is fixed — only one gaugino mass can be made real.

So

$$\mathcal{L}_{\substack{gluino\\mass}} = \frac{1}{2} M_3 \left[\tilde{G}^m \tilde{G}^m + \bar{\tilde{G}}^m \bar{\tilde{G}}^m \right]$$

Charginos & Neutralinos : $SU(2) \times U(1) \to U(1)_{em}$ results in states with the same electric charge and color (and spin) mixing.

The neutral Higgsinos and neutral gauginos will mix to form neutral mass eigenstates

Neutral Higgsinos : $\tilde{H}^0_u, \tilde{H}^0_d$ & $\bar{\tilde{H}}^0_u, \bar{\tilde{H}}^0_d$

Neutral Gauginos : $\tilde{A}^3, \tilde{B}$ & $\bar{\tilde{A}}^3, \bar{\tilde{B}}$

The corresponding mass terms comes from several sources:

soft SUSY Breaking 1) $\mathcal{L}_{\not{S}YM} \supset \frac{1}{2} M_1 \tilde{B}^2 + \frac{1}{2} \bar{M}_1 \bar{\tilde{B}}^2 + \frac{1}{2} M_2 \tilde{A}^{3\,2} + \frac{1}{2} \bar{M}_2 \bar{\tilde{A}}^{3\,2}$

Superpotential 2) $\int dS\, W + \int d\bar{S}\, \bar{W} \supset \int d^4x \left[-4\mu \tilde{H}^0_u \tilde{H}^0_d - 4\bar{\mu} \bar{\tilde{H}}^0_u \bar{\tilde{H}}^0_d \right]$

Kähler Potential 3) $\mathcal{L}_K \supset \sqrt{2}\, g_1 \left[\frac{1}{2} \frac{v_u}{\sqrt{2}} \tilde{B} \tilde{H}^0_u + \frac{1}{2} \frac{v_u}{\sqrt{2}} \bar{\tilde{H}}^0_u \bar{\tilde{B}} - \frac{1}{2} \frac{v_d}{\sqrt{2}} \tilde{B} \tilde{H}^0_d - \frac{1}{2} \frac{v_d}{\sqrt{2}} \bar{\tilde{H}}^0_d \bar{\tilde{B}} \right]$

$$+\sqrt{2}\, g_2 \Big[-\frac{1}{2}\frac{v_u}{\sqrt{2}} \tilde{A}^3 \tilde{H}_u^0 - \frac{1}{2}\frac{v_u}{\sqrt{2}} \bar{\tilde{H}}_u^0 \bar{\tilde{A}}^3$$

$$+ \frac{1}{2}\frac{v_d}{\sqrt{2}} \tilde{A}^3 \tilde{H}_d^0 + \frac{1}{2}\frac{v_d}{\sqrt{2}} \bar{\tilde{H}}_d^0 \bar{\tilde{A}}^3 \Big]$$

Hence the neutralino mass terms are

$$\mathcal{L}_{\text{neutralino}} = \Big[\frac{1}{2} M_1 \tilde{B}^2 + \frac{1}{2} M_2 \tilde{A}^{3\,2} - 4\mu \tilde{H}_u^0 \tilde{H}_d^0$$

$$+ \frac{1}{2} g_1 v_u \tilde{B} \tilde{H}_u^0 - \frac{1}{2} g_1 v_d \tilde{B} \tilde{H}_d^0$$

$$- \frac{1}{2} g_2 v_u \tilde{A}^3 \tilde{H}_u^0 + \frac{1}{2} g_2 v_d \tilde{A}^3 \tilde{H}_d^0$$

$$+ h.c. \Big]$$

$$= -\frac{1}{2} \left[\tilde{H}_u^0 \;\; \tilde{H}_d^0 \;\; \tilde{A}^3 \;\; \tilde{B} \right] M_{\text{neutralino}} \times \begin{bmatrix} \tilde{H}_u^0 \\ \tilde{H}_d^0 \\ \tilde{A}^3 \\ \tilde{B} \end{bmatrix}$$

with

$$M_{\text{neutralino}} = \begin{bmatrix} 0 & 4\mu & +\frac{g_2 v_u}{2} & -\frac{g_1 v_u}{2} \\ 4\mu & 0 & -\frac{g_2 v_d}{2} & +\frac{g_1 v_d}{2} \\ +\frac{g_2 v_u}{2} & -\frac{g_2 v_d}{2} & -M_2 & 0 \\ -\frac{g_1 v_u}{2} & +\frac{g_1 v_d}{2} & 0 & -M_1 \end{bmatrix}$$

The chargino mass matrix can be found similarly. The same charge Higgsinos and gauginos will mix to form mass eigenstates

Charged Higgsinos : $\tilde{H}_u^+, \tilde{H}_d^-$ $\left(\bar{\tilde{H}}_u^+, \bar{\tilde{H}}_d^- \right.$

Charged Gauginos : $\tilde{W}^+, \tilde{W}^-$ & $\left. \bar{\tilde{W}}^+, \bar{\tilde{W}}^- \right)$

with $\tilde{W}^{\pm} \equiv \frac{1}{\sqrt{2}}(\tilde{A}^1 \mp i \tilde{A}^2)$

The mass terms come from a few sources

soft-SUSY breaking 1) $\mathcal{L}_{\not{S}XM} \supset M_2 \tilde{W}^+ \tilde{W}^- + \bar{M}_2 \bar{\tilde{W}}^+ \bar{\tilde{W}}^-$

$$\left(= \frac{1}{2} M_2 (\tilde{A}^1 \tilde{A}^1 + \tilde{A}^2 \tilde{A}^2) + \frac{1}{2} \bar{M}_2 (\bar{\tilde{A}}^{1\,2} + \bar{\tilde{A}}^{2\,2}) \right)$$

Superpotential 2) $\int d^2\theta\, W + \int d^2\bar{\theta}\, \bar{W} \supset \int d^4x \left[4\mu \tilde{H}_u^+ \tilde{H}_d^- + 4\bar{\mu} \bar{\tilde{H}}_u^+ \bar{\tilde{H}}_d^- \right]$

Kähler Potential 3) $\mathcal{L}_K = \sqrt{2}\, g_2 \left(0 \;\; v_u/\sqrt{2} \right) \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & \tilde{W}^+ \\ \tilde{W}^- & 0 \end{bmatrix} \begin{pmatrix} \tilde{H}_u^+ \\ 0 \end{pmatrix}$

$$+ \sqrt{2}\, g_2 \left(\bar{\tilde{H}}_u^+ \;\; 0 \right) \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & \bar{\tilde{W}}^- \\ \bar{\tilde{W}}^+ & 0 \end{bmatrix} \begin{bmatrix} 0 \\ v_u/\sqrt{2} \end{bmatrix}$$

$$+ \sqrt{2}\, g_2 \left(\frac{v_d}{\sqrt{2}} \;\; 0 \right) \begin{bmatrix} 0 & \tilde{W}^+ \\ \tilde{W}^- & 0 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \tilde{H}_d^- \end{pmatrix}$$

$$+\sqrt{2}\, g_2 \begin{pmatrix} 0 & \tilde{H}_d^- \end{pmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & \tilde{W}^- \\ \tilde{W}^+ & 0 \end{bmatrix} \begin{pmatrix} v_d/\sqrt{2} \\ 0 \end{pmatrix}$$

$$= \frac{g_2 v_u}{\sqrt{2}} \left(\tilde{W}^- \tilde{H}_u^+ + \tilde{H}_u^+ \tilde{W}^- \right)$$

$$+ \frac{g_2 v_d}{\sqrt{2}} \left(\tilde{W}^+ \tilde{H}_d^- + \tilde{W}^+ \tilde{H}_d^- \right)$$

The chargino mass terms are

$$\mathcal{L}_{chargino} = \Big[M_2 \tilde{W}^+ \tilde{W}^- + 4_\mu \tilde{H}_u^+ \tilde{H}_d^- + \frac{g_2 v_u}{\sqrt{2}} \tilde{W}^- \tilde{H}_u^+ + \frac{g_2 v_d}{\sqrt{2}} \tilde{W}^+ \tilde{H}_d^- + h.c. \Big]$$

$$= -\frac{1}{2} \begin{pmatrix} \tilde{W}^+ & \tilde{H}_u^+ & \tilde{W}^- & \tilde{H}_d^- \end{pmatrix} M_{chargino} \begin{bmatrix} \tilde{W}^+ \\ \tilde{H}_u^+ \\ \tilde{W}^- \\ \tilde{H}_d^- \end{bmatrix}$$

with

$$M_{chargino} = \begin{bmatrix} 0 & 0 & -M_2 & -\frac{g_2 v_d}{\sqrt{2}} \\ 0 & 0 & -\frac{g_2 v_u}{\sqrt{2}} & -4\mu \\ -M_2 & -\frac{g_2 v_u}{\sqrt{2}} & 0 & 0 \\ -\frac{g_2 v_d}{\sqrt{2}} & -4\mu & 0 & 0 \end{bmatrix}$$

Expressing these in terms of M_W, M_Z, θ_W & β yields

$$M_{Neutralino} = \begin{bmatrix} 0 & 4\mu & +M_W s\beta & -M_Z s\theta_W s\beta \\ 4\mu & 0 & -M_W c\beta & +M_Z s\theta_W c\beta \\ M_W s\beta & -M_W c\beta & -M_2 & 0 \\ -M_Z s\theta_W s\beta & +M_Z s\theta_W s\beta & 0 & -M_1 \end{bmatrix}$$

$$M_{chargino} = \begin{bmatrix} 0 & M_{ch}^T \\ M_{ch} & 0 \end{bmatrix}$$

with

$$M_{ch} = \begin{bmatrix} -M_2 & -\sqrt{2} M_W \sin\beta \\ -\sqrt{2} M_W \cos\beta & -4\mu \end{bmatrix}$$

Likewise for the complex conjugate spinors

$$\overline{M}_{Neutralino} = M^*_{Neutralino}$$

$$\overline{M}_{chargino} = M^*_{chargino}$$

These are complex, <u>non</u>-Hermitian matrices hence they can be diagonalized by a singular value decomposition (just as we did in the CKM - Yukawa coupling quark case in the SM)

$$m_{\tilde{C}} = L^* M_{ch} R^{-1} \quad \text{with } L, R \text{ unitary } 2\times 2 \text{ matrices}$$

($m_{\tilde{C}}$ ↑ diagonal)

with the mass eigenstates given by

$$\begin{bmatrix} \tilde{C}_1^+ \\ \tilde{C}_2^+ \end{bmatrix} = R^* \begin{bmatrix} \tilde{W}^+ \\ \tilde{H}_u^+ \end{bmatrix} \; ; \; \begin{bmatrix} \tilde{C}_1^- \\ \tilde{C}_2^- \end{bmatrix} = L^* \begin{bmatrix} \hat{W}^- \\ \tilde{H}_d^- \end{bmatrix}$$

After diagonalization L & R matrix elements will appear in the interaction vertices of the chargino — mass eigenstate fields — like the CKM matrix in the charge current.

Note: The spinor free field equations are

$$i\bar{\not\partial}\psi_i = M_{ij}\psi_j \; ; \; i\not\partial\psi_i = \bar{M}_{ij}\bar{\psi}_j$$

$\Rightarrow$

$$i i\bar{\not\partial}\not\partial\bar{\psi}_i = M_{ij}\, i\not\partial\psi_j = M_{ij}\bar{M}_{jk}\bar{\psi}_k$$

$$\parallel$$

$$-\partial^2\bar{\psi}_i = (M\bar{M})_{ij}\bar{\psi}_j$$

$M\bar{M}$ acts as the (mass)^2 matrix, and it is Hermitian

Now

$$M_{\text{chargino}} \overline{M}_{\text{chargino}} = \begin{bmatrix} 0 & M_{ch}^T \\ M_{ch} & 0 \end{bmatrix} \begin{bmatrix} 0 & \overline{M}_{ch}^T \\ \overline{M}_{ch} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} M_{ch}^T \overline{M}_{ch} & 0 \\ 0 & M_{ch} \overline{M}_{ch}^T \end{bmatrix} = \begin{bmatrix} (M_{ch}^\dagger M_{ch})^T & 0 \\ 0 & M_{ch} M_{ch}^\dagger \end{bmatrix}$$

So we can easily find the eigenvalues of

$$m_{ch}^2 \equiv \left[M_{ch} \overline{M}_{ch}^T \right] = \begin{bmatrix} -M_2 & -\sqrt{2} M_W s\beta \\ -\sqrt{2} M_W c\beta & -4\mu \end{bmatrix} \times$$

$$\times \begin{bmatrix} -\overline{M}_2 & -\sqrt{2} M_W c\beta \\ -\sqrt{2} M_W s_\beta & -4\overline{\mu} \end{bmatrix}$$

$$m_{ch}^2 = \begin{bmatrix} |M_2|^2 + 2M_W^2 s_\beta^2 & \sqrt{2} M_W [M_2 c\beta + 4\overline{\mu} s\beta] \\ \sqrt{2} M_W [\overline{M}_2 c\beta + 4\mu s\beta] & 16|\mu|^2 + 2M_2^2 c^2\beta \end{bmatrix}$$

$$\det(m_{ch}^2 - \lambda \mathbb{1}) = 0$$

$$0 = \lambda^2 - \left[16|\mu|^2 + |M_2|^2 + 2M_W^2 \right] \lambda$$

$$+ |4\mu M_2 - M_W^2 \sin 2\beta|^2$$

$\Rightarrow$

$$m^2_{\tilde{c}_1} = \frac{1}{2}\left[16|\mu|^2 + |M_2|^2 + 2M_W^2 - \sqrt{(16|\mu|^2+|M_2|^2+2M_W^2)^2 - 4|4\mu M_2 - M_W^2 \sin 2\beta|^2}\right]$$

$$m^2_{\tilde{c}_2} = \frac{1}{2}\left[16|\mu|^2 + |M_2|^2 + 2M_W^2 + \sqrt{(16|\mu|^2+|M_2|^2+2M_W^2)^2 - 4|4\mu M_2 - M_W^2 \sin 2\beta|^2}\right]$$

$$m_{\tilde{c}_1} \leq m_{\tilde{c}_2}$$

Note: for $|4|\mu| \pm M_2| >> M_W$; $M_{ch} \approx \begin{bmatrix} |M_2|^2 & 0 \\ 0 & 16|\mu|^2 \end{bmatrix}$

so the charginos are a wino with mass $|M_2|$ and a higgsino with mass $4|\mu|$, approximately.

It is left to find L & R and hence the eigenvectors. $m_{\tilde{c}} = L^* M R^{-1}$, $m_{\tilde{c}} = R \bar{M}^T L^T$ $(= m_{\tilde{c}}^\dagger)$ (1×1, real; diagonal)

So

$$m^2_{\tilde{c}} = R\bar{M}^T M R^{-1} = L^* M \bar{M}^T L^T .$$

($m^2_{\tilde{c}}$: diagonal)