So we have the EWSB minimum of the MSSM and we can now shift the Higgs fields and expand ~~the~~ the action around the minimum

$$H_u = \begin{bmatrix} H_u^+ \\ H_u^0 + \frac{v_u}{\sqrt{2}} \end{bmatrix} ; H_d = \begin{bmatrix} H_d^0 + \frac{v_d}{\sqrt{2}} \\ H_d^- \end{bmatrix}$$

First consider the Higgs kinetic terms

$$\mathcal{L}_\phi + V_\phi = (D_\mu H_u)^\dagger (D^\mu H_u) + (D_\mu H_d)^\dagger (D^\mu H_d)$$

where

$$D_\mu H_u = \left(\partial_\mu - \frac{i g_2}{2} \vec{\sigma} \cdot \vec{A}_\mu - \frac{i g_1}{2} B_\mu\right) H_u$$

$$D_\mu H_d = \left(\partial_\mu - \frac{i g_2}{2} \vec{\sigma} \cdot \vec{A}_\mu + \frac{i g_1}{2} B_\mu\right) H_d$$

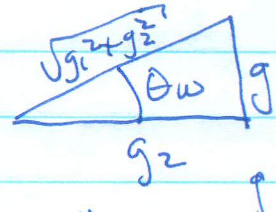
As in the SM we have

$$g_2 \vec{\sigma} \cdot \vec{A}_\mu \pm g_1 B_\mu = \begin{bmatrix} g_2 A_\mu^3 \pm g_1 B_\mu & g_2 (A_\mu^1 - i A_\mu^2) \\ g_2 (A_\mu^1 + i A_\mu^2) & -g_2 A_\mu^3 \pm g_1 B_\mu \end{bmatrix}$$

Define as in SM

$$\boxed{\begin{aligned} W^{\pm}_\mu &\equiv \frac{1}{\sqrt{2}}\left(A^1_\mu \mp i A^2_\mu\right) \\ Z_\mu &\equiv \frac{g_2 A^3_\mu - g_1 B_\mu}{\sqrt{g_1^2+g_2^2}} \\ A_\mu &\equiv \frac{g_2 B_\mu + g_1 A^3_\mu}{\sqrt{g_1^2+g_2^2}} \end{aligned}}$$

$$\tan\theta_W \equiv \frac{g_1}{g_2}$$



$$\sin\theta_W = \frac{g_1}{\sqrt{g_1^2+g_2^2}}$$

$$\cos\theta_W = \frac{g_2}{\sqrt{g_1^2+g_2^2}}$$

$$Z_\mu = \cos\theta_W A^3_\mu - \sin\theta_W B_\mu$$

$$A_\mu = \sin\theta_W A^3_\mu + \cos\theta_W B_\mu$$

So $\Rightarrow$

$$\boxed{\begin{aligned} A^3_\mu &= \cos\theta_W Z_\mu + \sin\theta_W A_\mu \\ B_\mu &= \cos\theta_W A_\mu - \sin\theta_W Z_\mu \end{aligned}}$$

Then

$$\frac{g_2 \vec{J}\cdot\vec{A}_\mu - g_1 B_\mu}{\sqrt{g_1^2+g_2^2}} = \begin{bmatrix} Z_\mu & \sqrt{2}\cos\theta_W W^+_\mu \\ \sqrt{2}\cos\theta_W W^-_\mu & -\cos 2\theta_W Z_\mu - \sin 2\theta_W A_\mu \end{bmatrix}$$

$$\frac{g_2 \vec{\sigma}\cdot\vec{A}_\mu + g_1 B_\mu}{\sqrt{g_1^2+g_2^2}} = \begin{bmatrix} \cos 2\theta_W Z_\mu + \sin 2\theta_W A_\mu & \sqrt{2}\cos\theta_W W_\mu^+ \\ \sqrt{2}\cos\theta_W W_\mu^- & -Z_\mu \end{bmatrix}$$

The $W^\pm, Z$ masses are found from just the $H_u \to \begin{bmatrix} 0 \\ v_u/\sqrt{2} \end{bmatrix}$; $H_d \to \begin{bmatrix} v_d/\sqrt{2} \\ 0 \end{bmatrix}$ substitution

$$\frac{D_\mu H_u}{\sqrt{g_1^2+g_2^2}} \to -\frac{i}{2}\begin{bmatrix} c2\theta_W Z_\mu + s2\theta_W A_\mu & \sqrt{2}\, c\theta_W W_\mu^+ \\ \sqrt{2}\, c\theta_W W_\mu^- & -Z_\mu \end{bmatrix}\begin{bmatrix} 0 \\ \frac{v_u}{\sqrt{2}} \end{bmatrix}$$

$$= -\frac{i}{2}\begin{bmatrix} v_u \cos\theta_W W_\mu^+ \\ -\frac{v_u}{\sqrt{2}} Z_\mu \end{bmatrix}$$

$\Rightarrow$

$$\frac{(D_\mu H_u)^\dagger (D^\mu H_u)}{(g_1^2+g_2^2)} \to \frac{1}{4}\begin{bmatrix} v_u \cos\theta_W W_\mu^- & -\frac{v_u}{\sqrt{2}} Z_\mu \end{bmatrix}\begin{bmatrix} v_u\, c\theta_W W^{+\mu} \\ -\frac{v_u}{\sqrt{2}} Z^\mu \end{bmatrix}$$

$$= \frac{1}{4} v_u^2 \left[\cos^2\theta_W W_\mu^+ W^{-\mu} + \frac{1}{2} Z_\mu Z^\mu\right]$$

$$\frac{D_\mu H_d}{\sqrt{g_1^2+g_2^2}} \to -\frac{i}{2}\begin{bmatrix} Z_\mu & \sqrt{2}\cos\theta_W W_\mu^+ \\ \sqrt{2}\cos\theta_W W_\mu^- & -\cos 2\theta_W Z_\mu - \sin 2\theta_W A_\mu \end{bmatrix}\begin{bmatrix} v_d/\sqrt{2} \\ 0 \end{bmatrix}$$

$$= -\frac{i}{2}\begin{bmatrix} \frac{v_d}{\sqrt{2}} Z_\mu \\ v_d \cos\theta_W W_\mu^- \end{bmatrix}$$

$\Rightarrow$

$$\frac{(D_\mu H_d)^\dagger (D^\mu H_d)}{(g_1^2+g_2^2)} \longrightarrow \frac{1}{4}\begin{bmatrix} \frac{v_d}{\sqrt{2}} Z_\mu & v_d \cos\theta_W W_\mu^+ \end{bmatrix}\begin{bmatrix} \frac{v_d}{\sqrt{2}} Z^\mu \\ v_d \cos\theta_W W^{-\mu} \end{bmatrix}$$

$$= \frac{1}{4} v_d^2 \left[\cos^2\theta_W W_\mu^+ W^{-\mu} + \frac{1}{2} Z_\mu Z^\mu\right]$$

So the Higgs kinetic terms contain the W & Z mass terms

$$(D_\mu H_u)^\dagger (D^\mu H_d) + (D_\mu H_d)^\dagger (D^\mu H_d)$$

$$\longrightarrow \frac{1}{4}(v_u^2 + v_d^2)\left[\frac{1}{2} Z_\mu Z^\mu + \cos^2\theta_W W_\mu^+ W^{-\mu}\right] \times (g_1^2+g_2^2)$$

$\Rightarrow$

$$M_W^2 = \frac{1}{4}(v_u^2 + v_d^2)\cos^2\theta_W (g_1^2+g_2^2)$$

$$M_Z^2 = \frac{1}{4}(v_u^2 + v_d^2)(g_1^2+g_2^2)$$

$$M_W^2 = \frac{1}{4} g_2^2 v^2 \quad ; \quad \boxed{M_W = \frac{g_2 v}{2}}$$

$$M_Z^2 = \frac{(g_1^2+g_2^2)v^2}{4} \quad ; \quad \boxed{\begin{aligned} M_Z &= \frac{\sqrt{g_1^2+g_2^2}\, v}{2} \\ &= \frac{M_W}{\cos\theta_W} \end{aligned}}$$

Consider the linear combinations with

$$\tilde{H}_d \equiv -i\sigma^2 H_d^* = \begin{bmatrix} 0 & -1 \\ +1 & 0 \end{bmatrix} \begin{bmatrix} H_d^{0\dagger} + \frac{v_d}{\sqrt{2}} \\ H_d^{+} \end{bmatrix}$$

$$= \begin{bmatrix} -H_d^{+} \\ +H_d^{0\dagger} + \frac{v_d}{\sqrt{2}} \end{bmatrix}$$

So

$$\phi \equiv H_u \sin\beta + \tilde{H}_d \cos\beta$$

$$= \begin{bmatrix} H_u^{+} \sin\beta - H_d^{+} \cos\beta \\ \left(H_u^0 + \frac{v_u}{\sqrt{2}}\right)\sin\beta + \left(H_d^{0\dagger} + \frac{v_d}{\sqrt{2}}\right)\cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} (H_u^{+} \sin\beta - H_d^{+} \cos\beta) \\ (H_u^0 \sin\beta + H_d^{0\dagger} \cos\beta) + \frac{\sqrt{v_u^2 + v_d^2}}{\sqrt{2}} \end{bmatrix} \equiv \begin{bmatrix} \phi^{+} \\ \phi^0 + \frac{v}{\sqrt{2}} \end{bmatrix}$$

$\langle\phi\rangle = \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix}$ is just the SM Higgs doublet.

On the other hand the orthogonal combo

$$\phi' = H_u \cos\beta - \tilde{H}_d \sin\beta$$

$$= \begin{bmatrix} (H_u^+ \cos\beta + H_d^{+} \sin\beta) \\ (H_u^0 \cos\beta - H_d^{0+} \sin\beta) \end{bmatrix} \equiv \begin{bmatrix} \phi'^+ \\ \phi'^0 \end{bmatrix}$$

and $\langle\phi'\rangle = 0$. ϕ' is just another scalar field that has nothing to do with symmetry breaking.