Neutral Scalar fields: There are 4 such fields. The charged scalars can be set to zero to find the neutral fields mass terms: p.-79-

$$V_{H\,\text{neutral mass}} = M_{H_u}^2 H_u^{0\dagger} H_u^0 + M_{H_d}^2 H_d^{0\dagger} H_d^0$$

$$+ b\left[H_u^0 H_d^0 + H_u^{0\dagger} H_d^{0\dagger}\right]$$

$$+ \frac{g_2^2+g_1^2}{4\cdot 2}\left[\frac{v_u^2}{2}\left(H_u^0+H_u^{0\dagger}\right)^2 + v_u^2\left(H_u^{0\dagger} H_u^0\right) + \frac{v_d^2}{2}\left(H_d^0+H_d^{0\dagger}\right)^2 + v_d^2 H_d^{0\dagger} H_d^0\right]$$

$$+ \frac{g_2^2-g_1^2}{2\cdot 2}\left[\frac{v_d^2}{2}|H_u^0|^2 + \frac{v_u^2}{2}|H_d^0|^2 + \frac{v_u v_d}{2}\left(H_u^0+H_u^{0\dagger}\right)\left(H_d^0+H_d^{0\dagger}\right)\right]$$

$$-\frac{g_2^2}{2}\left[\frac{v_u v_d}{2}\left(H_u^0 H_d^0 + H_u^{0\dagger} H_d^{0\dagger}\right) + \left(\frac{v_u}{\sqrt{2}} H_d^0 + \frac{v_d}{\sqrt{2}} H_u^0\right)\left(\frac{v_u}{\sqrt{2}} H_d^{0\dagger} + \frac{v_d}{\sqrt{2}} H_u^{0\dagger}\right)\right]$$

Since CP was good for the potential V_H the real (CP even) and imaginary (CP odd) components of the scalar fields do not mix. Hence the 4x4 mass matrix will break up into 2, 2x2 blocks. The neutral Goldstone boson will lie in the CP odd sector — so consider these pseudoscalar fields first — the imaginary components of the fields.

Express the fields as

$$H_u^0 = \frac{1}{\sqrt{2}}\left(H_u^R + i H_u^I\right) \; ; \; H_d^0 = \frac{1}{\sqrt{2}}\left(H_d^R + i H_d^I\right)$$

So

$$H_{\substack{u\\d}}^{0\dagger} H_{\substack{u\\d}}^{0} = \frac{1}{2}\left(H_{\substack{u\\d}}^{R\,2} + H_{\substack{u\\d}}^{I\,2}\right)$$

$$H_u^0 + H_u^{0\dagger} = \sqrt{2}\, H_u^R \; ; \; H_d^0 + H_d^{0\dagger} = \sqrt{2}\, H_d^R$$

$$\left(H_u^0 H_d^0 + H_u^{0\dagger} H_d^{0\dagger}\right) = H_u^R H_d^R - H_u^I H_d^I$$

$$\left(H_u^0 H_d^{0\dagger} + H_u^{0\dagger} H_d^0\right) = H_u^R H_d^R + H_u^I H_d^I$$

So

$$\begin{aligned}
V_{\substack{H\,\text{neutral}\\ \text{mass}}} &= \frac{1}{2} M_{H_u}^2 \left(H_u^{R2} + H_u^{I2}\right) + \frac{1}{2} M_{H_d}^2 \left(H_d^{R2} + H_d^{I2}\right) \\
&\quad + b\left[H_u^R H_d^R - H_u^I H_d^I\right] \\
&\quad + \frac{g_2^2 + g_1^2}{4\cdot 2}\Big[v_u^2 H_u^{R2} + v_u^2 \tfrac{1}{2}\left(H_u^{R2} + H_u^{I2}\right) \\
&\qquad + v_d^2 H_d^{R2} + \tfrac{1}{2} v_d^2 \left(H_d^{R2} + H_d^{I2}\right)\Big] \\
&\quad + \frac{g_2^2 - g_1^2}{2\cdot 2}\Big[\frac{v_d^2}{4}\left(H_u^{R2} + H_u^{I2}\right) + \frac{v_u^2}{4}\left(H_d^{R2} + H_d^{I2}\right) \\
&\qquad + v_u v_d \left(H_u^R\right)\left(H_d^R\right)\Big] \\
&\quad - \frac{g_2^2}{2}\Big[\frac{v_u v_d}{2}\left(H_u^R H_d^R - H_u^I H_d^I\right) \\
&\qquad + \frac{v_u^2}{4}\left(H_d^{R2} + H_d^{I2}\right) + \frac{v_d^2}{4}\left(H_u^{R2} + H_u^{I2}\right) \\
&\qquad + \frac{v_u v_d}{2}\left(H_u^R H_d^R + H_u^I H_d^I\right)\Big]
\end{aligned}$$

So consider the pseudoscalar terms

$$\boxed{\begin{aligned} V_{H_{ps}\,\text{mass}} &= \tfrac{1}{2}M_{Hu}^2\, H_u^{I\,2} + \tfrac{1}{2}M_{Hd}^2\, H_d^{I\,2} - b\, H_u^I H_d^I \\ &+ \frac{g_2^2+g_1^2}{4\cdot 2}\left[\tfrac{1}{2}v_u^2 H_u^{I\,2} + \tfrac{1}{2}v_d^2 H_d^{I\,2}\right] \\ &+ \frac{g_2^2-g_1^2}{2\cdot 2}\left[\tfrac{1}{4}v_d^2 H_u^{I\,2} + \tfrac{1}{4}v_u^2 H_d^{I\,2}\right] \\ &- \frac{g_2^2}{2}\left[\tfrac{1}{4}v_u^2 H_d^{I\,2} + \tfrac{1}{4}v_d^2 H_u^{I\,2}\right] \end{aligned}}$$

$$\boxed{\equiv \frac{1}{2}\begin{bmatrix} H_u^I & H_d^I \end{bmatrix} M_I^2 \begin{bmatrix} H_u^I \\ H_d^I \end{bmatrix}}$$

So

$$M_I^2 = \begin{bmatrix} \left.\dfrac{\partial^2 V_H}{\partial H_u^I \partial H_u^I}\right|_{vev} & \left.\dfrac{\partial^2 V_H}{\partial H_u^I \partial H_d^I}\right|_{vev} \\ \left.\dfrac{\partial^2 V_H}{\partial H_d^I \partial H_u^I}\right|_{vev} & \left.\dfrac{\partial^2 V_H}{\partial H_d^I \partial H_d^I}\right|_{vev} \end{bmatrix}$$

or from above reading off the (mass)^2 terms

$$M_I^2 = \begin{bmatrix} \left(M_{Hu}^2 + \frac{g_2^2+g_1^2}{8}v_u^2 + \frac{g_2^2-g_1^2}{8}v_d^2 - \frac{1}{4}g_2^2 v_d^2\right) & (-b) \\ (-b) & \left(M_{Hd}^2 + \frac{g_2^2+g_1^2}{8}v_d^2 + \frac{g_2^2-g_1^2}{8}v_u^2 - \frac{1}{4}g_2^2 v_u^2\right) \end{bmatrix}$$

Using the minimization conditions $\Rightarrow$

$$\boxed{M_I^2 = \begin{bmatrix} -b\cot\beta & -b \\ -b & -b\tan\beta \end{bmatrix}}$$

As previously we can diagonalize this with the same similarity transformation S

$$S M_I^2 S^T S \begin{bmatrix} H_u^I \\ H_d^I \end{bmatrix} = \underset{\text{diagonal}}{m_I^2} S \begin{bmatrix} H_u^I \\ H_d^I \end{bmatrix}$$

Recall: $\left(S = \begin{bmatrix} c\beta & s\beta \\ -s\beta & c\beta \end{bmatrix} \right)$

where

$$m_I^2 = \begin{bmatrix} -b(\cot\beta + \tan\beta) & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{2b}{\sin 2\beta} & 0 \\ 0 & 0 \end{bmatrix}$$

&

$$\boxed{S \begin{bmatrix} H_u^I \\ H_d^I \end{bmatrix} = \begin{bmatrix} A \\ \pi^0 \end{bmatrix} = \begin{bmatrix} H_u^I \cos\beta + H_d^I \sin\beta \\ H_d^I \cos\beta - H_u^I \sin\beta \end{bmatrix}}$$

So π^0 is the massless Goldstone boson that is eaten by the Z^0 gauge boson to give the Z mass. A is an additional pseudoscala particle (another Higgs particle) in the theory; it has mass

$$\boxed{m_A^2 \equiv \frac{-2b}{\sin 2\beta}}$$

Note: $\boxed{m_{H^\pm}^2 = m_A^2 + M_W^2}$

At tree level, the charged Higgs fields have mass

$$m_{H^\pm} \geq M_W \quad , \quad m_{H^\pm} \geq m_A$$

Finally consider the neutral scalar fields (real part). The scalar real part fields' potential is

$$V_{Hs}^{\text{mass}} = \frac{1}{2} M_u^2 H_u^{R^2} + \frac{1}{2} M_{Hd}^2 H_d^{R^2} + b H_u^R H_d^R$$
$$+ \frac{g_2^2 + g_1^2}{8}\left[\frac{3}{2} v_u^2 H_u^{R^2} + \frac{3}{2} v_d^2 H_d^{R^2}\right]$$
$$+ \frac{g_2^2 - g_1^2}{4}\left[\frac{1}{4} v_d^2 H_u^{R^2} + \frac{1}{4} v_u^2 H_d^{R^2} + v_u v_d H_u^R H_d^R\right]$$
$$- \frac{g_2^2}{2}\left[v_u v_d H_u^R H_d^R + \frac{1}{4} v_u^2 H_d^{R^2} + \frac{1}{4} v_d^2 H_u^{R^2}\right]$$