}\right]\int\frac{d^4k}{(2\pi)^4}\left[-2\frac{1}{[k^2-m_t^2]}+\frac{1}{[k^2-m_{\tilde t_1}^2]}+\frac{1}{[k^2-m_{\tilde t_2}^2]}\right.$$

$$\left.-\frac{2m_t^2}{[k^2-m_t^2]^2}+\frac{m_t^2}{[k^2-m_{\tilde t_1}^2]^2}+\frac{m_t^2}{[k^2-m_{\tilde t_2}^2]^2}\right]$$

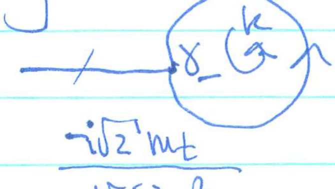
---

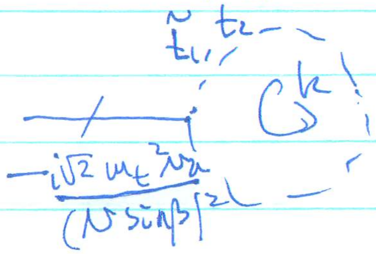
where $\displaystyle -\frac{2k^2}{[k^2-m_t^2]^2} = -2\,\frac{k^2-m_t^2+m_t^2}{[k^2-m_t^2]^2}$

$\displaystyle = -\frac{2}{[k^2-m_t^2]} - \frac{2m_t^2}{[k^2-m_t^2]^2}$ was used.

So we see then in the good SUSY limit all radiative corrections to the mass terms vanish and we have the good SUSY limit to the tree mass only.

The divergent terms will be cancelled by the renormalization of the model. In particular consider the Tadpole diagrams contributing to $\langle 0|H_u^0|0\rangle$

$i\langle 0|H_u^0|0\rangle^{t,\tilde{t}}_{1\text{-loop}} =$ 

$+$ 

$$= +\frac{i\sqrt{2}\,m_t}{v\sin\beta}\int\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left[\gamma_-\, i\frac{\not{k}+m_t}{k^2-m_t^2}\right]\cdot 3$$

$$-\frac{i\sqrt{2}\,m_t^2 v_u}{(v\sin\beta)^2}\int\frac{d^4k}{(2\pi)^4}\left[\frac{i}{k^2-m_{\tilde{t}_1}^2}+\frac{i}{k^2-m_{\tilde{t}_2}^2}\right]\cdot 3$$

$$= \frac{3\cdot m_t^2\sqrt{2}}{v\sin\beta}\int\frac{d^4k}{(2\pi)^4}\left[\frac{-2}{[k^2-m_t^2]}+\frac{1}{[k^2-m_{\tilde{t}_1}^2]}+\frac{1}{[k^2-m_{\tilde{t}_2}^2]}\right]$$

As usual, if Susy is good $m_{\tilde{t}_1}=m_{\tilde{t}_2}=m_t$, the 1-loop corrections vanish. However, the soft Susy breaking terms lead to a logarithmic divergence which contributes to the vev of the H_u^0 field. Hence we will need a counterterm for the vev

$$v_u \longrightarrow v_u + \delta v_u$$

and we will choose δv_u to cancel all the radiative tadpole diagrams.

In the approximation we ~~are~~ calculating this will just be these m_t^2 log divergent graphs.

$$\langle 0|H_u^0|0\rangle = \frac{v_u}{\sqrt{2}} + \frac{\delta v_u}{\sqrt{2}} + \langle 0|H_u^0|0\rangle^{t,\tilde{t}}_{1\text{-loop}}$$

$$\equiv \frac{v_u}{\sqrt{2}}$$

$$\Rightarrow \boxed{\begin{aligned}\delta v_u &= -\langle 0|H_u^0|0\rangle^{t,\tilde{t}}_{1\text{-loop}} \\ &= +i\frac{3\cdot 2\cdot m_t^2}{v\sin\beta}\int\frac{d^4k}{(2\pi)^4}\left[\frac{-2}{[k^2-m_t^2]} + \frac{1}{[k^2-m_{\tilde{t}_1}^2]} + \frac{1}{[k^2-m_{\tilde{t}_2}^2]}\right]\end{aligned}}$$

Note that these same divergent terms contribute to the $\langle 0|T\,\tilde{H}_u^0(0)\,H_u^{0\dagger}(0)|0\rangle^{1PI}$ self-energy on p.-165-

$$v\sin\beta\cdot\langle 0|T\,\tilde{H}_u^0(0)\,H_u^{0\dagger}(0)|0\rangle^{1PI}\Big|_{1\text{-loop div.}} = 3\cdot\frac{2m_t^2}{v\sin\beta}\int\frac{d^4k}{(2\pi)^4}\left[-\frac{2}{[k^2-m_t^2]} + \frac{1}{[k^2-m_{\tilde{t}_1}^2]} + \frac{1}{[k^2-m_{\tilde{t}_2}^2]}\right]$$

$$= -i\delta v_u$$

Consider the tadpole diagram:

$$\delta v_u = +i\frac{3\cdot 2 m_t^2}{v\sin\beta}\frac{(-i)}{(4\pi)^2}\frac{\Gamma(1-\frac{d}{2})}{\Gamma(1)}\left[-2\left(\frac{1}{m_t^2}\right)^{1-\frac{d}{2}} + \left(\frac{1}{m_{\tilde t_1}^2}\right)^{1-\frac{d}{2}} + \left(\frac{1}{m_{\tilde t_2}^2}\right)^{1-\frac{d}{2}}\right]$$

Recall $z\Gamma(z) = \Gamma(z+1) \Rightarrow$

$$\Gamma(1-\tfrac{d}{2}) = \frac{\Gamma(3-\frac{d}{2})}{(1-\frac{d}{2})(2-\frac{d}{2})} \xrightarrow{d\to 4} \frac{-\Gamma(1)}{(2-\frac{d}{2})}$$

So with $\frac{\epsilon}{2} = 2-\frac{d}{2}$ we find

$$\delta v_u = -\frac{3\cdot 2 m_t^2}{v\sin\beta}\frac{1}{16\pi^2}\cdot\underbrace{\left[m_{\tilde t_1}^2 + m_{\tilde t_2}^2 - 2m_t^2\right]}_{=\left[m_{\tilde q_t}^2 + m_{\tilde u_t}^2 + \frac{1}{2}M_Z^2\cos 2\beta\right]}\left(\frac{2}{\epsilon}\right)$$

log divergence

The complete, even one-loop, renormalization is complicated. Suffice it to say that by rescaling the fields with their $Z^{1/2}$ factors and adding mass counter-terms, etc., this divergent contribution to the $H_u^0 - H_u^{0+}$ 2 point function can be rendered finite and less dominant than the remaining m_t^4 terms. So ignoring this term we see that

Using dimensional regulation with the divergent term renormalized to a less dominant finite contribution we have the leading m_t^4 remaining finite result for each 1PI function. Recall the dimensional regulation of the integrals

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{[(p+k)^2-m^2][k^2-m^2]} = \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx \frac{1}{\left(x[(p+k)^2-m^2]+(1-x)[k^2-m^2]\right)^2}$$

$$= \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{1}{[\underbrace{k^2+2xp\cdot k+xp^2-m^2}_{=(k+xp)^2-m^2-x^2p^2+xp^2}]^2}$$

$$(\underbrace{k+xp}_{\equiv l})^2 \qquad \Rightarrow k = l - xp$$

$$= \int_0^1 dx \int \frac{d^4l}{(2\pi)^4} \frac{1}{[l^2-(m^2+x(x-1)p^2)]^2}$$

$$\overset{\lim_{d\to 4}}{=} \int_0^1 dx \int \frac{d^dl}{(2\pi)^d} \frac{1}{[l^2-\Delta_m]^2} \qquad (\Delta_m \equiv m^2+x(x-1)p^2)$$

$$\overset{d\to 4}{=} \int_0^1 dx \left[\frac{(-1)^2 i}{(4\pi)^{d/2}} \frac{\Gamma(2-\frac{d}{2})}{\Gamma(2)} \left(\frac{1}{\Delta_m}\right)^{2-\frac{d}{2}}\right]$$

$$\overset{\epsilon\to 0^+}{=} \int_0^1 dx \frac{i}{(4\pi)^2}\left[\frac{2}{\epsilon} - \ln\Delta_m - \gamma + \ln 4\pi + O(\epsilon)\right]$$

with $\epsilon = 4-d$

Now setting $p^2 = 0$ we have $\Delta_m = m^2$ & $\int_0^1 dx = 1$

$$\langle 0|T\tilde{H}_u^0(0) H_u^{0\dagger}(0)|0\rangle^{1PI} = \Gamma^{tree}_{H_u^0 H_u^{0\dagger}}(0)$$

$$+ 3\cdot\left[\frac{2m_t^2}{v^2\sin^2\beta}\right]\frac{im_t^2}{(4\pi)^2}\Big[-2\left(\cancel{\tfrac{2}{\epsilon}} - \ln m_t^2 - \gamma + \ln 4\pi\right)$$

$$+\left(\cancel{\tfrac{2}{\epsilon}} - \ln m_{t_1}^2 - \gamma + \ln 4\pi\right) + \left(\cancel{\tfrac{2}{\epsilon}} - \ln m_{t_2}^2 - \gamma + \ln 4\pi\right)$$

$$+ O(\epsilon)\Big]$$

Letting $\epsilon \to 0$ we have the finite result

$$\boxed{\begin{aligned}\langle 0|T\tilde{H}_u^0(p=0) H_u^{0\dagger}(0)|0\rangle^{1PI} &= \Gamma^{Tree}_{H_u^0 H_u^{0\dagger}}(p=0)\\ &- 3i\frac{2m_t^4}{(4\pi)^2 v^2\sin^2\beta}\ln\left[\frac{m_{t_1}^2 m_{t_2}^2}{m_t^4}\right]\end{aligned}}$$

Likewise for the other 2 1PI functions

$$\boxed{\begin{aligned}\langle 0|T\tilde{H}_u^0(p=0) H_u^{0}(0)|0\rangle^{1PI} &= \Gamma^{Tree}_{H_u^0 H_u^{0}}(p=0)\\ &- 3i\frac{2m_t^4}{16\pi^2 v^2\sin^2\beta}\ln\left[\frac{m_{t_1}^2 m_{t_2}^2}{m_t^4}\right]\end{aligned}}$$

and

$$\boxed{\langle 0|T\tilde{H_u^{0\dagger}}(p=0)\, H_u^{0\dagger}(0)|0\rangle^{1PI} = \Gamma^{Tree}_{H_u^{0\dagger} H_u^{0\dagger}}(p=0) - 3i\frac{2m_t^4}{16\pi^2 v^2 \sin^2\beta}\ln\left[\frac{m_{t_1}^2 m_{t_2}^2}{m_t^4}\right]}$$

Now recall the effective action generates the 1PI functions

$$\Gamma = \langle 0|T e^{\int d^4x\, \Phi_{cl}\phi}|0\rangle^{1PI}$$

$$= i\int d^4x\, \mathcal{L}_{eff}$$

For the $p=0$ case we have $\Gamma = -i\int d^4x\, V_{eff}$

and in our approximation

$$V_{effect.} = +i H_u^{0\dagger} H_u^0 \langle 0|T\hat{H}_u^{0\dagger}(0) H_u^0(0)|0\rangle^{1PI}$$
$$+\frac{i}{2} H_u^{0\dagger} H_u^{0\dagger}\langle 0|T\tilde{H}_u^0(0)\, H_u^0(0)|0\rangle^{1PI}$$
$$+\frac{i}{2} H_u^0 H_u^0 \langle 0|T\tilde{H}_u^{0\dagger}(0)\, H_u^{0\dagger}(0)|0\rangle^{1PI}$$

Converting to real and imaginary parts fields as before

$$H_u^0 = \frac{1}{\sqrt{2}}\left(H_u^R + iH_u^I\right),\quad H_u^{0\dagger} = \frac{1}{\sqrt{2}}\left(H_u^R - iH_u^I\right)$$

we find

$$V_{eff} = i\,\tfrac{1}{2}\left[H_u^R H_u^R + H_u^I H_u^I\right]\langle 0|T\,\tilde{H}_u^{0\dagger}(0)\,H_u^0(0)|0\rangle^{1PI}$$

$$+\tfrac{i}{2}\tfrac{1}{2}\left[H_u^R H_u^R - H_u^I H_u^I + 2i H_u^R H_u^I\right]\langle 0|T\,\tilde{H}_u^0(0)\,H_u^0(0)|0\rangle^{1PI}$$

$$+\tfrac{i}{2}\tfrac{1}{2}\left[H_u^R H_u^R - H_u^I H_u^I - 2i H_u^R H_u^I\right]\langle 0|T\,\tilde{H}_u^{0\dagger}(0)\,H_u^{0\dagger}(0)|0\rangle^{1PI}$$

$$= \tfrac{i}{2} H_u^R H_u^R\Big[\langle 0|T\,\tilde{H}_u^{0\dagger}(0)\,H_u^0(0)|0\rangle^{1PI} + \tfrac{1}{2}\langle 0|T\,\tilde{H}_u^0(0)\,H_u^0(0)|0\rangle^{1PI} + \tfrac{1}{2}\langle 0|T\,\tilde{H}_u^{0\dagger}(0)\,H_u^{0\dagger}(0)|0\rangle^{1PI}\Big]$$

$$+\tfrac{i}{2} H_u^I H_u^I\Big[\langle 0|T\,\tilde{H}_u^{0\dagger}(0)\,H_u^0(0)|0\rangle^{1PI} - \tfrac{1}{2}\langle 0|T\,\tilde{H}_u^0(0)\,H_u^0(0)|0\rangle^{1PI} - \tfrac{1}{2}\langle 0|T\,\tilde{H}_u^{0\dagger}(0)\,H_u^{0\dagger}(0)|0\rangle^{1PI}\Big]$$

---

where as in the tree case the mixed $H_u^R H_u^I$ terms vanish (cancel).

Also we see that the loop contributions to the imaginary parts cancel to leave only the tree contribution. Hence m_A^2 is still the same as in tree level

$$m_A^2 = -\frac{2b}{\sin 2\beta} \quad \text{and} \quad m_{H^\pm}^2 = m_A^2 + M_W^2$$

is still intact.

Now we are interested in corrections to the light higgs h mass. The h-H mass matrix is given by

(see p.-89-)

$$M_R^2 = \begin{bmatrix} \left.\dfrac{\partial^2 V_{eff}}{\partial H_u^R \partial H_u^R}\right|_{vev} & \left.\dfrac{\partial^2 V_{eff}}{\partial H_u^R \partial H_d^R}\right|_{vev} \\ \left.\dfrac{\partial^2 V_{eff}}{\partial H_d^R \partial H_u^R}\right|_{vev} & \left.\dfrac{\partial^2 V_{eff}}{\partial H_d^R \partial H_d^R}\right|_{vev} \end{bmatrix}$$

The stop & top correction we just calculated is in the $\left.\dfrac{\partial^2 V_{eff}}{\partial H_u^R \partial H_u^R}\right|_{vev}$ entry to the matrix.

The remainder is given by the tree potential So we have

p.-88- & p.-89- tree potential $V_{HS\,mass} = V_H$

$$V_{eff} = V_{HS\,mass} + \frac{1}{2} H_u^R H_u^R \underbrace{\left[3 \cdot \frac{4 \cdot m_t^4}{16\pi^2 v^2 \sin^2\beta} \ln\left[\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}\right]\right]}_{\equiv\, \delta}$$

Thus the H-h mass matrix becomes

$$\boxed{M_R^2 = \begin{bmatrix} (m_A^2 \cos^2\beta + M_Z^2 \sin^2\beta + \delta) & (-(m_A^2 + M_Z^2)\sin\beta\cos\beta) \\ (-(m_A^2 + M_Z^2)\sin\beta\cos\beta) & (m_A^2 \sin^2\beta + M_Z^2 \cos^2\beta) \end{bmatrix}}$$

The eigenvalues now become

$$m_h^2 = \frac{1}{2}\left[(m_A^2 + M_Z^2 + \delta) - \xi^{1/2}\right]$$

$$m_H^2 = \frac{1}{2}\left[(m_A^2 + M_Z^2 + \delta) + \xi^{1/2}\right]$$

where

$$\xi = \left[(m_A^2 - M_Z^2)\cos 2\beta + \delta\right]^2 + \left[m_A^2 + M_Z^2\right]^2 \sin^2 2\beta$$

with

$$\delta = 3\frac{g_2^2\, m_t^4}{16\pi^2 M_W^2 \sin^2\beta} \ln\left[\frac{m_{t_1}^2\, m_{t_2}^2}{m_t^4}\right]$$

$$= 3\cdot\frac{\alpha}{4\pi}\frac{m_t^4}{M_W^2 \sin^2\theta_W \sin^2\beta} \ln\left[\frac{m_{t_1}^2\, m_{t_2}^2}{m_t^4}\right]$$

So we have that

$$m_h^2 = m_{h\,tree}^2 + \delta\left(1 - \frac{(m_A^2 - M_Z^2)\cos 2\beta}{\sqrt{(m_A^2 - M_Z^2)^2\cos^2 2\beta + (m_A^2 + M_Z^2)^2 \sin^2 2\beta}}\right)$$

$$= m_{h\,tree}^2 + \delta\left[1 - \frac{1}{\sqrt{1 + \frac{(m_A^2 + M_Z^2)^2}{(m_A^2 - M_Z^2)^2}\tan^2 2\beta}}\right]$$

Now suppose $\frac{m_A^2+M_Z^2}{m_A^2-M_Z^2} \approx 1$, then

$$m_h^2 = m_{h\,tree}^2 + \delta(1-|\cos 2\beta|)$$

$$= m_{h\,tree}^2 + \delta(2\sin^2\beta)$$

$$\Rightarrow \boxed{m_h^2 = m_{h\,tree}^2 + 2\cdot 3\frac{\alpha}{4\pi}\frac{m_t^4}{M_W^2\sin^2\theta_W}\ln\left[\frac{m_{t_1}^2 m_{t_2}^2}{m_t^4}\right]}$$

So if $m_{t_1} \approx m_{t_2} \approx 350\,GeV$; $m_t \approx 174\,GeV$

then $$\delta \approx \frac{3}{4\pi}\frac{1}{137}\frac{174^4}{80^2(0.23)}\ln\left[\frac{350^4}{174^4}\right]\frac{GeV^2}{\sin^2\beta}$$

$$\approx \frac{3{,}033\,GeV^2}{\sin^2\beta}$$

Now
$$m_h = m_{h\,tree}\sqrt{1+2\frac{\delta\sin^2\beta}{m_{h\,tree}^2}}$$

$$\boxed{m_h \approx m_{h\,tree} + \frac{\delta\sin^2\beta}{m_{h\,tree}}}$$

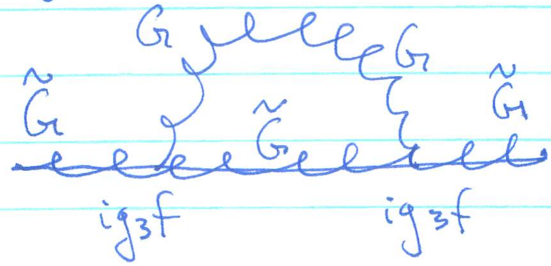
$$\boxed{\approx m_{h\,tree} + \frac{3{,}033\,GeV^2}{m_{h\,tree}}}$$

For $m_{h\,tree} = 90\,GeV \Rightarrow$ $\boxed{m_h \approx 124\,GeV}$.

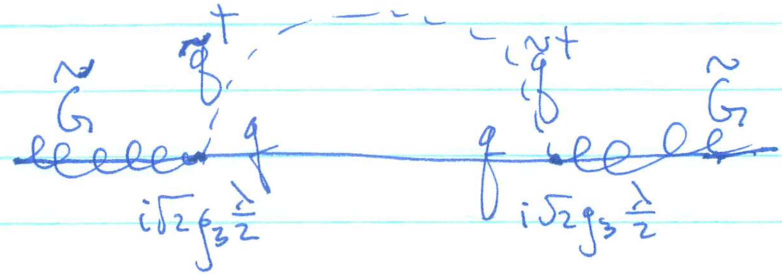
Beyond the reach of LEP II but within that of LHC!

Besides the significant radiative corrections to the Higgs mass, it has also been found that the gluino mass receives large corrections of the order of 25% due to loop diagrams (S. Martin & M. Vaughn Physics letters B318 (1993) 331.)

There are gluon exchange graphs



and quark-squark loops (in Weyl notation) neglecting squark mixing



These are a sum over a large number of graphs since there are 12 different quark-squark multiplets.

The dominant corrections to squark masses come from the strong interactions also and can have a complicated structure if intra-generation mixing is not negligible refer to D. Pierce, et al. Nucl. Phys. B491 (1997) 3 for details on spectra corrections.

Also the chargino & neutralino masses can be radiatively corrected. In fact under certain circumstances which particle is the LSP can depend on radiative corrections.

Finally at the one loop level h^0 can couple to down quarks via its couplings to up- & down-type squarks. For large $\tan\beta$ this can lead to masses depending on v_u that are significant corrections.

So we see radiative corrections can make a significant difference to the masses and acceptable parameter region.

Indeed FCNC and CP violating processes will further constrain SUSY parameter space as areas of the 105 parameter parameter space already violate experiment.