Squarks & Sleptons: This is the most complex sector since all scalars with the same quantum numbers can mix — so we expect to diagonalize 6×6 matrices! Let's begin by gathering all the sources of squark & slepton mass terms:

Auxiliary fields D-terms

$$\mathcal{L}_{YM} \supset \frac{1}{2} \underset{(2)}{D_A^i} \underset{(2)}{D_A^i} + \frac{1}{2} \underset{(1)}{D_B} \underset{(1)}{D_B} + \frac{1}{2} \underset{(3)}{D_G^m} \underset{(3)}{D_G^m}$$

Kähler Potential D-terms & F-terms

$$\mathcal{L}_K \supset \sum_{\substack{\text{chiral}\\ \text{fields}\\ x}} F_x^\dagger F_x$$

$$-g_1 D_B \Big[-\tfrac{1}{2} \tilde{l}^\dagger \tilde{l} + \tfrac{1}{6} \tilde{q}^{a\dagger} \tilde{q}^a + \tilde{e}^{c\dagger} \tilde{e}^c - \tfrac{2}{3} \tilde{u}^{ca\dagger} \tilde{u}^{ca} + \tfrac{1}{3} \tilde{d}^{ca\dagger} \tilde{d}^{ca} + \tfrac{1}{4}\left(v_u^2 - v_d^2\right) \Big]$$

$$-g_2 D_A^i \Big[\tilde{l}^\dagger \frac{\sigma^i}{2} \tilde{l} + \tilde{q}^{a\dagger} \frac{\sigma^i}{2} \tilde{q}^a + \tfrac{1}{4} \delta^{i3} \left(v_d^2 - v_u^2\right) \Big]$$

Superpotential "Yukawa" F-couplings

(Let μ be real here for simplicity)

$$\mathcal{L}_W \supset F_{H_u} \cdot \left[-4\mu H_d - 4 \tilde{q}\, y_u \tilde{u}^c \right] + F_{H_d} \cdot \left[4\mu H_u - 4 \tilde{q}\, y_d \tilde{d}^c - 4 \tilde{l}\, y_e \tilde{e}^c \right]$$
$$+ F_q \cdot \left[4 H_u y_u \tilde{u}^c + 4 H_d y_d \tilde{d}^c \right] + F_l \cdot \left[4 H_d y_e \tilde{e}^c \right]$$
$$+ F_{u^c}\left[4 H_u \cdot \tilde{q}\, y_u \right] + F_{d^c}\left[4 H_d \cdot \tilde{q}\, y_d \right] + F_{e^c}\left[4 H_d \cdot \tilde{l}\, y_e \right] + h.c.$$

And the many SUSY breaking terms:

Kähler soft-SUSY Breaking

$$\mathcal{L}_{\not{S}K} \supset -\tilde{l}^{\dagger} m_l^2 \tilde{l} - \tilde{q}^{a\dagger} m_q^2 \tilde{q}^a - \tilde{e}^{c\dagger} m_{e^c}^2 \tilde{e}^c - \tilde{u}^{ca\dagger} m_{u^c}^2 \tilde{u}^{ca} - \tilde{d}^{ca\dagger} m_{d^c}^2 \tilde{d}^{ca}$$

(these are 3×3 Family matrix mass terms)

Superpotential A,B soft SUSY breaking (only A-terms)

$$\mathcal{L}_{\not{S}W} \supset -H_u\cdot\tilde{q}^a A_u \tilde{u}^{ca} - H_d\cdot\tilde{q}^a A_d \tilde{d}^{ca} - H_d\cdot\tilde{l} A_l \tilde{e}^c + h.c.$$

(these are 3×3 Family matrix mass terms)

(ignore C-type inter-generational mixing)

So we must expand the auxiliary terms & keep only the mass term contributions:

From D-terms

$$\boxed{\begin{aligned}\mathcal{L}_{D\text{-term mass}} = &+\tfrac{1}{2}g_1^2\tfrac{1}{2}(v_d^2-v_u^2)\Big[-\tfrac{1}{2}\tilde{\nu}_e^{\dagger}\tilde{\nu}_e - \tfrac{1}{2}\tilde{e}^{\dagger}\tilde{e} + \tfrac{1}{6}\tilde{u}^{a\dagger}\tilde{u}^a + \tfrac{1}{6}\tilde{d}^{a\dagger}\tilde{d}^a + \tilde{e}^{c\dagger}\tilde{e}^c - \tfrac{2}{3}\tilde{u}^{ca\dagger}\tilde{u}^{ca} + \tfrac{1}{3}\tilde{d}^{ca\dagger}\tilde{d}^{ca}\Big]\\ &-\tfrac{1}{2}g_2^2\tfrac{1}{2}(v_d^2-v_u^2)\Big[\tfrac{1}{2}\tilde{\nu}_e^{\dagger}\tilde{\nu}_e - \tfrac{1}{2}\tilde{e}^{\dagger}\tilde{e} + \tfrac{1}{2}\tilde{u}^{a\dagger}\tilde{u}^a - \tfrac{1}{2}\tilde{d}^{a\dagger}\tilde{d}^a\Big]\end{aligned}}$$

if μ complex

From F-terms

$$\mathcal{L}_{\text{F-term mass}} = -\Big\{ \frac{16\mu}{\sqrt{2}} v_d \left(\tilde{u}\, y_u\, \tilde{u}^c + \tilde{u}^{c\dagger} y_u^\dagger \tilde{u}^\dagger\right)$$
$$- \frac{16\mu}{\sqrt{2}} v_u \left(\tilde{d}\, y_d\, \tilde{d}^c + \tilde{e}\, y_l\, \tilde{e}^c + \tilde{d}^{c\dagger} y_d^\dagger \tilde{d}^\dagger + \tilde{e}^{c\dagger} y_l^\dagger \tilde{e}^\dagger\right)$$
$$+ \left(8 v_u^2\, \tilde{u}^{c\dagger} y_u^\dagger y_u\, \tilde{u}^c + 8 v_d^2\, \tilde{d}^{c\dagger} y_d^\dagger y_d\, \tilde{d}^c\right)$$
$$+ 8\left(v_d^2\, \tilde{e}^{c\dagger} y_l^\dagger y_l\, \tilde{e}^c\right) + 8\left(\tilde{u}\, y_u y_u^\dagger\, \tilde{u}^\dagger v_u^2\right)$$
$$+ 8\left(v_d^2\, \tilde{d}\, y_d y_d^\dagger\, \tilde{d}^\dagger\right) + 8 v_d^2 \left(\tilde{e}\, y_l y_l^\dagger\, \tilde{e}^\dagger\right) \Big\}$$

SUSY Breaking mass terms

Kähler + Superpotential Breaking Terms

$$\mathcal{L}_{\not{s}\,\text{mass}} = -\Big[\tilde{\nu}_e^\dagger m_l^2 \tilde{\nu}_e + \tilde{e}^\dagger m_l^2 \tilde{e}$$
$$+ \tilde{u}^{\dagger a} m_q^2 \tilde{u}^a + \tilde{d}^{\dagger a} m_q^2 \tilde{d}^a$$
$$+ \tilde{e}^{c\dagger} m_{e^c}^2 \tilde{e}^c + \tilde{u}^{ca\dagger} m_{u^c}^2 \tilde{u}^{ca}$$
$$+ \tilde{d}^{ca\dagger} m_{d^c}^2 \tilde{d}^{ca} \Big]$$
$$- \Big[-\frac{v_u}{\sqrt{2}} \tilde{u}^a A_u \tilde{u}^{ca} + \frac{v_d}{\sqrt{2}} \tilde{d}^a A_d \tilde{d}^{ca}$$
$$+ \frac{v_d}{\sqrt{2}} \tilde{e} A_l \tilde{e}^c - \frac{v_u}{\sqrt{2}} \tilde{u}^{ca\dagger} A_u^\dagger \tilde{u}^{a\dagger}$$
$$+ \frac{v_d}{\sqrt{2}} \tilde{d}^{ca\dagger} A_d^\dagger \tilde{d}^{a\dagger} + \frac{v_d}{\sqrt{2}} \tilde{e}^{c\dagger} A_l^\dagger \tilde{e}^\dagger \Big]$$

The fields split according to their character so we only have mixing among the same species ex. $\tilde{u}$ & $\tilde{u}^c$, $\tilde{d}$ and $\tilde{d}^c$, $\tilde{e}$ and $\tilde{e}^c$ and finally $\tilde{\nu}_e$. So for these fields we find

$$\boxed{\begin{aligned} \mathcal{L}_{\tilde{\nu}_e\,mass} = &-\tfrac{1}{8}(g_1^2+g_2^2)(v_d^2-v_u^2)\tilde{\nu}_e^\dagger\tilde{\nu}_e \\ &-\tilde{\nu}_e^\dagger m_\ell^2\tilde{\nu}_e \end{aligned}}$$

$$\boxed{\begin{aligned} \mathcal{L}_{\tilde{e}\,mass} = &-\tfrac{1}{8}(g_1^2-g_2^2)(v_d^2-v_u^2)\tilde{e}^\dagger\tilde{e} \\ &+\tfrac{1}{4}g_1^2(v_d^2-v_u^2)\tilde{e}^{c\dagger}\tilde{e}^c \\ &-8v_d^2\tilde{e}^\dagger(y_e y_e^\dagger)^T\tilde{e} - 8v_d^2\tilde{e}^{c\dagger}(y_e^\dagger y_e)\tilde{e}^c \\ &+\frac{16\mu}{\sqrt{2}}v_u\left(\tilde{e}\,y_e\,\tilde{e}^c\frac{\bar{\mu}}{\mu}+\tilde{e}^{c\dagger}y_e^\dagger\tilde{e}^\dagger\right) \quad \text{if } \mu \text{ complex} \\ &-\tilde{e}^\dagger m_\ell^2\tilde{e}-\tilde{e}^{c\dagger}m_{ec}^2\tilde{e}^c \\ &-\frac{v_d}{\sqrt{2}}\tilde{e}A_\ell\tilde{e}^c-\frac{v_d}{\sqrt{2}}\tilde{e}^{c\dagger}A_\ell^\dagger\tilde{e}^\dagger \end{aligned}}$$

$$\mathcal{L}_{\tilde{u}\,mass} = \frac{1}{4}(v_d^2 - v_u^2)\left(\frac{1}{6}g_1^2 - \frac{1}{2}g_2^2\right)\tilde{u}^{a\dagger}\tilde{u}^a$$
$$+\frac{1}{4}(v_d^2 - v_u^2)\left(-\frac{2}{3}g_1^2\right)\tilde{u}^{ca\dagger}\tilde{u}^{ca}$$

if μ complex

$$-\frac{16\mu}{\sqrt{2}}v_d\left(\tilde{u}\,y_u\,\tilde{u}^c\,\frac{\mu^*}{\mu} + \tilde{u}^{c\dagger}y_u^\dagger\tilde{u}^\dagger\right)$$
$$-8v_u^2\,\tilde{u}^{c\dagger}y_u^\dagger y_u\,\tilde{u}^c - 8v_u^2\,\tilde{u}\,y_u y_u^\dagger\,\tilde{u}^\dagger$$
$$-\tilde{u}^{\dagger a}m_q^2\tilde{u}^a - \tilde{u}^{ca\dagger}m_{u^c}^2\tilde{u}^{ca}$$
$$+\frac{v_u}{\sqrt{2}}\tilde{u}^a A_u\tilde{u}^{ca} + \frac{v_u}{\sqrt{2}}\tilde{u}^{ca\dagger}A_u^\dagger\tilde{u}^{a\dagger}$$

$$\mathcal{L}_{\tilde{d}\,mass} = +\frac{1}{4}(v_d^2 - v_u^2)\left(\frac{1}{6}g_1^2 + \frac{1}{2}g_2^2\right)\tilde{d}^{a\dagger}\tilde{d}^a$$
$$+\frac{1}{4}(v_d^2 - v_u^2)\left(\frac{1}{3}g_1^2\right)\tilde{d}^{ca\dagger}\tilde{d}^{ca}$$
$$+\frac{16\mu}{\sqrt{2}}v_u\left(\frac{\mu^*}{\mu}\tilde{d}\,y_d\,\tilde{d}^c + \tilde{d}^{c\dagger}y_d^\dagger\tilde{d}^\dagger\right)$$
$$-8v_d^2\,\tilde{d}^{c\dagger}y_d^\dagger y_d\,\tilde{d}^c - 8v_d^2\,\tilde{d}\,y_d y_d^\dagger\,\tilde{d}^\dagger$$
$$-\tilde{d}^{\dagger a}m_q^2\tilde{d}^a - \tilde{d}^{ca\dagger}m_{d^c}^2\tilde{d}^{ca}$$
$$-\frac{v_d}{\sqrt{2}}\tilde{d}^a A_d\tilde{d}^{ca} - \frac{v_d}{\sqrt{2}}\tilde{d}^{ca\dagger}A_d^\dagger\tilde{d}^{a\dagger}$$

Note: The D-term contribution to the masses can be expressed in terms of the fields' electric charge and weak isospin T_3 recall $Q = T_3 + Y$. The D-term mass matrix contribution is

$$\mathcal{L}_{D\text{-term}} = -M_Z^2 \cos 2\beta \left[T_3 - Q \sin^2\theta_W \right] \tilde{f}^\dagger \tilde{f}$$

ex. $$\mathcal{L}_{D\text{-term}} = -M_Z^2 \cos 2\beta \Big[\underbrace{\tfrac{1}{2}}_{(=T_3^{\tilde{u}})} - \underbrace{\tfrac{2}{3}}_{(=Q^{\tilde{u}})} \sin^2\theta_W \Big] \tilde{u}^\dagger \tilde{u}$$

$$- M_Z^2 \cos 2\beta \Big[+ \underbrace{\tfrac{2}{3}}_{(=Q^{\tilde{u}^c})} \sin^2\theta_W \Big] \tilde{u}^{c\dagger} \tilde{u}^c$$

$$- M_Z^2 \cos 2\beta \Big[\underbrace{-\tfrac{1}{2}}_{(=T_3^d)} + \underbrace{\tfrac{1}{3}}_{(=Q)} \sin^2\theta_W \Big] \tilde{d}^\dagger \tilde{d}$$

$$- M_Z^2 \cos 2\beta \left[-\tfrac{1}{3} \sin^2\theta_W \right] \tilde{d}^{c\dagger} \tilde{d}^c$$

$$- M_Z^2 \cos 2\beta \left[-\tfrac{1}{2} + 1 \sin^2\theta_W \right] \tilde{e}^\dagger \tilde{e}$$

$$- M_Z^2 \cos 2\beta \left[-1 \sin^2\theta_W \right] \tilde{e}^{c\dagger} \tilde{e}^c$$

$$- M_Z^2 \cos 2\beta \left[\tfrac{1}{2} \right] \tilde{\nu}_e^\dagger \tilde{\nu}_e$$

To see what to expect for squark & slepton masses let's ignore inter-generational mixing and first focus on the $\tilde{u}$-type squark mass matrix as $\tilde{u}$ & $\tilde{u}^{c\dagger}$ still mix.

$$\boxed{\begin{aligned}\mathcal{L}_{\tilde{u}\,mass} = &-M_Z^2\cos 2\beta\left[\tfrac{1}{2}-\tfrac{2}{3}\sin^2\theta_W\right]\tilde{u}^\dagger\tilde{u}\\ &-M_Z^2\cos 2\beta\left[\tfrac{2}{3}\sin^2\theta_W\right]\tilde{u}^{c\dagger}\tilde{u}^c\\ &-4\mu\cot\beta\left[\tilde{u}\,m_u^{diag.}\cdot\tilde{u}^c\tfrac{\mu^*}{\mu}+\tilde{u}^{c\dagger}m_u^{diag}\tilde{u}^\dagger\right] \quad \text{if } \mu \text{ complex}\\ &-\tilde{u}^\dagger m_u^{diag\,2}\tilde{u}-\tilde{u}^{c\dagger}m_u^{diag\,2}\tilde{u}^c\\ &-\tilde{u}^\dagger m_q^2\tilde{u}-\tilde{u}^{c\dagger}m_{uc}^2\tilde{u}^c\\ &-\tilde{u}\,m_u^{diag.}A_u^{diag.}\tilde{u}^c-\tilde{u}^{c\dagger}m_u^{diag}\overline{A}_u^{diag}\tilde{u}^\dagger \quad \text{if } A_u^{diag} \text{ complex}\end{aligned}}$$

where we have Assumed $m_u^{diag}=\begin{bmatrix} m_u & & 0\\ & m_c & \\ 0 & & m_t\end{bmatrix}$

as well the tri-linear breaking is also diagonal

$$A_u^{diag}=\begin{bmatrix} A_u & & 0\\ & A_c & \\ 0 & & A_t\end{bmatrix}$$

~~real~~ numbers for simplicity

write

$$\frac{v_u}{\sqrt{2}}\underset{3\times 3\text{ matrix}}{A_u}\equiv -m_u^{diag.}\cdot A_u^{diag}$$, likewise for A_d, A_e.

and recall that $M^u = -\frac{1}{\sqrt{2}}\Gamma^u v \sin\beta = \frac{4}{\sqrt{2}} y_u^* v \sin\beta$

$$M^d = +\frac{1}{\sqrt{2}}\Gamma^d v \cos\beta = -\frac{4}{\sqrt{2}} y_d^* v \cos\beta$$
$$M^e = +\frac{1}{\sqrt{2}}\Gamma^e v \cos\beta = -\frac{4}{\sqrt{2}} y_\ell^* v \cos\beta$$

and when diagonalized $M^{u,d,e}$ become

$$M^u_{diag} = \begin{bmatrix} m_u & & 0 \\ & m_c & \\ 0 & & m_t \end{bmatrix} = m_u^{diag}; \quad M^d_{diag} = \begin{bmatrix} m_d & & 0 \\ & m_s & \\ 0 & & m_b \end{bmatrix} = m_d^{diag}$$

$$M^e_{diag} = \begin{bmatrix} m_e & & 0 \\ & m_\mu & \\ 0 & & m_\tau \end{bmatrix} = m_e^{diag}.$$

Also the soft SUSY breaking masses will be assumed family diagonal so that

$$m_q^2 = \begin{bmatrix} m_{qu}^2 & & 0 \\ & m_{qc}^2 & \\ 0 & & m_{qt}^2 \end{bmatrix}; \quad m_{u^c}^2 = \begin{bmatrix} m_{u^c u}^2 & & 0 \\ & m_{u^c c}^2 & \\ 0 & & m_{u^c t}^2 \end{bmatrix}$$

$$m_{d^c}^2 = \begin{bmatrix} m_{d^c d}^2 & & 0 \\ & m_{d^c s}^2 & \\ 0 & & m_{d^c b}^2 \end{bmatrix}$$

$$m_\ell^2 = \begin{bmatrix} m_{\ell e}^2 & & 0 \\ & m_{\ell \mu}^2 & \\ 0 & & m_{\ell \tau}^2 \end{bmatrix}; \quad m_{e^c}^2 = \begin{bmatrix} m_{e^c e}^2 & & 0 \\ & m_{e^c \mu}^2 & \\ 0 & & m_{e^c \tau}^2 \end{bmatrix}.$$

Hence for each squark & slepton we obtain their corresponding mass matrix — again focusing on the $\tilde{u}$ families we have

$$\mathcal{L}_{\tilde{u}\text{-mass}} = -\begin{bmatrix}\tilde{u}^{+} & \tilde{u}^{c}\end{bmatrix} \mathcal{M}^2_{\tilde{u}} \begin{bmatrix}\tilde{u} \\ \tilde{u}^{c+}\end{bmatrix}$$

$$\mathcal{L}_{\tilde{c}\text{-mass}} = -\begin{bmatrix}\tilde{c}^{+} & \tilde{c}^{c}\end{bmatrix} \mathcal{M}^2_{\tilde{c}} \begin{bmatrix}\tilde{c} \\ \tilde{c}^{c+}\end{bmatrix}$$

$$\mathcal{L}_{\tilde{t}\text{-mass}} = -\begin{bmatrix}\tilde{t}^{+} & \tilde{t}^{c}\end{bmatrix} \mathcal{M}^2_{\tilde{t}} \begin{bmatrix}\tilde{t} \\ \tilde{t}^{c+}\end{bmatrix}$$

with say for the $\tilde{t}$ case, and similar results for the others

$$\mathcal{M}^2_{\tilde{t}} = \begin{bmatrix} (m_t^2 + m_{qt}^2 + \overbrace{M_Z^2\cos 2\beta(\tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta_W)}^{\equiv D(\tilde{t})}) & m_t(4\mu\cot\beta + \bar{A}_t) \\ m_t(4\bar{\mu}\cot\beta + A_t) & (M_t^2 + m_{uct}^2 + \underbrace{M_Z^2\cos 2\beta(\tfrac{2}{3}\sin^2\theta_W)}_{\equiv D(\tilde{t}^c)}) \end{bmatrix}$$

The $\tilde{t}$ & $\tilde{t}^{c+}$ squarks mix with mass eigenstates given by $\tilde{t}_1$, $\tilde{t}_2$ with $m_{\tilde{t}_1} < m_{\tilde{t}_2}$

(if μ, A_t complex need Unitary Matrix not just rotation)

$$\begin{pmatrix}\tilde{t}_1 \\ \tilde{t}_2\end{pmatrix} = \underbrace{\begin{bmatrix}\cos\theta_t & -\sin\theta_t \\ \sin\theta_t & \cos\theta_t\end{bmatrix}}_{\equiv S_{\tilde{t}}} \begin{bmatrix}\tilde{t} \\ \tilde{t}^{c+}\end{bmatrix}$$

(μ, A_t Real)

Recall 2x2 matrices: // $\underline{M \text{ real}}$

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} a & b \\ b & d \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \equiv \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

$$= \begin{bmatrix} ac^2+ds^2-bs2\theta & \frac{1}{2}a\sin 2\theta - \frac{1}{2}d\sin 2\theta + b\cos 2\theta \\ \frac{1}{2}(a-d)\sin 2\theta + b\cos 2\theta & as^2+dc^2+b\sin 2\theta \end{bmatrix}$$

So $m_1+m_2 = a+d$, $m_1 m_2 = ad-b^2$
which we solved for m_1 & m_2. Also

$$\boxed{\begin{aligned} m_1 &= a\cos^2\theta + d\sin^2\theta - b\sin 2\theta \\ m_2 &= a\sin^2\theta + d\cos^2\theta + b\sin 2\theta \\ &\& \\ 2b\cos 2\theta &= (d-a)\sin 2\theta \end{aligned}}$$

$$\Rightarrow \boxed{\boxed{\tan\theta = \frac{a-m_1}{b}}}$$

i.e.

$$\boxed{\begin{aligned} m_1 &= \frac{TrM - \sqrt{(TrM)^2 - 4\det M}}{2} = \frac{(a+d) - \sqrt{(a-d)^2+4b^2}}{2} \\ m_2 &= \frac{TrM + \sqrt{(TrM)^2 - 4\det M}}{2} = \frac{(a+d) + \sqrt{(a-d)^2+4b^2}}{2} \end{aligned}}$$

Suppose $M = M^\dagger = \begin{pmatrix} a & b \\ b^* & d \end{pmatrix}$ a, d real, $b \in \mathbb{C}$, M Hermitian

Then unitary matrix diagonalizes M, $U^{-1} = U^\dagger$

$$U M U^\dagger = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$ with $m_{1,2}$ real eigenvalues

Recall the eigenvalue problem:

$$M_{ij} |e_j^{(k)}\rangle = \lambda^{(k)} e_i^{(k)} \qquad i,j,k = 1,2$$

$$\Rightarrow \det[M - \lambda \mathbb{1}] = 0 \Rightarrow$$

$$\boxed{\begin{aligned} m_1 &= \lambda^{(1)} = \tfrac{1}{2}\left[a + d - \sqrt{(a-d)^2 + 4|b|^2}\right] \\ m_2 &= \lambda^{(2)} = \tfrac{1}{2}\left[a + d + \sqrt{(a-d)^2 + 4|b|^2}\right] \end{aligned}}$$

Let

$$e^{(k)} = N^{(k)} \begin{pmatrix} 1 \\ f^{(k)} \end{pmatrix}$$ with Normalization:

$$\langle e^{(i)} | e^{(j)} \rangle \equiv \delta^{ij} = \bar{N}^{(i)} N^{(j)} \left[1 + \bar{f}^{(i)} f^{(j)}\right]$$

So $\boxed{|N^{(i)}|^2 = \dfrac{1}{1 + |f^{(i)}|^2}}$ (no sum on i)

So $M |e^{(i)}\rangle = \lambda^{(i)} |e^{(i)}\rangle$

$\Rightarrow$

$$a + b f^{(i)} = \lambda^{(i)}$$

(redundant) $b^* + d f^{(i)} = f^{(i)} \lambda^{(i)}$

$\Rightarrow$

$$f^{(i)} = \frac{\lambda^{(i)} - a}{b} = \boxed{\frac{m_i - a}{b} = f^{(i)}}$$

Now $U^{-1} = U^{\dagger} = \left[\begin{pmatrix} e_1^{(1)} \\ e_2^{(1)} \end{pmatrix} \begin{pmatrix} e_1^{(2)} \\ e_2^{(2)} \end{pmatrix} \right] \Rightarrow U = \begin{bmatrix} \bar{e}_1^{(1)} & \bar{e}_2^{(1)} \\ \bar{e}_1^{(2)} & \bar{e}_2^{(2)} \end{bmatrix}$ — 114'' —

So $UU^{\dagger} = \begin{bmatrix} \langle e^{(1)} | e^{(1)} \rangle & \langle e^{(1)} | e^{(2)} \rangle \\ \langle e^{(2)} | e^{(1)} \rangle & \langle e^{(2)} | e^{(2)} \rangle \end{bmatrix} = \underline{1}$ ✓

Hence the mass eigenfields are given by the linear combinations

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = U \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad \text{where}$$

$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$ are the interaction basis fields and $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$ the mass eigenfields

i.e. $M_{diag} \begin{pmatrix} \phi_1 \\ 0 \end{pmatrix} = m_1 \begin{pmatrix} \phi_1 \\ 0 \end{pmatrix}$ etc.

So for instance $\begin{pmatrix} \tilde{E}_1 \\ \tilde{E}_2 \end{pmatrix} = U_{\tilde{E}} \begin{pmatrix} \tilde{E} \\ \tilde{E}^{c\dagger} \end{pmatrix}$.

$$U_{\tilde{E}} = \begin{bmatrix} \bar{N}^{(1)} & \bar{N}^{(1)} \bar{f}^{(1)} \\ \bar{N}^{(2)} & \bar{N}^{(2)} \bar{f}^{(2)} \end{bmatrix}$$

with $f^{(i)} = \dfrac{m^2_{\tilde{E}_i} - a}{b}$; $|N^{(i)}|^2 = \dfrac{1}{1 + |f^{(i)}|^2}$

Alternatively, the general 2x2 Unitary matrix can be put in the form

$$U = e^{i\frac{\phi}{2}} \begin{bmatrix} e^{i\varphi}\cos\theta & -e^{+i\omega}\sin\theta \\ +e^{-i\omega}\sin\theta & e^{-i\varphi}\cos\theta \end{bmatrix}$$

$\phi, \varphi, \omega, \theta \in \mathbb{R}$ (this follows from $U^{\dagger} = U^{-1}$)

So then $\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = U_{\tilde{t}} \begin{pmatrix} \tilde{t} \\ \tilde{t}^{c\dagger} \end{pmatrix}$

$$U_{\tilde{t}} = e^{i\frac{\phi_t}{2}} \begin{bmatrix} e^{i\varphi_t}\cos\theta_t & -e^{i\omega_t}\sin\theta_t \\ +e^{-i\omega_t}\sin\theta_t & e^{i\varphi_t}\cos\theta_t \end{bmatrix}$$

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and

$$U_{\tilde{t}} M^2_{\tilde{t}} U^{\dagger}_{\tilde{t}} = M^2_{\tilde{t}\,diag.} = \begin{pmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{pmatrix}$$

it is left to find $\phi_t, \varphi_t, \omega_t, \theta_t$ in terms of $M^2_{\tilde{t}}$ matrix elements as in the real case.

---

with $S_{\tilde{t}} \mathcal{M}^2_{\tilde{t}} S^T_{\tilde{t}} = \mathcal{M}^2_{\tilde{t}\,diag.} = \begin{bmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{bmatrix}$

The eigenvalues are $\det(\mathcal{M}_{\tilde{t}} - \lambda \mathbb{1}) = 0 \Rightarrow$

$$0 = (m_t^2 + m_{q_t}^2 + D(\tilde{t}) - \lambda)(m_t^2 + m_{u^c t}^2 + D(\tilde{t}^c) - \lambda) - m_t^2(4\mu\cot\beta + A_t)^2$$

$\Rightarrow$

$$0 = \lambda^2 - [2m_t^2 + m_{q_t}^2 + m_{u^c t}^2 + D(\tilde{t}) + D(\tilde{t}^c)]\lambda + (m_t^2 + m_{q_t}^2 + D(\tilde{t}))(m_t^2 + m_{u^c t}^2 + D(\tilde{t}^c)) - m_t^2(4\mu\cot\beta + A_t)^2$$

$\Rightarrow$

$$\boxed{m^2_{\tilde{t}_1} = m_t^2 + \tfrac{1}{2}(m_{q_t}^2 + m_{u^c t}^2) + \tfrac{1}{4}M_Z^2\cos 2\beta - \sqrt{\left(\left(\frac{m_{q_t}^2 - m_{u^c t}^2}{2}\right) + M_Z^2\cos 2\beta\left(\tfrac{1}{4} - \tfrac{2}{3}\sin^2\theta_W\right)\right)^2 + m_t^2|4\mu\cot\beta + A_t|^2}}$$

↑ $|\,|^2$ if μ, A_t complex

$$\boxed{m^2_{\tilde{t}_2} = m_t^2 + \tfrac{1}{2}(m_{q_t}^2 + m_{u^c t}^2) + \tfrac{1}{4}M_Z^2\cos 2\beta + \sqrt{\left(\left(\frac{m_{q_t}^2 - m_{u^c t}^2}{2}\right) + M_Z^2\cos 2\beta\left(\tfrac{1}{4} - \tfrac{2}{3}\sin^2\theta_W\right)\right)^2 + m_t^2|4\mu\cot\beta + A_t|^2}}$$

↑ $|\,|^2$ if μ, A_t complex

And

$$\left(\mu, A_t \text{ real}\right) \qquad \boxed{\tan\theta_t = \frac{m_t^2 + m_{\tilde{q}_t}^2 + M_Z^2\cos 2\beta\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right) - m_{\tilde{t}_1}^2}{m_t\left(4\mu\cot\beta + A_t\right)}}$$

Likewise, with analogous assumptions, the $\tilde{d}$-type squark mass matrix has $\tilde{d}$ & $\tilde{d}^{c\dagger}$ mixing

$$\mathcal{L}_{\tilde{d}\text{-mass}} = -D(\tilde{d})\,\tilde{d}^\dagger\tilde{d} - D(\tilde{d}^c)\,\tilde{d}^{c\dagger}\tilde{d}^c$$

$$-4\mu\tan\beta\left[\frac{\bar{\mu}}{\mu}\tilde{d}\, m_d^{diag.}\,\tilde{d}^c + \tilde{d}^{c\dagger}\, m_d^{diag.}\,\tilde{d}^\dagger\right]$$

$$-\tilde{d}^\dagger\, m_d^{diag.\,2}\,\tilde{d} - \tilde{d}^{c\dagger}\, m_d^{diag.\,2}\,\tilde{d}^c$$

$$-\tilde{d}^\dagger\, m_{\tilde{q}}^2\,\tilde{d} - \tilde{d}^{c\dagger}\, m_{\tilde{d}^c}^2\,\tilde{d}^c$$

$$-\tilde{d}\, m_d^{diag.} A_d^{diag.}\,\tilde{d}^c - \tilde{d}^{c\dagger}\, m_d^{diag.}\,\bar{A}_d^{diag.}\,\tilde{d}^\dagger$$

---

where as before

$$D(\tilde{d}) = M_Z^2\cos 2\beta\left[-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W\right]$$

$$D(\tilde{d}^c) = M_Z^2\cos 2\beta\left[-\frac{1}{3}\sin^2\theta_W\right]$$

&

$$m_d^{diag.} = \begin{bmatrix} m_d & & 0 \\ & m_s & \\ 0 & & m_b \end{bmatrix}; \quad A_d^{diag} = \begin{bmatrix} A_d & & 0 \\ & A_s & \\ 0 & & A_b \end{bmatrix}$$

and

$$\frac{Y_d v_d}{\sqrt{2}} A_d \equiv m_d^{diag.} A_d^{diag.}$$

Again for each down type squark we have a mass matrix:

$$\mathcal{L}_{\tilde{d}\text{-mass}} = - \begin{bmatrix} \tilde{d}^{+} & \tilde{d}^{c} \end{bmatrix} \mathcal{M}^2_{\tilde{d}} \begin{bmatrix} \tilde{d} \\ \tilde{d}^{c+} \end{bmatrix}$$

$$\mathcal{L}_{\tilde{s}\text{-mass}} = - \begin{bmatrix} \tilde{s}^{+} & \tilde{s}^{c} \end{bmatrix} \mathcal{M}^2_{\tilde{s}} \begin{bmatrix} \tilde{s} \\ \tilde{s}^{c+} \end{bmatrix}.$$

$$\mathcal{L}_{\tilde{b}\text{-mass}} = - \begin{bmatrix} \tilde{b}^{+} & \tilde{b}^{c} \end{bmatrix} \mathcal{M}^2_{\tilde{b}} \begin{bmatrix} \tilde{b} \\ \tilde{b}^{c+} \end{bmatrix}$$

where focusing on the bottom squark we have

$$\mathcal{M}^2_{\tilde{b}} = \begin{bmatrix} \left(m_b^2 + m_{\tilde{t}_L}^2 + M_Z^2 \cos 2\beta\left[-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W\right]\right) & \left(m_b\left(4\mu\tan\beta + \bar{A}_b\right)\right) \\ \left(m_b\left(+4\bar{\mu}\tan\beta + A_b\right)\right) & \left(m_b^2 + m_{\tilde{d}^c_b}^2 + M_Z^2 \cos 2\beta\left[-\frac{1}{3}\sin^2\theta_W\right]\right) \end{bmatrix}$$

Defining the sbottom mass eigenstates $\tilde{b}_1$, $\tilde{b}_2$ with $\tilde{b}_1$ the lighter

(if μ, A_b complex need unitary matrix $U_{\tilde{b}}$)

$$\begin{bmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta_b & -\sin\theta_b \\ \sin\theta_b & \cos\theta_b \end{bmatrix}}_{=S_{\tilde{b}}} \begin{bmatrix} \tilde{b} \\ \tilde{b}^{c+} \end{bmatrix} \quad (\mu, A_b \text{ real})$$

we find that the masses are $S_{\tilde{b}} \mathcal{M}^2_{\tilde{b}} S^T_{\tilde{b}} = \mathcal{M}^2_{\tilde{b}\,diag.} = \begin{bmatrix} m^2_{\tilde{b}_1} & 0 \\ 0 & m^2_{\tilde{b}_2} \end{bmatrix}$

$$m^2_{\tilde{b}_1} = m_b^2 + \frac{1}{2}(m^2_{\tilde{q}_L} + m^2_{\tilde{d}^c_b}) - \frac{1}{4}M_Z^2\cos 2\beta - \sqrt{\left[\left(\frac{1}{2}(m^2_{\tilde{q}_L} - m^2_{\tilde{d}^c_b}) + M_Z^2\cos 2\beta\left(-\frac{1}{4} + \frac{1}{3}\sin^2\theta_W\right)\right)^2 + m_b^2\left|4\mu\tan\beta + \bar{A}_b\right|^2\right]}$$

$|\,|^2$ if μ, A_b complex

$$m^2_{\tilde{b}_2} = m_b^2 + \frac{1}{2}(m^2_{\tilde{q}_L} + m^2_{\tilde{d}^c_b}) - \frac{1}{4}M_Z^2\cos 2\beta + \sqrt{\left(\frac{(m^2_{\tilde{q}_L} - m^2_{\tilde{d}^c_b})}{2} + M_Z^2\cos 2\beta\left(-\frac{1}{4} + \frac{1}{3}\sin^2\theta_W\right)\right)^2 + m_b^2\left|4\mu\tan\beta + \bar{A}_b\right|^2}$$

$|\,|^2$ if μ, A_b complex

with mixing angle

$$\tan\theta_b = \frac{m_b^2 + m^2_{\tilde{q}_L} + M_Z^2\cos 2\beta\left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W\right) - m^2_{\tilde{b}_1}}{m_b(4\mu\tan\beta + A_b)}$$

(μ, A_b real)

Finally considering the slepton masses, assuming lepton flavor is conserved so that they do not mix, we have for the sneutrinos

$$\mathcal{L}_{\tilde{\nu}\text{-mass}} = -\left(\tfrac{1}{2}M_Z^2\cos 2\beta + m_\ell^2\right)\tilde{\nu}_e^\dagger \tilde{\nu}_e$$

So each sneutrino has (mass)^2

$$\boxed{\begin{aligned} m_{\tilde{\nu}_e}^2 &= m_{\ell_e}^2 + \tfrac{1}{2}M_Z^2\cos 2\beta \\ m_{\tilde{\nu}_\mu}^2 &= m_{\ell_\mu}^2 + \tfrac{1}{2}M_Z^2\cos 2\beta \\ m_{\tilde{\nu}_\tau}^2 &= m_{\ell_\tau}^2 + \tfrac{1}{2}M_Z^2\cos 2\beta \end{aligned}}$$

Meanwhile, the down-like sleptons have mass terms

$$\begin{aligned} \mathcal{L}_{\tilde{e}\,\text{mass}} = &-M_Z^2\cos 2\beta\left[-\tfrac{1}{2}+\sin^2\theta_W\right]\tilde{e}^\dagger\tilde{e} \\ &-M_Z^2\cos 2\beta\left[-\sin^2\theta_W\right]\tilde{e}^{c\dagger}\tilde{e}^c \\ &-\mu\tan\beta\left[\bar{\mu}\,\tilde{e}\, m_e^{\text{diag.}}\,\tilde{e}^c - \tilde{e}^{c\dagger}\, m_e^{\text{diag.}}\,\tilde{e}^\dagger\right] \\ &-\tilde{e}^\dagger\, m_e^{\text{diag.}\,2}\,\tilde{e} - \tilde{e}^{c\dagger}\, m_e^{\text{diag.}\,2}\,\tilde{e}^c \\ &-\tilde{e}^\dagger\, m_\ell^2\,\tilde{e} - \tilde{e}^{c\dagger}\, m_{ec}^2\,\tilde{e}^c \\ &-\tilde{e}\, m_e^{\text{diag.}}\cdot A_\ell^{\text{diag.}}\cdot\tilde{e}^c - \tilde{e}^{c\dagger}\, m_e^{\text{diag.}}\cdot\bar{A}_\ell^{\text{diag.}}\,\tilde{e}^\dagger \end{aligned}$$

And for each type of charged slepton we have

$$\mathcal{L}_{\tilde e\text{-mass}} = -\begin{bmatrix}\tilde e^{\dagger} & \tilde e^{c}\end{bmatrix} \mathcal{M}^2_{\tilde e} \begin{bmatrix}\tilde e \\ \tilde e^{c\dagger}\end{bmatrix}$$

$$\mathcal{L}_{\tilde \mu\text{-mass}} = -\begin{bmatrix}\tilde \mu^{\dagger} & \tilde \mu^{c}\end{bmatrix} \mathcal{M}^2_{\tilde \mu} \begin{bmatrix}\tilde \mu \\ \tilde \mu^{c\dagger}\end{bmatrix}$$

$$\mathcal{L}_{\tilde \tau\text{-mass}} = -\begin{bmatrix}\tilde \tau^{\dagger} & \tilde \tau^{c}\end{bmatrix} \mathcal{M}^2_{\tilde \tau} \begin{bmatrix}\tilde \tau \\ \tilde \tau^{c\dagger}\end{bmatrix}$$

where for the tau-slepton we have

$$\mathcal{M}^2_{\tilde\tau} = \begin{bmatrix} \left(m_\tau^2 + m_{l\tau}^2 + M_Z^2\cos2\beta\left[-\tfrac{1}{2}+\sin^2\theta_W\right]\right) & \left(m_\tau(4\mu\tan\beta + \bar{A}_\tau)\right) \\ \left(m_\tau(4\bar{\mu}\tan\beta + A_\tau)\right) & \left(m_\tau^2 + m_{ec\tau}^2 - M_Z^2\cos2\beta\sin^2\theta_W\right) \end{bmatrix}$$

Defining the mass eigenstates $\tilde\tau_1, \tilde\tau_2$ with $m_{\tilde\tau_1} < m_{\tilde\tau_2}$

(if μ, A_τ complex need unitary matrix $U_{\tilde\tau}$)

$$\begin{bmatrix}\tilde\tau_1 \\ \tilde\tau_2\end{bmatrix} = \underbrace{\begin{bmatrix}\cos\theta_\tau & -\sin\theta_\tau \\ \sin\theta_\tau & \cos\theta_\tau\end{bmatrix}}_{=S_{\tilde\tau}} \begin{bmatrix}\tilde\tau \\ \tilde\tau^{c\dagger}\end{bmatrix}$$

(μ, A_τ real)

$$S_{\tilde{\tau}} \mathcal{M}^2_{\tilde{\tau}} S^T_{\tilde{\tau}} = \mathcal{M}^2_{\tilde{\tau}\,diag.} = \begin{bmatrix} m^2_{\tilde{\tau}_1} & 0 \\ 0 & m^2_{\tilde{\tau}_2} \end{bmatrix}$$

Consequently we find the masses

$$m^2_{\tilde{\tau}_1} = m^2_\tau + \tfrac{1}{2}\left(m^2_{\ell\tau} + m^2_{e^c\tau}\right) - \tfrac{1}{4} M^2_Z \cos 2\beta$$
$$- \left[\left(\frac{(m^2_{\ell\tau} - m^2_{e^c\tau})}{2} + M^2_Z \cos 2\beta\left(-\tfrac{1}{4} + \sin^2\theta_W\right)\right)^2 + m^2_\tau \left|4\mu \tan\beta + \bar{A}_\tau\right|^2\right]^{1/2}$$

$|\,|^2$ if μ, A_τ complex

$$m^2_{\tilde{\tau}_2} = m^2_\tau + \tfrac{1}{2}\left(m^2_{\ell\tau} + m^2_{e^c\tau}\right) - \tfrac{1}{4} M^2_Z \cos 2\beta$$
$$+ \left[\left(\frac{(m^2_{\ell\tau} - m^2_{e^c\tau})}{2} + M^2_Z \cos 2\beta\left(-\tfrac{1}{4} + \sin^2\theta_W\right)\right)^2 + m^2_\tau \left|4\mu \tan\beta + \bar{A}_\tau\right|^2\right]^{1/2}$$

$|\,|^2$ if μ, A_τ complex

with mixing angle

$$\tan\theta_\tau = \frac{m^2_\tau + m^2_{\ell\tau} + M^2_Z \cos 2\beta\left[-\tfrac{1}{2} + \sin^2\theta_W\right] - m^2_{\tilde{\tau}_1}}{m_\tau\left(4\mu \tan\beta + A_\tau\right)}$$

(μ, A_τ real)

Now for the first 2 generations of squarks and sleptons the corresponding quark and lepton masses are much smaller than the SUSY breaking masses — hence we can ignore the mixing within each family — i.e. the mass matrix is diagonal (to a high degree of accuracy) and the $\tilde{f}$, $\tilde{f}^{c+}$ are mass eigenstates already with

$$m^2_{\tilde{u}} = m^2_u + m^2_{q_u} + M^2_Z \cos 2\beta \left(\tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta_W\right)$$

$$m^2_{\tilde{u}^c} = m^2_u + m^2_{u^c_u} + M^2_Z \cos 2\beta \left(\tfrac{2}{3}\sin^2\theta_W\right)$$

$$m^2_{\tilde{d}} = m^2_d + m^2_{q_u} + M^2_Z \cos 2\beta \left[-\tfrac{1}{2} + \tfrac{1}{3}\sin^2\theta_W\right]$$

$$m^2_{\tilde{d}^c} = m^2_d + m^2_{d^c_d} + M^2_Z \cos 2\beta \left[-\tfrac{1}{3}\sin^2\theta_W\right]$$

$$m^2_{\tilde{e}} = m^2_e + m^2_{l_e} + M^2_Z \cos 2\beta \left[-\tfrac{1}{2} + \sin^2\theta_W\right]$$

$$m^2_{\tilde{e}^c} = m^2_e + m^2_{e^c_e} - M^2_Z \cos 2\beta \left[\sin^2\theta_W\right]$$

$$m^2_{\tilde{\nu}_e} = m^2_{l_e} + \tfrac{1}{2} M^2_Z \cos 2\beta$$

with similar expressions for the second generation.

Note that the mass splitting in the doublet fields is an SU(2) relation, no superpotential terms contribute to the masses (negligibly small). So for instance

$$m_{\tilde{u}}^2 - m_{\tilde{d}}^2 = m_u^2 - m_d^2 + M_Z^2 \cos 2\beta \left[1 - \sin^2\theta_W\right]$$

$$m_{\tilde{\nu}_e}^2 - m_{\tilde{e}}^2 = -m_e^2 + M_Z^2 \cos 2\beta \left[1 - \sin^2\theta_W\right] ,$$

This is then a model independent splitting since it is due to SU(2) only. The splitting cannot be too large.

With the particle & sparticle masses determined - the possible interaction vertices are delineated next.