In order to determine the interactions with the mass eigenstate fields we will first recall the Lagrangian in general — then rotate the fields to the mass eigenstates.

Essentially we are applying our master formula for the Lagrangian (p. ~~84~~ ) to $SU(3)\times SU(2)\times U(1)$

$$\mathcal{L}_{susy} = -\frac{1}{4}F^{i}_{\mu\nu}F^{i\mu\nu} + \frac{i}{2}\bar{\lambda}\bar{\sigma}^{\mu}\overleftrightarrow{D}_{\mu}\lambda$$

$$+(D_{\mu}A)^{\dagger}(D^{\mu}A) + \frac{i}{2}\bar{\psi}\bar{\sigma}^{\mu}\overleftrightarrow{D}_{\mu}\psi$$

$$+\sqrt{2}g\left[A^{\dagger}(\lambda\cdot T)\psi + \bar{\psi}(\bar{\lambda}\cdot T)A\right]$$

$$+2\bar{\psi}^{a}\frac{\partial^{2}\bar{W}(\bar{A})}{\partial\bar{A}^{a}\partial\bar{A}^{b}}\bar{\psi}^{b} + 2\psi^{a}\frac{\partial^{2}W(A)}{\partial A^{a}\partial A^{b}}\psi^{b}$$

$$-\frac{1}{2}g^{2}(A^{\dagger}T^{i}A)(A^{\dagger}T^{i}A) - 16\left|\frac{\partial W}{\partial A^{a}}\right|^{2}$$

plus soft susy breaking terms $\mathcal{L}_{\not{s}} = \frac{1}{2}M\lambda\lambda + \frac{1}{2}\bar{M}\bar{\lambda}\bar{\lambda}$ $- A^{\dagger}m^{2}A - \mu B\frac{\partial W}{\partial\mu}\Big| - A\frac{\partial W}{\partial y}\Big| - \bar{\mu}\bar{B}\frac{\partial\bar{W}}{\partial\bar{\mu}}\Big| - \bar{A}\frac{\partial\bar{W}}{\partial\bar{y}}$.

Now we already showed that the partner independent part of the Lagrangian re-produces the SM Lagrangian but with two Higgs doublets and a restricted Higgs potential for them. But let's repeat all!

Consider the good SUSY Lagrangian first

$$\mathcal{L}_{SUSY} = \mathcal{L}_{YM} + \mathcal{L}_K + \mathcal{L}_Y + \mathcal{L}_D + \mathcal{L}_F + \mathcal{L}_{\tilde{Y}}$$

with now

$$\mathcal{L}_{YM} = -\tfrac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} - \tfrac{1}{4} B_{\mu\nu} B^{\mu\nu} - \tfrac{1}{4} G^m_{\mu\nu} G^{m\mu\nu}$$
$$+ \tfrac{i}{2} \tilde{A} \sigma^\mu \overleftrightarrow{D}_\mu \bar{\tilde{A}} + \tfrac{i}{2} \tilde{B} \overleftrightarrow{\partial\!\!\!/} \bar{\tilde{B}} + \tfrac{i}{2} \tilde{G} \sigma^\mu \overleftrightarrow{D}_\mu \bar{\tilde{G}}$$

with $F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g_2 \epsilon_{ijk} A^j_\mu A^k_\nu$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$G^m_{\mu\nu} = \partial_\mu G^m_\nu - \partial_\nu G^m_\mu + g_3 f_{mnp} G^n_\mu G^p_\nu$$

$$(D_\mu \tilde{A})^i = \partial_\mu \tilde{A}^i + g_2 \epsilon_{ijk} A^j_\mu \tilde{A}^k$$

$$(D_\mu \tilde{G})^m = \partial_\mu \tilde{G}^m + g_3 f_{mnp} G^n_\mu \tilde{G}^p$$

---

Now the Kähler potential leads to many terms with gauge interactions & Yukawa couplings with gauginos and D-terms

So matter kinetic terms have the usual form

$$\mathcal{L}_K = i\bar{l}\,\bar{\sigma}^\mu D_\mu l + i\bar{q}\,\bar{\sigma}^\mu D_\mu q$$
$$+ i\bar{e}^c\bar{\sigma}^\mu D_\mu e^c + i\bar{u}^c\bar{\sigma}^\mu D_\mu u^c$$
$$+ i\bar{d}^c\bar{\sigma}^\mu D_\mu d^c + i\bar{\tilde{H}}_u\bar{\sigma}^\mu D_\mu \tilde{H}_u$$
$$+ i\bar{\tilde{H}}_d\bar{\sigma}^\mu D_\mu \tilde{H}_d$$
$$+ (D_\mu\tilde{l})^\dagger(D^\mu\tilde{l}) + (D_\mu\tilde{q})^\dagger(D^\mu\tilde{q})$$
$$+ (D_\mu\tilde{e}^c)^\dagger(D^\mu\tilde{e}^c) + (D_\mu\tilde{u}^c)^\dagger(D^\mu\tilde{u}^c)$$
$$+ (D_\mu\tilde{d}^c)^\dagger(D^\mu\tilde{d}^c) + (D_\mu H_u)^\dagger(D^\mu H_u)$$
$$+ (D_\mu H_d)^\dagger(D^\mu H_d)$$

---

where p. -37- & -38- are re-listed for the covariant derivatives. Recall we still must shift the Higgs fields about their vacuum values which affects the last two terms.

Next integrating out the D terms we find

$$\mathcal{L}_D = -\frac{1}{2}g_1^2\Big[-\frac{1}{2}\tilde{l}^\dagger\tilde{l} + \frac{1}{6}\tilde{q}^\dagger\tilde{q} + \tilde{e}^{c\dagger}\tilde{e}^c - \frac{2}{3}\tilde{u}^{c\dagger}\tilde{u}^c + \frac{1}{3}\tilde{d}^{c\dagger}\tilde{d}^c + \frac{1}{2}H_u^\dagger H_u - \frac{1}{2}H_d^\dagger H_d\Big]^2$$

$$-\frac{1}{2}g_2^2\Big[\tilde{l}^\dagger\frac{\sigma^i}{2}\tilde{l} + \tilde{q}^\dagger\frac{\sigma^i}{2}\tilde{q} + H_u^\dagger\frac{\sigma^i}{2}H_u + H_d^\dagger\frac{\sigma^i}{2}H_d\Big]^2$$

$$-\frac{1}{2}g_3^2\Big[\tilde{q}^\dagger\frac{\lambda^a}{2}\tilde{q} - \tilde{u}^{c\dagger}\frac{\lambda^a}{2}\tilde{u}^c - \tilde{d}^{c\dagger}\frac{\lambda^a}{2}\tilde{d}^c\Big]^2$$

where again $H_u$ & $H_d$ will be shifted about their vev's

---

Finally we have the gaugino Yukawa terms

$$\mathcal{L}_{\tilde{Y}} = \sqrt{2}\,g_3\Big[\tilde{q}^\dagger(\tilde{G}\cdot\frac{\lambda}{2})q + \bar{q}(\bar{\tilde{G}}\cdot\frac{\lambda}{2})\tilde{q} - \tilde{u}^{c\dagger}(\tilde{G}\cdot\frac{\lambda}{2})u^c - \overline{u^c}(\bar{\tilde{G}}\cdot\frac{\lambda}{2})\tilde{u}^c - \tilde{d}^{c\dagger}(\tilde{G}\cdot\frac{\lambda}{2})d^c - \overline{d^c}(\bar{\tilde{G}}\cdot\frac{\lambda}{2})\tilde{d}^c\Big]$$

$$+\sqrt{2}\,g_2\Big[\tilde{l}^\dagger(\tilde{A}\cdot\frac{\sigma}{2})l + \bar{l}(\bar{\tilde{A}}\cdot\frac{\sigma}{2})\tilde{l} + \tilde{q}^\dagger(\tilde{A}\cdot\frac{\sigma}{2})q + \bar{q}(\bar{\tilde{A}}\cdot\frac{\sigma}{2})\tilde{q} + H_u^\dagger(\tilde{A}\cdot\frac{\sigma}{2})\tilde{H}_u + \bar{\tilde{H}}_u(\bar{\tilde{A}}\cdot\frac{\sigma}{2})H_u + H_d^\dagger(\tilde{A}\cdot\frac{\sigma}{2})\tilde{H}_d + \bar{\tilde{H}}_d(\bar{\tilde{A}}\cdot\frac{\sigma}{2})H_d\Big]$$

$$+\sqrt{2}\, g_1 \Big[ -\tfrac{1}{2}\tilde{l}^\dagger \tilde{B}\, l + \tfrac{1}{6}\tilde{q}^\dagger \tilde{B}\, q - \tfrac{1}{2}\bar{l}\,\tilde{\bar{B}}\,\tilde{l} + \tfrac{1}{6}\bar{q}\,\tilde{\bar{B}}\,\tilde{q}$$

$$+\tilde{e}^{c\dagger}\tilde{B}\, e^c + \bar{e^c}\,\tilde{\bar{B}}\,\tilde{e}^c - \tfrac{2}{3}\tilde{u}^{c\dagger}\tilde{B}\, u^c - \tfrac{2}{3}\bar{u^c}\,\tilde{\bar{B}}\,\tilde{u}^c$$

$$+\tfrac{1}{3}\tilde{d}^{c\dagger}\tilde{B}\, d^c + \tfrac{1}{3}\bar{d^c}\,\tilde{\bar{B}}\,\tilde{d}^c + \tfrac{1}{2}H_u^\dagger \tilde{B}\,\tilde{H}_u$$

$$+\tfrac{1}{2}\tilde{\bar{H}}_u\,\tilde{\bar{B}}\, H_u - \tfrac{1}{2}H_d^\dagger \tilde{B}\,\tilde{H}_d - \tfrac{1}{2}\tilde{\bar{H}}_d\,\tilde{\bar{B}}\, H_d \Big]$$

---

Finally we have the (s)matter Yukawa and F-terms. Consider the F-terms first:

$$\mathcal{L}_F = -\left[4\bar{\mu} H_d^\dagger + 4\tilde{u}^{c\dagger} y_u \tilde{q}^\dagger\right]\left[4\mu H_d + 4\tilde{q}\, y_u \tilde{u}^c\right]$$

$$-\left[4\bar{\mu} H_u^\dagger - 4\tilde{d}^{c\dagger} y_d^\dagger \tilde{q}^\dagger - 4\tilde{e}^{c\dagger} y_e^\dagger \tilde{l}^\dagger\right]\left[4\mu H_u - 4\tilde{q}\, y_d \tilde{d}^c - 4\tilde{l}\, y_e \tilde{e}^c\right]$$

$$-\left[4\tilde{u}^{c\dagger} y_u^\dagger H_u^\dagger + 4\tilde{d}^{c\dagger} y_d^\dagger H_d^\dagger\right]\left[4 H_u y_u \tilde{u}^c + 4 H_d y_d \tilde{d}^c\right]$$

$$-\left[16 H_d^\dagger H_d\, \tilde{e}^{c\dagger} y_e^\dagger y_e \tilde{e}^c\right].$$

$$-\left[16 H_u\cdot\tilde{q}\, y_u y_u^\dagger H_u^\dagger\cdot\tilde{q}^\dagger\right]$$

$$-\left[16 H_d\cdot\tilde{q}\, y_d y_d^\dagger H_d^\dagger\cdot\tilde{q}^\dagger\right]$$

$$-\left[16 H_d\cdot\tilde{l}\, y_e y_e^\dagger H_d^\dagger\cdot\tilde{l}^\dagger\right]$$

---

Finally the mass & Yukawa terms from the superpotential

$$\mathcal{L}_Y = 4\mu \tilde{H}_u \cdot \tilde{H}_d + 4\bar{\mu}\, \tilde{H}_u^\dagger \cdot \tilde{H}_d^\dagger$$

$$+ 4\tilde{H}_u \cdot q\, y_u\, \tilde{u}^c + 4\tilde{H}_u \cdot \tilde{q}\, y_u\, u^c + 4 H_u \cdot q\, y_u\, u^c$$

$$+ 4\tilde{H}_d \cdot q\, y_d\, \tilde{d}^c + 4\tilde{H}_d \cdot \tilde{q}\, y_d\, d^c + 4 H_d \cdot q\, y_d\, d^c$$

$$+ 4\tilde{H}_d \cdot l\, y_e\, \tilde{e}^c + 4\tilde{H}_d \cdot \tilde{l}\, y_e\, e^c + 4 H_d \cdot l\, y_e\, e^c$$

$$+ 4\tilde{u}^{c\dagger} y_u^\dagger\, \bar{\tilde{H}}_u \cdot \bar{q} + 4\bar{u}^c y_u^\dagger\, \bar{\tilde{H}}_u \cdot \tilde{q}^\dagger + 4\bar{u}^c y_u^\dagger H_u^\dagger \cdot \bar{q}$$

$$+ 4\tilde{d}^{c\dagger} y_d^\dagger\, \bar{\tilde{H}}_d \cdot \bar{q} + 4\bar{d}^c y_d^\dagger\, \bar{\tilde{H}}_d \cdot \tilde{q}^\dagger + 4\bar{d}^c y_d^\dagger H_d^\dagger \cdot \bar{q}$$

$$+ 4\tilde{e}^{c\dagger} y_e^\dagger\, \bar{\tilde{H}}_d \cdot \bar{l} + 4\bar{e}^c y_e^\dagger\, \bar{\tilde{H}}_d \cdot \tilde{l}^\dagger + 4\bar{e}^c y_e^\dagger H_d^\dagger \cdot \bar{l}$$

---

This completes the supersymmetric part of the MSSM Lagrangian.

(Note: $\tilde{H}_u \cdot \tilde{H}_d \equiv \tilde{H}^\alpha_{u\,i}\, \epsilon_{ij}\, \tilde{H}_{d\,j\alpha}$; $\epsilon_{ij} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ etc.)

---

Finally consider the soft SUSY breaking terms. (We are ignoring C-type breaking (trilinear) and $m_K, \bar{m}_K$ Kähler type breaking since this last leads to the same corrections as $M^2$ Kähler breaking). The contributions are

$$\mathcal{L}_{\not S} = \mathcal{L}_{\not S YM} + \mathcal{L}_{\not S K} + \mathcal{L}_{\not S W}$$

with the gaugino masses given by

$$\mathcal{L}_{\not S YM} = \frac{1}{2}\left[M_1 \tilde{B}\tilde{B} + \bar{M}_1 \bar{\tilde{B}}\bar{\tilde{B}}\right]$$
$$+ \frac{1}{2}\left[M_2 \tilde{A}^i\tilde{A}^i + \bar{M}_2 \bar{\tilde{A}}^i\bar{\tilde{A}}^i\right]$$
$$+ \frac{1}{2} M_3\left[\tilde{G}^m\tilde{G}^m + \bar{\tilde{G}}^m\bar{\tilde{G}}^m\right]$$

recalling that we made $M_3$ = real by a re-phasing definition of $\tilde{G}^m$ and the squark fields, this is a conventional choice.

---

Likewise, the Kähler potential soft-SUSY breaking yields the squark & slepton and Higgs scalar soft SUSY breaking (masses)$^2$

$$\mathcal{L}_{\not{S}K} = -\tilde{l}^{\dagger} m_l^2 \tilde{l} - \tilde{q}^{\dagger} m_q^2 \tilde{q} - \tilde{e}^{c\dagger} m_{e^c}^2 \tilde{e}^c$$

$$- \tilde{u}^{c\dagger} m_{u^c}^2 \tilde{u}^c - \tilde{d}^{c\dagger} m_{d^c}^2 \tilde{d}^c$$

$$- m_{H_u}^2 H_u^{\dagger} H_u - m_{H_d}^2 H_d^{\dagger} H_d$$

---

we have analyzed these mass breaking terms already along with the mass terms from the superpotential breaking

$$\mathcal{L}_{\not{S}W} = -\mu B H_u \cdot H_d - \bar{\mu} \bar{B} H_u^{\dagger} \cdot H_d^{\dagger}$$

$$- H_u \cdot \tilde{q} A_u \tilde{u}^c - \tilde{u}^{c\dagger} A_u^{\dagger} H_u^{\dagger} \cdot \tilde{q}^{\dagger}$$

$$- H_d \cdot \tilde{q} A_d \tilde{d}^c - \tilde{d}^{c\dagger} A_d^{\dagger} H_d^{\dagger} \cdot \tilde{q}^{\dagger}$$

$$- H_d \cdot \tilde{l} A_l \tilde{e}^c - \tilde{e}^{c\dagger} A_l^{\dagger} H_d^{\dagger} \cdot \tilde{l}^{\dagger}$$

---

Thus from $\mathcal{L}_{\not{S}}$ only the last tri-linear terms yield new interactions of the Higgs and SUSY partner fields.

---

So we have the complete MSSM lagrangian with soft-SUSY breaking terms. It remains to shift the Higgs scalars by their vev's $H_u \to H_u + \begin{pmatrix} 0 \\ v_u/\sqrt{2} \end{pmatrix}$, $H_d \to H_d + \begin{pmatrix} v_d/\sqrt{2} \\ 0 \end{pmatrix}$. This does not affect the dimension $= 4$ terms only the dimension 1, 2, 3. Dimension 1 yields the minimum of the potential - hence zero. The dimension 2 yielded our lengthy mass analysis. Dimension 3 are interactions due to the EWSB and vanish if $v_u$ and $v_d = 0$. As we saw, the weak interaction fields are not the mass eigenstate fields - Also the charge conjugate spinors can be replaced by right handed spinors and combined with the left-handed similar fields to become components of Dirac spinors (and neutrinos are just left-handed so far). In this way we recovered the SM as part of the MSSM.

As we had earlier when analyzing the SM interactions, for example, the Dirac spinor of quark fields was given by

$$\underset{\text{Dirac upquark}}{u} = \begin{pmatrix} u \\ \bar{u}^c \end{pmatrix} \leftarrow \text{Weyl} \qquad \text{so that } u_L = \gamma_- u = \begin{pmatrix} u \\ 0 \end{pmatrix}$$

$$u_R = \gamma_+ u = \begin{pmatrix} 0 \\ \bar{u}^c \end{pmatrix}$$

and $\bar{u}_L = [\,0 \;\; \bar{u}\,]$ ; $\bar{u}_R = [\,u^c \;\; 0\,]$

(Recall also $u^c = \begin{bmatrix} u^c \\ \bar{u} \end{bmatrix}$ $\quad \underset{\text{Dirac}}{u^c_L} = \begin{bmatrix} u^c \\ 0 \end{bmatrix} \quad u^c_R = \begin{bmatrix} 0 \\ \bar{u} \end{bmatrix}$

and so $\bar{u}^c_L = [\,0 \;\; \bar{u}^c\,]$ ; $\bar{u}^c_R = [\,u \;\; 0\,]$ )

So we have as previously, p. -42- to -50-

$$\bar{u}_L \gamma^\mu \partial_\mu u_L = \bar{u}\bar{\sigma}^\mu \partial_\mu u$$

$$\underbrace{\bar{u}^c_L \gamma^\mu \partial_\mu u^c_L}_{\text{Dirac}} = \underbrace{\bar{u}^c \bar{\sigma}^\mu \partial_\mu u^c}_{\text{Weyl}} = \underbrace{-\partial_\mu \bar{u}_R \gamma^\mu u_R}_{\text{Dirac}}$$

So we can do likewise for the down quarks & leptons

$$\underset{\text{Dirac}}{d} = \begin{pmatrix} d \\ \bar{d}^c \end{pmatrix} \leftarrow \text{Weyl} \quad ; \quad \underset{\text{Dirac}}{e} = \begin{pmatrix} e \\ \bar{e}^c \end{pmatrix} \leftarrow \text{Weyl}$$

Now the neutrino is only left handed (Majorana) so far so we have

$$\nu_e = \begin{pmatrix} \nu_e \\ \bar{\nu}_e \end{pmatrix} \quad \text{and} \quad \nu_{eL} = \begin{pmatrix} \nu_e \\ 0 \end{pmatrix}$$

Majorana — Weyl

$$\nu_{eR} = \begin{pmatrix} 0 \\ \bar{\nu}_e \end{pmatrix}$$

$$\nu_e^c = \begin{pmatrix} \nu_e \\ \bar{\nu}_e \end{pmatrix} = \nu_e$$

self-charge-conjugate (Majorana)

---

So $\underbrace{\bar{\nu}_{eL} \not{\partial} \nu_{eL}}_{\text{4 component Dirac}} = \underbrace{\bar{\nu}_e \bar{\sigma}^\mu \partial_\mu \nu_e}_{\text{Weyl}}$.

---

Now correspondingly we squark & slepton scalar fields creating and annihilating smatter For instance

$\tilde{u}^\dagger$ creates a scalar up-quark partner to $\bar{u}_L$

$\tilde{u}^c$ creates a scalar charge conjugate up-quark partner to $\bar{u}_R$

or $\tilde{u}$ annihilates the scalar partner to $u_L$

$\tilde{u}^{c\dagger}$ annihilates the scalar partner to $u_R$

---

The smaller scalars will be kept as they are since in the end we must rotate them into their mass eigenfields. Recall the results of that analysis:

Gauge fields: $W_\mu^\pm = \frac{1}{\sqrt{2}}[A_\mu^1 \mp i A_\mu^2]$

$$Z_\mu = A_\mu^3 \cos\theta_W - B_\mu \sin\theta_W$$

$$A_\mu = A_\mu^3 \sin\theta_W + B_\mu \cos\theta_W$$

$$\tan\theta_W \equiv \frac{g_1}{g_2}$$

and so $A_\mu^3 = Z_\mu \cos\theta_W + A_\mu \sin\theta_W$

$$B_\mu = A_\mu \cos\theta_W - Z_\mu \sin\theta_W$$

---

where $A_\mu$ is massless, $W_\mu^\pm$ has mass $M_W = \frac{g_2 v}{2}$

$Z_\mu$ has mass $M_Z = \frac{\sqrt{g_1^2+g_2^2}\, v}{2} = \frac{M_W}{\cos\theta_W}$

---

Matter Fermion Masses: as in the SM

$$M^u = -\frac{1}{\sqrt{2}}\Gamma^u v \sin\beta = \frac{4}{\sqrt{2}} y_u^* v \sin\beta$$
$$M^d = +\frac{1}{\sqrt{2}}\Gamma^d v \cos\beta = -\frac{4}{\sqrt{2}} y_d^* v \cos\beta$$
$$M^e = +\frac{1}{\sqrt{2}}\Gamma^e v \cos\beta = -\frac{4}{\sqrt{2}} y_\ell^* v \cos\beta$$

In general then

$$M_{diagonal} = \begin{pmatrix} m_1 & & 0 \\ & m_2 & \\ 0 & & m_3 \end{pmatrix} = A_L^\dagger M A_R$$

The weak basis fields (now with superscript W) are related to the mass eigenfields (no superscript) by the A matrices

$$u_L^W = A_L^u u_L \quad , \quad u_R^W = A_R^u u_R$$

$$d_L^W = A_L^d d_L \quad , \quad d_R^W = A_R^d d_R$$

$$e_L^W = A_L^e e_L \quad , \quad e_R^W = A_R^e e_R$$

$(\nu_L \equiv A_L^{e\dagger} \nu_L^W$ since massless at this point$)$

---

The SM lagrangian only depended on the CKM combination

$(V = A_{CKM}^\dagger)$

$$A_{CKM} \equiv (A_L^{d\dagger} A_L^u)$$

---

The diagonal mass matrices just give the mass eigenvalues:

$$M_{diagonal}^u = \begin{pmatrix} m_u & & 0 \\ & m_c & \\ 0 & & m_t \end{pmatrix} \quad ; \quad M_{diagonal}^d = \begin{pmatrix} m_d & & 0 \\ & m_s & \\ 0 & & m_b \end{pmatrix}$$

$$M_{diagonal}^e = \begin{pmatrix} m_e & & 0 \\ & m_\mu & \\ 0 & & m_\tau \end{pmatrix}$$

---

## Higgs Fields: $H_u^\pm, H_u^0, H_u^{0\dagger}; H_d^\pm, H_d^0, H_d^{0\dagger}$

a) Charged Higgs Fields: $H^\pm, \pi^\pm$

$$\begin{bmatrix} H^- \\ \pi^- \end{bmatrix} = S \begin{bmatrix} H_u^- \\ H_d^- \end{bmatrix} \quad ; \quad S = \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

$$\begin{bmatrix} H^+ \\ \pi^+ \end{bmatrix} = S \begin{bmatrix} H_u^+ \\ H_d^+ \end{bmatrix}$$

diagonal

$$S M_{H^\pm}^2 S^T = m_{H^\pm}^2 = \begin{bmatrix} m_{H^\pm}^2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$m_{H^\pm}^2 = \frac{-2b}{\sin 2\beta} + M_W^2$$

$$= m_A^2 + M_W^2 \qquad (\text{tree level: } m_{H^\pm} \geq M_W, \ m_{H^\pm} \geq m_A)$$

$$m_{\pi^\pm}^2 = 0 \quad (\text{Goldstone bosons eaten by } W_\mu^\pm)$$

---

b) Neutral PseudoScalar Higgs: $A, \pi^0$

$$\begin{bmatrix} A \\ \pi^0 \end{bmatrix} = S \begin{bmatrix} H_u^I \\ H_d^I \end{bmatrix}$$

$$\boxed{\begin{aligned} H_u^0 &= \frac{1}{\sqrt{2}}\left[H_u^R + i H_u^I\right] \\ H_d^0 &= \frac{1}{\sqrt{2}}\left[H_d^R + i H_d^I\right] \end{aligned}}$$

$$S M_I^2 S^T = m_I^2 = \begin{bmatrix} m_A^2 & 0 \\ 0 & 0 \end{bmatrix}$$

diagonal

$$m_A^2 = \frac{-2b}{\sin 2\beta} \quad ; \quad m_{\pi^0}^2 = 0 \quad (\text{Goldstone boson eaten by } Z_\mu)$$

---

c) Neutral Scalar Higgs: $H$, $h$

$$\begin{bmatrix} h \\ H \end{bmatrix} = R \begin{bmatrix} H_u^R \\ H_d^R \end{bmatrix} \qquad R \equiv \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$R M_R^2 R^T = \underbrace{m_R^2}_{\text{diagonal}} = \begin{bmatrix} m_h^2 & 0 \\ 0 & m_H^2 \end{bmatrix}$$

where

$$\tan\alpha = \frac{(m_A^2 - M_Z^2)\cos 2\beta + \sqrt{(m_A^2+M_Z^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta}}{(m_A^2+M_Z^2)\sin 2\beta}$$

and the mass eigenvalues are:

$$\boxed{\begin{aligned} m_h^2 &= \tfrac{1}{2}\left[(m_A^2+M_Z^2) - \sqrt{(m_A^2+M_Z^2)^2 - 4m_A^2 M_Z^2\cos^2 2\beta}\right] \\ m_H^2 &= \tfrac{1}{2}\left[(m_A^2+M_Z^2) + \sqrt{(m_A^2+M_Z^2)^2 - 4m_A^2 M_Z^2\cos^2 2\beta}\right] \end{aligned}}$$

$m_h^2 \leq m_H^2$ ; $h$ is the light Higgs scalar

$$m_h \leq m_A|\cos 2\beta| \leq m_H$$

$$m_h \leq M_Z|\cos 2\beta| \leq m_H$$

$\Rightarrow$ $\underline{m_h \leq M_Z}$ tree level (radiative corrections are important to put $m_h > M_Z$ !!)

## Sparticle Masses:

Gluinos: gluinos do not mix. SU(3) octet of gluinos $\tilde{G}^m_\alpha, \bar{\tilde{G}}^m_{\dot\alpha}$ (real) mass $M_3$.

---

Charginos: $\tilde{H}^+_u, \tilde{H}^-_d$ $(\bar{\tilde{H}}^+_u, \bar{\tilde{H}}^-_d)$ charged Higgsinos

$\tilde{W}^+, \tilde{W}^-$ $(\bar{\tilde{W}}^+, \bar{\tilde{W}}^-)$ charged Gauginos

with $\tilde{W}^\pm \equiv \frac{1}{\sqrt{2}}(\tilde{A}^1 \mp i\tilde{A}^2)$

---

$$\begin{bmatrix} \tilde{C}^+_1 \\ \tilde{C}^+_2 \\ \tilde{C}^-_1 \\ \tilde{C}^-_2 \end{bmatrix} = \begin{bmatrix} R^* & 0 \\ 0 & L^* \end{bmatrix} \begin{bmatrix} \tilde{W}^+ \\ \tilde{H}^+_u \\ \tilde{W}^- \\ \tilde{H}^-_d \end{bmatrix}$$

and

$$M_{chargino} = \begin{bmatrix} 0 & M^T_{ch} \\ M_{ch} & 0 \end{bmatrix} \quad ; \quad \underset{\uparrow \text{diagonal}}{m_{\tilde{C}}} = L^* M_{ch} R^{-1}$$

Now $M_{chargino} \bar{M}_{chargino} = \begin{bmatrix} M^T_{ch} \bar{M}_{ch} & 0 \\ 0 & M_{ch} \bar{M}^T_{ch} \end{bmatrix} = m^2_{ch}$

The eigenvalues are repeated in each 2x2 submatrix

$$m_{\tilde{C}_1}^2 = \frac{1}{2}\Big[16|\mu|^2 + |M_2|^2 + 2M_W^2 - \sqrt{(16|\mu|^2 + |M_2|^2 + 2M_W^2)^2 - 4|4\mu M_2 - M_W^2 \sin 2\beta|^2}\,\Big]$$

$$m_{\tilde{C}_2}^2 = \frac{1}{2}\Big[16|\mu|^2 + |M_2|^2 + 2M_W^2 + \sqrt{(16|\mu|^2 + |M_2|^2 + 2M_W^2)^2 - 4|4\mu M_2 - M_W^2 \sin 2\beta|^2}\,\Big]$$

---

It is left to find the eigenvectors, that is L & R

$$m_{\tilde{C}} = L^* M_{ch} R^{-1} \qquad m_{\tilde{C}} = m_{\tilde{C}}^\dagger = R \bar{M}^T L^T$$

diagonal

So $$m_{\tilde{C}}^2 = R \bar{M}_{ch}^T M_{ch} R^{-1} = L^* M_{ch} \bar{M}_{ch}^T L^T$$

diagonal

---

Neutralinos: Neutral Higgsinos: $\tilde{H}_u^0, \tilde{H}_d^0$ & $\bar{\tilde{H}}_u^0, \bar{\tilde{H}}_d^0$

Neutral Gauginos: $\tilde{A}^3, \tilde{B}$ & $\bar{\tilde{A}}^3, \bar{\tilde{B}}$

$M_{Neutralino}$ = 4x4 symmetric complex (not hermitean) matrix (L=R case)

diagonal $\rightarrow$ $m_{\tilde{N}} = U^* M_{Neutralino} U^{-1}$

$\Rightarrow$

$$\underset{\uparrow \text{diagonal}}{m_{\tilde{N}}^2} = U^* \underbrace{M_{Neut.} M^\dagger_{Neut}}_{\text{Hermitean}} U^T$$

Eigenstates

$$\begin{bmatrix} \tilde{H}_u^0 \\ \tilde{H}_d^0 \\ \tilde{A}^3 \\ \tilde{B} \end{bmatrix} = \begin{bmatrix} \left(u^{(1)}\right) & \left(u^{(2)}\right) & \left(u^{(3)}\right) & \left(u^{(4)}\right) \end{bmatrix} \begin{bmatrix} \tilde{Z}_1 \\ \tilde{Z}_2 \\ \tilde{Z}_3 \\ \tilde{Z}_4 \end{bmatrix}$$

$u^{(i)} = 4$ eigenvectors of $U = \begin{pmatrix} u_1^{(i)} \\ u_2^{(i)} \\ u_3^{(i)} \\ u_4^{(i)} \end{pmatrix}$

$m_{\tilde{Z}_1} \leq m_{\tilde{Z}_2} \leq m_{\tilde{Z}_3} \leq m_{\tilde{Z}_4}$ by convention.

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Squarks & sleptons: $\left.\begin{matrix} \tilde{u}, \tilde{u}^{c\dagger}; \tilde{c}, \tilde{c}^{c\dagger}; \tilde{t}, \tilde{t}^{c\dagger} \\ \tilde{d}, \tilde{d}^{c\dagger}; \tilde{s}, \tilde{s}^{c\dagger}; \tilde{b}, \tilde{b}^{c\dagger} \end{matrix}\right\}$ squarks

$\left.\begin{matrix} \tilde{e}, \tilde{e}^{c\dagger}; \tilde{\mu}, \tilde{\mu}^{c\dagger}; \tilde{\tau}, \tilde{\tau}^{c\dagger} \\ \tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau \end{matrix}\right\}$ sleptons

all complex scalar fields.

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We will 1) ignore interfamily mixing; 2) 1st & 2nd generation have no mixing; 3) only stop, sbottom, stau have intrafamily mixing

1) First generation: all fields are mass eigenfields with masses

$$m_{\tilde{u}}^2 = m_u^2 + m_{q_u}^2 + M_Z^2 \cos 2\beta \left(\tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta_W\right)$$

$$m_{\tilde{u}^c}^2 = m_u^2 + m_{u^c_u}^2 + M_Z^2 \cos 2\beta \left(\tfrac{2}{3}\sin^2\theta_W\right)$$

$$m_{\tilde{d}}^2 = m_d^2 + m_{q_u}^2 + M_Z^2 \cos 2\beta \left(-\tfrac{1}{2} + \tfrac{1}{3}\sin^2\theta_W\right)$$

$$m_{\tilde{d}^c}^2 = m_d^2 + m_{d^c_d}^2 + M_Z^2 \cos 2\beta \left(-\tfrac{1}{3}\sin^2\theta_W\right)$$

$$m_{\tilde{e}}^2 = m_e^2 + m_{l_e}^2 + M_Z^2 \cos 2\beta \left(-\tfrac{1}{2} + \sin^2\theta_W\right)$$

$$m_{\tilde{e}^c}^2 = m_e^2 + m_{e^c_e}^2 - M_Z^2 \cos 2\beta \left(\sin^2\theta_W\right)$$

$$m_{\tilde{\nu}_e}^2 = m_{l_e}^2 + \tfrac{1}{2} M_Z^2 \cos 2\beta$$

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2) Second generation: all fields are mass eigenfields with masses

$$m_{\tilde{c}}^2 = m_c^2 + m_{q_c}^2 + M_Z^2 \cos 2\beta \left(\tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta_W\right)$$

$$m_{\tilde{c}^c}^2 = m_c^2 + m_{u^c_c}^2 + M_Z^2 \cos 2\beta \left(\tfrac{2}{3}\sin^2\theta_W\right)$$

$$m_{\tilde{s}}^2 = m_s^2 + m_{q_c}^2 + M_Z^2 \cos 2\beta \left(-\tfrac{1}{2} + \tfrac{1}{3}\sin^2\theta_W\right)$$

$$m_{\tilde{s}^c}^2 = m_s^2 + m_{d^c_s}^2 + M_Z^2 \cos 2\beta \left(-\tfrac{1}{3}\sin^2\theta_W\right)$$

2)

$$m^2_{\tilde{\mu}} = m^2_{\mu} + m^2_{l_\mu} + M_Z^2 \cos 2\beta\left(-\tfrac{1}{2} + \sin^2\theta_W\right)$$

$$m^2_{\tilde{\mu}^c} = m^2_{\mu} + m^2_{e^c_\mu} - M_Z^2 \cos 2\beta\left(\sin^2\theta_W\right)$$

$$m^2_{\tilde{\nu}_\mu} = m^2_{l_\mu} + \tfrac{1}{2} M_Z^2 \cos 2\beta$$

3) Third generation: Intragenerational mixing since the masses of top, bottom & tau are heavy compared to soft SUSY breaking masses (although SUSY breaking masses are larger)

a.) stop mixing:

$$\begin{bmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{bmatrix} = S_{\tilde{t}} \begin{bmatrix} \tilde{t} \\ \tilde{t}^{c\,t} \end{bmatrix}$$

$$S_{\tilde{t}} = \begin{bmatrix} \cos\theta_t & -\sin\theta_t \\ \sin\theta_t & \cos\theta_t \end{bmatrix} \text{ and } S_{\tilde{t}} M^2_{\tilde{t}} S^T_{\tilde{t}} = M^2_{\tilde{t}\,diag.} = \begin{bmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{bmatrix}$$

where the eigenmasses are

(If $\mu, A_t$ are complex $\begin{bmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{bmatrix} = U_{\tilde{t}} \begin{bmatrix} \tilde{t} \\ \tilde{t}^{c\,t} \end{bmatrix}$ where the $2\times 2$ mixing matrix $U_{\tilde{t}}$ is unitary instead of orthogonal)

3.a.) given by (including $\mu$, $A_{\tilde{t}}$ being complex)

$$m^2_{\tilde{t}_1} = m_t^2 + \frac{1}{2}(m^2_{q_t} + m^2_{u^c_t}) + \frac{1}{4} M_Z^2 \cos 2\beta$$

$$- \sqrt{\left(\frac{1}{2}(m^2_{q_t} - m^2_{u^c_t}) + M_Z^2 \cos 2\beta\left(\frac{1}{4} - \frac{2}{3}\sin^2\theta_W\right)\right)^2 + m_t^2 \left|4\bar{\mu}\cot\beta + A_t\right|^2}$$

$$m^2_{\tilde{t}_2} = m_t^2 + \frac{1}{2}(m^2_{q_t} + m^2_{u^c_t}) + \frac{1}{4} M_Z^2 \cos 2\beta$$

$$+ \sqrt{\left(\frac{1}{2}(m^2_{q_t} - m^2_{u^c_t}) + M_Z^2 \cos 2\beta\left(\frac{1}{4} - \frac{2}{3}\sin^2\theta_W\right)\right)^2 + m_t^2 \left|4\bar{\mu}\cot\beta + A_t\right|^2}$$

In the $\mu$, $A_t$ real case the mixing angle is

$$\tan\theta_t = \frac{m_t^2 + m^2_{q_t} + M_Z^2 \cos 2\beta\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right) - m^2_{\tilde{t}_1}}{m_t(4\mu\cot\beta + A_t)}$$

## 3.b.) sBottom mixing:

(real $\mu$, $A_b$ case)

$$\begin{bmatrix}\tilde{b}_1\\ \tilde{b}_2\end{bmatrix} = S_{\tilde{b}}\begin{bmatrix}\tilde{b}\\ \tilde{b}^{c\dagger}\end{bmatrix} = \begin{bmatrix}\cos\theta_b & -\sin\theta_b\\ \sin\theta_b & \cos\theta_b\end{bmatrix}\begin{bmatrix}\tilde{b}\\ \tilde{b}^{c\dagger}\end{bmatrix}$$

and $S_{\tilde{b}} M^2_{\tilde{b}} S^T_{\tilde{b}} = M^2_{\tilde{b}\,\text{diag.}} = \begin{bmatrix} m^2_{\tilde{b}_1} & 0\\ 0 & m^2_{\tilde{b}_2}\end{bmatrix}$

(As previously, if $\mu$, $A_b$ are complex $\begin{bmatrix}\tilde{b}_1\\ \tilde{b}_2\end{bmatrix} = U_{\tilde{b}}\begin{bmatrix}\tilde{b}\\ \tilde{b}^{c\dagger}\end{bmatrix}$ where $U_{\tilde{b}}$ is unitary.)

where the eigenmasses are given by, in general,

$$m^2_{\tilde{b}_1} = m^2_b + \tfrac{1}{2}(m^2_{qt} + m^2_{d^c b}) - \tfrac{1}{4}M^2_Z\cos 2\beta - \sqrt{\left(\tfrac{1}{2}(m^2_{qt} - m^2_{d^c b}) + M^2_Z\cos 2\beta\left(-\tfrac{1}{4}+\tfrac{1}{3}\sin^2\theta_W\right)\right)^2 + m_b^2\left|4\mu\tan\beta + \bar{A}_b\right|^2}$$

$$m^2_{\tilde{b}_2} = m^2_b + \tfrac{1}{2}(m^2_{qt} + m^2_{d^c b}) - \tfrac{1}{4}M^2_Z\cos 2\beta + \sqrt{\left(\tfrac{1}{2}(m^2_{qt} - m^2_{d^c b}) + M^2_Z\cos 2\beta\left(-\tfrac{1}{4}+\tfrac{1}{3}\sin^2\theta_W\right)\right)^2 + m_b^2\left|4\mu\tan\beta + \bar{A}_b\right|^2}$$

And in the $\mu$, $A_b$ real case the mixing angle is

$$\tan\theta_b = \frac{m_b^2 + m^2_{qt} + M^2_Z\cos 2\beta\left(-\tfrac{1}{2}+\tfrac{1}{3}\sin^2\theta_W\right) - m^2_{\tilde{b}_1}}{m_b(4\mu\tan\beta + A_b)}$$

3.c.) <u>Sneutrino</u>: (tau) (No mixing) $\tilde{\nu}_\tau, \tilde{\nu}_\tau^{\dagger}$

$$m^2_{\tilde{\nu}_\tau} = m^2_{\ell\tau} + \frac{1}{2} M_Z^2 \cos 2\beta$$

3.d.) <u>stau</u>:

(real $\mu, A_\tau$ case) $\begin{bmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{bmatrix} = S_{\tilde{\tau}} \begin{bmatrix} \tilde{\tau} \\ \tilde{\tau}^{c\dagger} \end{bmatrix} = \begin{bmatrix} \cos\theta_\tau & -\sin\theta_\tau \\ \sin\theta_\tau & \cos\theta_\tau \end{bmatrix} \begin{bmatrix} \tilde{\tau} \\ \tilde{\tau}^{c\dagger} \end{bmatrix}$

and

$$S_{\tilde{\tau}} \mathcal{M}^2_{\tilde{\tau}} S^T_{\tilde{\tau}} = \mathcal{M}^2_{\tilde{\tau}\,diag.} = \begin{bmatrix} m^2_{\tilde{\tau}_1} & 0 \\ 0 & m^2_{\tilde{\tau}_2} \end{bmatrix}$$

(As previously, if $\mu, A_\tau$ are complex $\begin{bmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{bmatrix} = U_{\tilde{\tau}} \begin{bmatrix} \tilde{\tau} \\ \tilde{\tau}^{c\dagger} \end{bmatrix}$ where $U_{\tilde{\tau}}$ is unitary.)

where the eigenmasses are given by, in general,

$$m^2_{\tilde{\tau}_1} = m^2_\tau + \frac{1}{2}(m^2_{\ell\tau} + m^2_{e^c\tau}) - \frac{1}{4} M_Z^2 \cos 2\beta - \sqrt{\left(\frac{1}{2}(m^2_{\ell\tau} + m^2_{e^c\tau}) + M_Z^2 \cos 2\beta\left(-\frac{1}{4} + \sin^2\theta_W\right)\right)^2 + m^2_\tau \left|4\mu\tan\beta + \bar{A}_\tau\right|^2}$$

$$m^2_{\tilde{\tau}_2} = m^2_\tau + \frac{1}{2}(m^2_{\ell\tau} + m^2_{e^c\tau}) - \frac{1}{4} M_Z^2 \cos 2\beta + \sqrt{\left(\frac{1}{2}(m^2_{\ell\tau} + m^2_{e^c\tau}) + M_Z^2 \cos 2\beta\left(-\frac{1}{4} + \sin^2\theta_W\right)\right)^2 + m^2_\tau \left|4\mu\tan\beta + \bar{A}_\tau\right|^2}$$

And in the $\mu, A_\tau$ real case the mixing angle is

$$\tan\theta_\tau = \frac{m_\tau^2 + m_{l\tau}^2 + M_Z^2\cos 2\beta\left[-\frac{1}{2}+\sin^2\theta_W\right] - m_{\tilde\tau}^2}{m_\tau\left(4\mu\tan\beta + A_\tau\right)}$$

---

Now we have the daunting but straightforward task of re-expressing the MSSM Lagrangian in terms of the mass eigenstate fields.

We shall just do this for a few terms, leaving the rest for the reader!

For example: 1) Consider the quark-squark-gluino coupling. It is contained in the smatter-gaugino Yukawa couplings $\mathcal{L}_{\tilde Y}$

$$\begin{aligned}\mathcal{L}_{\tilde Y}^{q\tilde q\tilde G} = \sqrt{2}\, g_3 \Big[ & \tilde q^\dagger\left(\tilde G\cdot\tfrac{\lambda}{2}\right) q + \bar q\left(\bar{\tilde G}\cdot\tfrac{\lambda}{2}\right)\tilde q \\ & - \tilde u^{c\dagger}\left(\tilde G\cdot\tfrac{\lambda}{2}\right) u^c - \bar u^c\left(\bar{\tilde G}\cdot\tfrac{\lambda}{2}\right)\tilde u^c \\ & - \tilde d^{c\dagger}\left(\tilde G\cdot\tfrac{\lambda}{2}\right) d^c - \bar d^c\left(\bar{\tilde G}\cdot\tfrac{\lambda}{2}\right)\tilde d^c \Big].\end{aligned}$$

Recall p. -133- $\underset{\text{Dirac}}{u} = \underset{\text{Weyl}}{\begin{pmatrix} u \\ \bar u^c \end{pmatrix}}$, etc.

So the SU(2) doublet $q = \underset{\text{Weyl}}{\begin{pmatrix} u \\ d \end{pmatrix}} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$ ← left-handed Dirac $u_L = \gamma_- u$ (↑ Dirac)

Likewise the gluino is Majorana-Weyl. So, with a slight abuse of notation,

$$\hat{\tilde{G}} = \begin{pmatrix} \tilde{G} \\ \bar{\tilde{G}} \end{pmatrix}$$

4-component Majorana ($\hat{\tilde{G}}$); Weyl ($\tilde{G}$, $\bar{\tilde{G}}$)

So

$$\underbrace{\tilde{G}q}_{\substack{\text{Weyl}\\ \text{2 component}}} = \underbrace{\hat{\tilde{G}}^T q_L}_{\substack{\text{Dirac}\\ \text{4 component}}} \; ; \; \underbrace{\bar{q}\bar{\tilde{G}}}_{\substack{\text{2 comp.}\\ \text{Weyl}}} = \underbrace{\bar{q}_L \hat{\tilde{G}}}_{\substack{\text{4 comp.}\\ \text{Dirac}}}$$

---

Hence in 4-component notation we have

$$\mathcal{L}_{\tilde{Y}}^{\tilde{q}q\tilde{G}} = \sqrt{2}\, g_3 \Big[ \tilde{q}^{\dagger}\big(\hat{\tilde{G}}^T \cdot \tfrac{\lambda}{2}\big) q_L + \bar{q}_L \big(\hat{\tilde{G}} \cdot \tfrac{\lambda}{2}\big)\tilde{q}$$

$$- \tilde{u}^{c\dagger}\big(\hat{\tilde{G}}^T \cdot \tfrac{\lambda}{2}\big)\bar{u}_R^T - u_R^T\big(\hat{\tilde{G}} \cdot \tfrac{\lambda}{2}\big)\tilde{u}^c$$

$$- \tilde{d}^{c\dagger}\big(\hat{\tilde{G}}^T \cdot \tfrac{\lambda}{2}\big)\bar{d}_R^T - d_R^T\big(\hat{\tilde{G}} \cdot \tfrac{\lambda}{2}\big)\tilde{d}^c \Big]$$

---

(SUSY Flavor Problem $\tilde{A} \neq A$)

Now it remains to transform to mass eigenstates. Assume that the squarks' and sleptons' matrices are diagonal in the same basis that the fermions' Yukawa couplings are diagonal ex. $u_L^w = A_L^u u_L$ i.e. $\tilde{A}_L^u = A_L^u$ so that $\tilde{u}^w = A_L^u \tilde{u}$, etc.

So consequently we only draw the intra-generational mixing to account for in these terms since the $A^{\text{family}}_{L,R,\Lambda}$ matrices are unitary.

$$\begin{bmatrix} \tilde{u} \\ \tilde{u}^{c\dagger} \end{bmatrix} = S^T_{\tilde{u}} \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \end{bmatrix} = \begin{bmatrix} \tilde{u}_1 \cos\theta_u + \tilde{u}_2 \sin\theta_u \\ -\tilde{u}_1 \sin\theta_u + \tilde{u}_2 \cos\theta_u \end{bmatrix}$$

$$\begin{bmatrix} \tilde{d} \\ \tilde{d}^{c\dagger} \end{bmatrix} = S^T_{\tilde{d}} \begin{bmatrix} \tilde{d}_1 \\ \tilde{d}_2 \end{bmatrix} = \begin{bmatrix} \tilde{d}_1 \cos\theta_d + \tilde{d}_2 \sin\theta_d \\ -\tilde{d}_1 \sin\theta_d + \tilde{d}_2 \cos\theta_d \end{bmatrix}$$

So

$$\begin{aligned}
\mathcal{L}^{q\tilde{q}\tilde{G}}_{\tilde{Y}} = \sqrt{2}\, g_3 \Big[ & \left(\tilde{u}_1^\dagger \cos\theta_u + \tilde{u}_2^\dagger \sin\theta_u\right)\left(\tilde{G}^T \cdot \tfrac{\lambda}{2}\right) u_L \\
& + \left(\tilde{d}_1^\dagger \cos\theta_d + \tilde{d}_2^\dagger \sin\theta_d\right)\left(\tilde{G}^T \cdot \tfrac{\lambda}{2}\right) d_L \\
& + \bar{u}_L \left(\tilde{G} \cdot \tfrac{\lambda}{2}\right)\left(\tilde{u}_1 \cos\theta_u + \tilde{u}_2 \sin\theta_u\right) \\
& + \bar{d}_L \left(\tilde{G} \cdot \tfrac{\lambda}{2}\right)\left(\tilde{d}_1 \cos\theta_d + \tilde{d}_2 \sin\theta_d\right) \\
& - \left(-\tilde{u}_1 \sin\theta_u + \tilde{u}_2 \cos\theta_u\right)\left(\tilde{G}^T \cdot \tfrac{\lambda}{2}\right) \bar{u}_R^T \\
& - u_R^T \left(\tilde{G} \cdot \tfrac{\lambda}{2}\right)\left(-\tilde{u}_1^\dagger \sin\theta_u + \tilde{u}_2^\dagger \cos\theta_u\right) \\
& - \left(-\tilde{d}_1 \sin\theta_d + \tilde{d}_2 \cos\theta_d\right)\left(\tilde{G}^T \cdot \tfrac{\lambda}{2}\right) \bar{d}_R^T \\
& - d_R^T \left(\tilde{G} \cdot \tfrac{\lambda}{2}\right)\left(-\tilde{d}_1^\dagger \sin\theta_d + \tilde{d}_2^\dagger \cos\theta_d\right) \Big]
\end{aligned}$$

Grouping similar terms

$$\mathcal{L}_{\tilde{Y}}^{q\tilde{q}\tilde{G}} = \sqrt{2}\, g_3 \Big[ \tilde{u}_1^{\dagger} \cos\theta_u \left(\tilde{G}^T \cdot \tfrac{\lambda}{2}\right) u_L$$
$$- \tilde{u}_1^{\dagger} \sin\theta_u \left(\tilde{G}^T \cdot \tfrac{\lambda}{2}\right) u_R$$
$$+ \tilde{u}_2^{\dagger} \sin\theta_u \left(\tilde{G}^T \cdot \tfrac{\lambda}{2}\right) u_L$$
$$+ \tilde{u}_2^{\dagger} \cos\theta_u \left(\tilde{G}^T \cdot \tfrac{\lambda}{2}\right) u_R$$
$$+ \tilde{d}_1^{\dagger} \cos\theta_d \left(\tilde{G}^T \cdot \tfrac{\lambda}{2}\right) d_L$$
$$- \tilde{d}_1^{\dagger} \sin\theta_d \left(\tilde{G}^T \cdot \tfrac{\lambda}{2}\right) d_R$$
$$+ \tilde{d}_2^{\dagger} \sin\theta_d \left(\tilde{G}^T \cdot \tfrac{\lambda}{2}\right) d_L$$
$$+ \tilde{d}_2^{\dagger} \cos\theta_d \left(\tilde{G}^T \cdot \tfrac{\lambda}{2}\right) d_R \Big] + h.c.$$

---

$$\mathcal{L}_{\tilde{Y}}^{q\tilde{q}\tilde{G}} = \sqrt{2}\, g_3 \Big[ \tilde{u}_1^{\dagger} \left(\tilde{G}^T \cdot \tfrac{\lambda}{2}\right) \left[\cos\theta_u\, u_L - \sin\theta_u\, u_R\right]$$
$$+ \tilde{u}_2^{\dagger} \left(\tilde{G}^T \cdot \tfrac{\lambda}{2}\right) \left[\sin\theta_u\, u_L + \cos\theta_u\, u_R\right]$$
$$+ \tilde{d}_1^{\dagger} \left(\tilde{G}^T \cdot \tfrac{\lambda}{2}\right) \left[\cos\theta_d\, d_L - \sin\theta_d\, d_R\right]$$
$$+ \tilde{d}_2^{\dagger} \left(\tilde{G}^T \cdot \tfrac{\lambda}{2}\right) \left[\sin\theta_d\, d_L + \cos\theta_d\, d_R\right] \Big] + h.c.$$

---

So we can write this as

$$\boxed{\mathcal{L}_{\tilde{\chi}}^{q\tilde{q}\tilde{G}} = \sqrt{2}\, g_3 \sum_{q=\{u,d,c,s,t,b\}} \Big[ \tilde{q}_1^{\dagger} \left(\tilde{G}^T \cdot \frac{\lambda}{2}\right)\left[\cos\theta_q\, q_L - \sin\theta_q\, q_R\right] + \tilde{q}_2^{\dagger}\left(\tilde{G}^T \cdot \frac{\lambda}{2}\right)\left[\sin\theta_q\, q_L + \cos\theta_q\, q_R\right]\Big] + h.c.}$$

(Recall squark mixing only significant for 3rd generation)

2) Similarly the coupling of the light higgs h to top and stop quarks can be found. The coupling of the Higgs to (s)quarks comes from the D-term Lagrangian $\mathcal{L}_D$, the Yukawa Lagrangian $\mathcal{L}_Y$, the F-term Lagrangian $\mathcal{L}_F$ and the tri-linear breaking Lagrangian $\mathcal{L}_{SW}$.

1) $$\mathcal{L}_D^{h,t,\tilde{t}} = -\frac{1}{2}g_1^2\left[\frac{1}{6}\tilde{u}^\dagger\tilde{u} - \frac{2}{3}\tilde{u}^{c\dagger}\tilde{u}^c + \frac{1}{2}H_u^{0\dagger}H_u^0 - \frac{1}{2}H_d^{0\dagger}H_d^0\right]^2$$

$$-\frac{1}{2}g_2^2\left[\tilde{u}^\dagger\tilde{u}\left(\frac{\sigma^i}{2}\right)_{11} + H_u^{0\dagger}H_u^0\left(\frac{\sigma^i}{2}\right)_{22} + H_d^{0\dagger}H_d^0\left(\frac{\sigma^i}{2}\right)_{11}\right]^2$$

$\left(\begin{array}{l} H_u^0 \to \frac{1}{\sqrt{2}}v_u + H_u^0 \\ H_d^0 \to \frac{1}{\sqrt{2}}v_d + H_d^0 \end{array}\right)$

$$= -\frac{1}{2}g_1^2\left[\frac{1}{6}\tilde{u}^\dagger\tilde{u} - \frac{2}{3}\tilde{u}^{c\dagger}\tilde{u}^c + \frac{1}{2}\left[\frac{v_u}{\sqrt{2}} + H_u^0\right]^\dagger\left[\frac{v_u}{\sqrt{2}} + H_u^0\right] - \frac{1}{2}\left[\frac{v_d}{\sqrt{2}} + H_d^0\right]^\dagger\left[\frac{v_d}{\sqrt{2}} + H_d^0\right]\right]^2$$

only $\sigma^3$ has diagonal els $\neq 0$

$$-\frac{1}{2}g_2^2\left[\frac{1}{2}\tilde{u}^\dagger\tilde{u} - \frac{1}{2}\left[\frac{v_u}{\sqrt{2}} + H_u^0\right]^\dagger\left[\frac{v_u}{\sqrt{2}} + H_u^0\right] + \frac{1}{2}\left[\frac{v_d}{\sqrt{2}} + H_d^0\right]^\dagger\left[\frac{v_d}{\sqrt{2}} + H_d^0\right]\right]^2$$

Now recall that
$$H_u^0 = \frac{1}{\sqrt{2}}\left[H_u^R + iH_u^I\right]$$
$$H_d^0 = \frac{1}{\sqrt{2}}\left[H_d^R + iH_d^I\right]$$

and
$$\begin{bmatrix} H_u^R \\ H_d^R \end{bmatrix} = \begin{bmatrix} h\cos\alpha - H\sin\alpha \\ h\sin\alpha + H\cos\alpha \end{bmatrix}$$

$$\begin{bmatrix} H_u^I \\ H_d^I \end{bmatrix} = \begin{bmatrix} A\cos\beta - \pi^0\sin\beta \\ A\sin\beta + \pi^0\cos\beta \end{bmatrix}$$

2.1.) Since we are interested in the h couplings we have

$$H_u^0 = \frac{1}{\sqrt{2}}\left[h\cos\alpha - H\sin\alpha + iA\cos\beta - i\Pi^0\sin\beta\right]$$

$$\longrightarrow \frac{1}{\sqrt{2}}h\cos\alpha$$

$$H_d^0 = \frac{1}{\sqrt{2}}\left[h\sin\alpha + H\cos\alpha + iA\sin\beta + i\Pi^0\cos\beta\right]$$

$$\longrightarrow \frac{1}{\sqrt{2}}h\sin\alpha$$

---

$$\boxed{\begin{aligned}\mathcal{L}_D^{h,t,\tilde{t}} = &-\tfrac{1}{2}g_1^2\Big[\tfrac{1}{6}\tilde{u}^\dagger\tilde{u} - \tfrac{2}{3}\tilde{u}^{c\dagger}\tilde{u}^c \\ &\qquad + \tfrac{1}{4}\left[v_u + h\cos\alpha\right]\left[v_u + h\cos\alpha\right]\Big]^2 \\ &-\tfrac{1}{2}g_2^2\Big[\tfrac{1}{2}\tilde{u}^\dagger\tilde{u} - \tfrac{1}{4}\left[v_u + h\cos\alpha\right]^2 \\ &\qquad + \tfrac{1}{4}\left[v_d + h\sin\alpha\right]^2\Big]^2\end{aligned}}$$

2) $$\mathcal{L}_Y^{h,t,\tilde{t}} = -4H_u^0\,u\,y_u\,u^c - 4H_u^{0\dagger}\,\bar{u}^c\,y_u^\dagger\,\bar{u}$$

(Recall $y_u^* = \frac{\sqrt{2}}{4}\frac{1}{v\sin\beta}M^u$)

$$= -\frac{4}{\sqrt{2}}\left(u\,y_u\,u^c + \bar{u}^c y_u^\dagger \bar{u}\right)h\cos\alpha$$

$$= -\frac{\cos\alpha}{v\sin\beta}h\left[u^c M^{u\dagger} u + \bar{u}M^u\bar{u}^c\right]$$

(Dirac 4-comp. notation)

$$= -\frac{\cos\alpha}{v\sin\beta}h\left[\bar{u}_R M^{u\dagger} u_L + \bar{u}_L M^u u_R\right]$$

2) Taking the up type quarks to the mass eigenbasis

$$u_L \to A_L^u u_L \; ; \; u_R \to A_R^u u_R$$

yields

$$\mathcal{L}_Y^{h,t,\tilde{t}} = -\frac{\cos\alpha}{v\sin\beta} h \Big[ \bar{u}_R (A_R^{u\dagger} M^{u\dagger} A_L^u) u_L + \bar{u}_L (A_L^{u\dagger} M^u A_R^u) u_R \Big]$$

mass eigen-fields now

and $A_L^{u\dagger} M^u A_R^u = \begin{bmatrix} m_u & & 0 \\ & m_c & \\ 0 & & m_t \end{bmatrix} = M^u_{diag.}$

So finally in terms of mass eigenstates

$$\boxed{\mathcal{L}_Y^{h,t,\tilde{t}} = -\frac{\cos\alpha}{v\sin\beta} h \left[ \bar{u}_L u_R + \bar{u}_R u_L \right] m_u \quad \left(\text{sum over } u = u, c, t\right)}$$

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3) $$\mathcal{L}_F^{h,t,\tilde{t}} = -16\mu\, \tilde{u}^{c\dagger} y_u^\dagger \tilde{u}^\dagger H_d^0 - 16\bar{\mu}\, H_d^{0\dagger} \tilde{u}\, y_u \tilde{u}^c - 16\, \tilde{u}^{c\dagger} y_u^\dagger y_u \tilde{u}^c H_u^{0\dagger} H_u^0 - 16\, H_u^0 H_u^{0\dagger} \tilde{u}\, y_u y_u^\dagger \tilde{u}^\dagger$$

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3) $$\mathcal{L}_F^{h,t,\tilde{t}} = -4\mu\,\tilde{u}^{\dagger}\frac{\sqrt{2}}{4}\frac{4H_d^0}{v\sin\beta}M^u\tilde{u}^{c\dagger} - 4\bar{\mu}\,\tilde{u}^c\frac{\sqrt{2}}{4}\frac{H_d^{0\dagger}\cdot 4}{v\sin\beta}M^{u\dagger}\tilde{u}$$

$$-H_u^{0\dagger}H_u^0\,2\left(\frac{1}{v\sin\beta}\right)^2\tilde{u}^c M^{u\dagger}M^u\tilde{u}^{c\dagger}$$

$$-H_u^{0\dagger}H_u^0\,2\left(\frac{1}{v\sin\beta}\right)^2\tilde{u}M^{u\dagger}M^u\tilde{u}^{\dagger}$$

$$\boxed{\mathcal{L}_F^{h,t,\tilde{t}} = -4\mu\frac{\sin\alpha}{v\sin\beta}h\,\tilde{u}^{\dagger}M^u\tilde{u}^{c\dagger} - 4\bar{\mu}\frac{\sin\alpha}{v\sin\beta}h\,\tilde{u}^c M^{u\dagger}\tilde{u} - \frac{(v_u+h\cos\alpha)^2}{(v\sin\beta)^2}\left[\tilde{u}^c M^{u\dagger}M^u\tilde{u}^{c\dagger} + \tilde{u}M^{u\dagger}M^u\tilde{u}^{\dagger}\right]}$$

Now we can mimick what we did in the quark sector i.e. $\tilde{u} \equiv \tilde{u}_L \to \tilde{A}^u_L\tilde{u}_L$

$\tilde{u}_c \equiv \tilde{u}_R^{\dagger}$

$\tilde{u}_c^{\dagger} = \tilde{u}_R \to \tilde{A}^u_R\tilde{u}_R$

Or ignore intergenerational mixing

3) So

$$\boxed{\mathcal{L}_F^{h,t,\tilde{t}} = -4\mu \frac{\sin\alpha}{v\sin\beta} h m_u \tilde{u}^{\dagger} \tilde{u}^{c\dagger} - 4\bar{\mu} \frac{\sin\alpha}{v\sin\beta} h m_u \tilde{u}^c \tilde{u} - \frac{(v_u + h\cos\alpha)^2}{(v\sin\beta)^2} \left[m_u^2\right]\left[\tilde{u}^c \tilde{u}^{c\dagger} + \tilde{u}\tilde{u}^{\dagger}\right]}$$

(Sum over all up quarks)

Finally

$$\boxed{\begin{aligned} 4)\ \mathcal{L}_{\not{S}W}^{h,t,\tilde{t}} &= + H_u^0 \tilde{u} A_u \tilde{u}^c + H_u^{0\dagger} \tilde{u}^{c\dagger} A_u^{\dagger} \tilde{u}^{\dagger} \\ &= \frac{1}{\sqrt{2}} h \left[\tilde{u} A_u \tilde{u}^c + \tilde{u}^{c\dagger} A_u^{\dagger} \tilde{u}^{\dagger}\right] \cos\alpha \end{aligned}}$$

As before we consider $A_u$ to be diagonal here so $A_u = \begin{bmatrix} A_u & & 0 \\ & A_c & \\ 0 & & A_t \end{bmatrix}$ and sum over squarks.

(Recall p.-112- $\frac{v_u}{\sqrt{2}} A_u = -m_u^{\text{diag.}} A_u^{\text{diag.}}$)

This is a cumbersome task — we will find the other interaction terms as needed.

Finally let's isolate the coupling to the (s)top quarks —

$$\mathcal{L}_h^{t,\tilde{t}} = -\frac{1}{2} g_1^2 \left[ \frac{1}{6} \tilde{t}^\dagger \tilde{t} - \frac{2}{3} \tilde{t}^{c\dagger} \tilde{t}^c + \frac{1}{4} (v_u + h\cos\alpha)^2 \right]^2$$

$$- \frac{1}{2} g_2^2 \left[ \frac{1}{2} \tilde{t}^\dagger \tilde{t} - \frac{1}{4} (v_u + h\cos\alpha)^2 + \frac{1}{4} (v_d + h\sin\alpha)^2 \right]^2$$

$$- \frac{m_t \cos\alpha}{v \sin\beta} h \left[ \underbrace{\bar{t}_L t_R + \bar{t}_R t_L}_{= \bar{t} t \ \text{(Dirac)}} \right]$$

$$- \frac{m_t \sin\alpha}{v \sin\beta} h \left[ 4\mu \tilde{t}^\dagger \tilde{t}^{c\dagger} + 4\bar{\mu} \tilde{t}^c \tilde{t} \right]$$

$$- \frac{m_t^2 (v_u + h\cos\alpha)^2}{(v \sin\beta)^2} \left[ \tilde{t}^c \tilde{t}^{c\dagger} + \tilde{t} \tilde{t}^\dagger \right]$$

$$+ \frac{A_t}{\sqrt{2}} h \left[ \tilde{t} \tilde{t}^c + \tilde{t}^{c\dagger} \tilde{t}^\dagger \right] \cos\alpha$$