Return to the MSSM in order to consider the minimum of the full potential — There are many parameters:

Raw #

5 Gauge: $g_1, g_2, g_3, \Theta_{QCD}, \xi$

2+54 Superpotential: $\mu, \bar{\mu}$; y_u, y_d, y_l; $y_u^\dagger, y_d^\dagger, y_l^\dagger$ (3 lepton masses + 6 quark masses, 3 quark mixing ∡'s + 1 CP phase)

6 Soft breaking: $M_{1,2,3}, \bar{M}_{1,2,3}$ (one imaginary component can be removed by phase transf.)

$m_{KHd}, \bar{m}_{KHd}$, $m_{KHu}, \bar{m}_{KHu}$ } effectively equiv. to $m^2_{Hd,u}$

2 $B, \bar{B}$ (real using Higgs rephasing)

2 m^2_{Hu}, m^2_{Hd}

45
$m^2_l, m_{Kl}, \bar{m}_{Kl}$
$m^2_{e^c}, m_{Ke^c}, \bar{m}_{Ke^c}$
$m^2_q, m_{Kq}, \bar{m}_{Kq}$
$m^2_{u^c}, m_{Ku^c}, m_{Ku^c}$
$m^2_{d^c}, m_{Kd^c}, \bar{m}_{Kd^c}$
} (effectively take m_K's $= 0$; m^2 are 3×3, Hermitian $\Rightarrow 6 + 3$ phases $= 9\times5 = 45$)

54
A_u
A_d
A_l
} 3×3 complex $= 18$ each

170

Now we discussed SUSY breaking & will some more shortly - but we F-I term yields light scalars as does O'R. So take $\boxed{\xi_3 = 0}$

We can rephase Higgs field $\Rightarrow$ B = real
" gluino " $\rightarrow$ M_3 = real

Also the Kähler and gauge actions are $U(3)^5$ globally invariant one for each $L_F, Q_F, E^c_F, U^c_F, D^c_F$ but the superpotential ~~and~~ Yukawa terms and the SUSY breaking A-terms are not invariant. Hence these transformations can be used to set some of these parameters to zero. U(3) has 3 angles and six phases parameters = 3+6 = 9 parameters each $\Rightarrow$ total of 5×9 = 45 parameters of the 170 should be removable. 2 of the phases however correspond to B and L which are invariances of the Lagrangian hence cannot be removed.

So we actually have

$$170 - 3 - (45 - 2)$$

$$= 170 - 46 = \boxed{124 \text{ parameters!}}$$

(Note: There are additional soft breaking terms allowed for the MSSM which are in most models surpressed or zero. They are the so called c-terms of the form AAA^{+} and correspond to

$$C_{u_{FG}} \tilde{q}_F \cdot H_d^{+} \tilde{u}^c_G + C_{d_{FG}} \tilde{q}_F \cdot H_u^{+} \tilde{d}^c_G + C_{\ell_{FG}} \tilde{l}_F H_u^{+} \tilde{e}^c_G + h.c.$$

These would provide another (3x3, complex) x 3 set of parameters $18 \times 3 = 54$

For a total of $124 + 54 = 178$ parameters.

we will take the C's $= 0$)

Recall the SM has 19 free parameters

$g_1, g_2, g_3, \theta_{QCD}$: 4 gauge parameters
μ, λ : 2 Higgs potential parameters
m_q, m_ℓ : 6 quark mass, 3 lepton masses
θ_{ij} : 3 mixing angles
δ_E : 1 cp phase

19 parameters!

The MSSM has <u>105</u> more parameters!!

Besides these enormity — consider the minimization of the scalar potential — How many directions in moduli space are there — that is how many real scalar fields are there in the MSSM

There are 2 real scalars for each chiral superfield

Superfields			#
$\begin{pmatrix} \nu_F \\ e_F \end{pmatrix}$	3×2	$\tilde{\nu}_F, \tilde{\nu}_F^+$	6
	3×2	$\tilde{e}_F, \tilde{e}_F^+$	6
$\begin{pmatrix} u_F \\ d_F \end{pmatrix}$	$3\times 2\times 3$	$\tilde{u}^a_F, \tilde{u}^{a+}_F$	18
	$3\times 2\times 3$	$\tilde{d}^a_F, \tilde{d}^{a+}_F$	18
E^c_F	3×2	$\tilde{e}^c_F, \tilde{e}^{c+}_F$	6
U^c_F	$3\times 2\times 3$	$\tilde{u}^{ca}_F, \tilde{u}^{ca+}_F$	18
D^c_F	$3\times 2\times 3$	$\tilde{d}^{ca}_F, \tilde{d}^{ca+}_F$	18
$\begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$	2	H_u^+, H_u^-	2
	2	H_u^0, H_u^{0+}	2
$\begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$	2	H_d^0, H_d^{0+}	2
	2	H_d^-, H_d^+	2
			98 real fields!

So $V = V(98 \text{ field directions!})$

Now the vacuum we desire should preserve electric charge, color or lepton number

So we will assume that no deeper minimum occurs for charged or colored or $\not{L}$ (lepton #) fields getting a vev. Only the Higgs fields can get a vev and yield the minimum of the potential.

Now we can use the SU(2) gauge invariance to rotate the H_u vev to the lower component H_u^0. So only $\langle H_u^0 \rangle \neq 0$

This will imply $\langle H_d^- \rangle = 0$ by minimizing the potential.

So let
$$H_u = \begin{bmatrix} h_u^+ \\ \frac{v_u}{\sqrt{2}} \end{bmatrix} \; ; \; H_d = \begin{bmatrix} v_d/\sqrt{2} \\ h_d^- \end{bmatrix}$$

$$H_u^\dagger H_u = h_u^+ h_u^- + \frac{|v_u|^2}{2}$$

$$H_d^\dagger H_d = h_d^+ h_d^- + \frac{|v_d|^2}{2}$$

$$H_u \cdot H_d = h_u^+ h_d^- - \frac{1}{2} v_u v_d$$

So
$$V_H = M_{H_u}^2 \left[h_u^+ h_u^- + \tfrac{1}{2}|v_u|^2 \right] + M_{H_d}^2 \left[h_d^+ h_d^- + \tfrac{1}{2}|v_d|^2 \right]$$
$$- b \left[h_u^+ h_d^- - \tfrac{1}{2} v_u v_d + h_u^- h_d^+ - \tfrac{1}{2} v_u^* v_d^* \right]$$
$$+ \frac{g_2^2 + g_1^2}{4 \cdot 2} \left[\left(|h_u^+|^2 + \tfrac{1}{2}|v_u|^2\right)^2 + \left(|h_d^-|^2 + \tfrac{1}{2}|v_d|^2\right)^2 \right]$$
$$+ \frac{g_2^2 - g_1^2}{2 \cdot 2} \left[|h_u^+|^2 + \tfrac{1}{2}|v_u|^2 \right] \left[|h_d^-|^2 + \tfrac{1}{2}|v_d|^2 \right]$$

$$-\frac{g_2^2}{2}\left| h_u^+ h_d^- - \tfrac{1}{2} v_u v_d \right|^2$$

Now we can use SU(2) transformation to set the $h_u^+ = 0$ but first

$$0 = \left.\frac{\partial V_H}{\partial h_u^+}\right|_{h_u^+=0=h_u^-} = M_{Hu}^2 h_u^- - b h_d^- + \frac{g_2^2+g_1^2}{2\cdot 2} h_u^- \left[|h_u^+|^2 + \tfrac{1}{2}|v_u|^2 \right]$$

$$+ \frac{g_2^2-g_1^2}{2\cdot 2} h_u^- \left[|h_d^-|^2 + \tfrac{1}{2}|v_d|^2 \right]$$

$$-\frac{g_2^2}{2}\cdot 2 h_d^- \left[h_u^- h_d^+ - \tfrac{1}{2} v_u^* v_d^* \right]$$

$$= -b h_d^- - g_2^2 h_d^- \left[-\tfrac{1}{2} v_u^* v_d^* \right]$$

$$\Rightarrow \boxed{h_d^- = 0 = h_d^+}$$

So we consider the minimum of the Higgs potential for neutral components v_u, v_d only

$$\boxed{\begin{aligned} V_H = {} & \tfrac{1}{2} M_{Hu}^2 |v_u|^2 + \tfrac{1}{2} M_{Hd}^2 |v_d|^2 \\ & + \tfrac{1}{2} b \left(v_u v_d + v_u^* v_d^* \right) \\ & + \frac{g_2^2+g_1^2}{4\cdot 2} \left[\tfrac{1}{4}|v_u|^4 + \tfrac{1}{4}|v_d|^4 \right] \\ & + \frac{g_2^2-g_1^2}{2\cdot 2} \left(\tfrac{1}{4}|v_u|^2 |v_d|^2 \right) - \tfrac{1}{8} g_2^2 |v_u|^2 |v_d|^2 \end{aligned}}$$

$$\boxed{\begin{aligned} V_H = \tfrac{1}{2} M_{Hu}^2 |v_u|^2 + \tfrac{1}{2} M_{Hd}^2 |v_d|^2 \\ + \tfrac{1}{2} b \left(v_u v_d + v_u^* v_d^* \right) \\ + \frac{g_1^2 + g_2^2}{32} \left[|v_u|^2 - |v_d|^2 \right]^2 \end{aligned}}$$

Minimum given by

1) $0 = \frac{\partial V_H}{\partial v_u^*} = \frac{1}{2} M_{Hu}^2 v_u + \frac{1}{2} b v_d^* + \frac{g_1^2 + g_2^2}{8 \cdot 2} v_u \left[|v_u|^2 - |v_d|^2 \right]$

2) $0 = \frac{\partial V_H}{\partial v_d^*} = \frac{1}{2} M_{Hd}^2 v_d + \frac{1}{2} b v_u^* - \frac{g_1^2 + g_2^2}{8 \cdot 2} v_d \left[|v_u|^2 - |v_d|^2 \right]$

and the conjugate equations.

Note $v_u = 0 = v_d$ is a solution — no electroweak symmetry breaking. Now for the origin of field space to be a maximum rather than a minimum we require that we have a negative eigenvalue at the origin: $\det \left| \frac{\partial^2 V_H}{\partial v_i \partial v_j} \right|_{v_{u,d}=0} < 0$

So

$$\left.\frac{\partial^2 V_H}{\partial v_i \partial v_j}\right|_{v_u=0=v_d} = \begin{array}{c|cc} & v_d^* & v_u \\ \hline v_d & \frac{1}{2}M_{Hd}^2 & \frac{1}{2}b \\ v_u^* & \frac{1}{2}b & \frac{1}{2}M_{Hu}^2 \end{array}$$

$$\det V'' = \frac{1}{4}\left(M_{Hd}^2 M_{Hu}^2 - b^2\right) < 0$$

$$\Rightarrow \boxed{b^2 > M_{Hd}^2 M_{Hu}^2}$$

Now also the potential must be bounded from below for a stable ground state. In general the quartic field terms in the potential provide the stability for large values of the field i.e. the potential grows quarticly

However here we have a D-flat direction in field space where $|v_u| = |v_d|$, the quartic terms vanish. In this direction $|v_u| = |v_d|$ the potential must be positive

$$\left. V_H \right|_{|v_u|=|v_d|} = \frac{1}{2}\left(M_{Hu}^2 + M_{Hd}^2\right)|v_u|^2 + \frac{1}{2}b|v_u|^2\left(e^{i\theta_u}e^{i\theta_d} + e^{-i\theta_u}e^{-i\theta_d}\right)$$

So for the $V_H\Big|_{|v_u|=|v_d|} > 0$ i.e. the potential to be bounded below since for $b>0$ that term can grow negative arbitrarily in the $|v_u|=|v_d|$ direction. we find

$$(M_{H_u}^2 + M_{H_d}^2) + 2b\cos(\theta_u + \theta_d) > 0$$

$$\Rightarrow \quad (M_{H_u}^2 + M_{H_d}^2) - 2|b| > 0$$

$$\Rightarrow \quad \boxed{2|b| < (M_{H_u}^2 + M_{H_d}^2)}$$

When these conditions are satisfied the electroweak symmetry $SU(2)\times U(1)$ breaksdown to a conserved $U(1)$ of electromagnetism.

Remarks: 1) The minima conditions imply v_u & v_d have opposite phases

$$v_u = |v_u| e^{i\theta_u} \; ; \; v_d = |v_d| e^{+i\theta_d}$$

Only solve equations if $\theta_u = -\theta_d = \theta$

But H_u & H_d have opposite $U(1)$ hypercharges $\pm\frac{1}{2}$. So we can use the hypercharge gauge transformation to gauge away this phase. Hence v_u, v_d are real and positive. Since $v_{u,d}$ are real — the vacuum is CP invariant — CP is not spontaneously broken in MSSM.

2) Recall $M_{Hu}^2 = m_{Hu}^2 + 16|\mu|^2$

$$M_{Hd}^2 = m_{Hd}^2 + 16|\mu|^2$$

The minimum stability relates soft-SUSY breaking parameters m_{Hu}^2, m_{Hd}^2 and the good SUSY superpotential mass μ. These parameters should have nothing to do with eachother.

$$b^2 > (m_{Hd}^2 + 16|\mu|^2)(m_{Hu}^2 + 16|\mu|^2)$$

$$2|b| < (m_{Hd}^2 + m_{Hu}^2 + 32|\mu|^2)$$

Now in general m_{Hd}^2 & m_{Hu}^2 are taken not equal with opposite signs. Note: if $m_{Hd}^2 = m_{Hu}^2 = m^2 \Rightarrow$

a) $$b^2 > (m^2 + 16|\mu|^2)^2$$
$$2|b| < 2(m^2 + 16|\mu|^2)$$

b) $$\Rightarrow b^2 < (m^2 + 16|\mu|^2)^2$$

Hence a) & b) imply no stable minimum — no solution.

So $m_{Hd}^2 \neq m_{Hu}^2$.

Suppose we take the typical relation

$$m_{Hu}^2 = -\frac{1}{2} m_d^2 = -\frac{1}{2} m^2$$

Then we have

$$b^2 > (m^2 + 16|\mu|^2)(16|\mu|^2 - \tfrac{1}{2}m^2)$$

or a) $\boxed{\frac{b^2}{m^2} > \left(16\frac{|\mu|^2}{m^2} + 1\right)\left(16\frac{|\mu|^2}{m^2} - \frac{1}{2}\right)}$

&

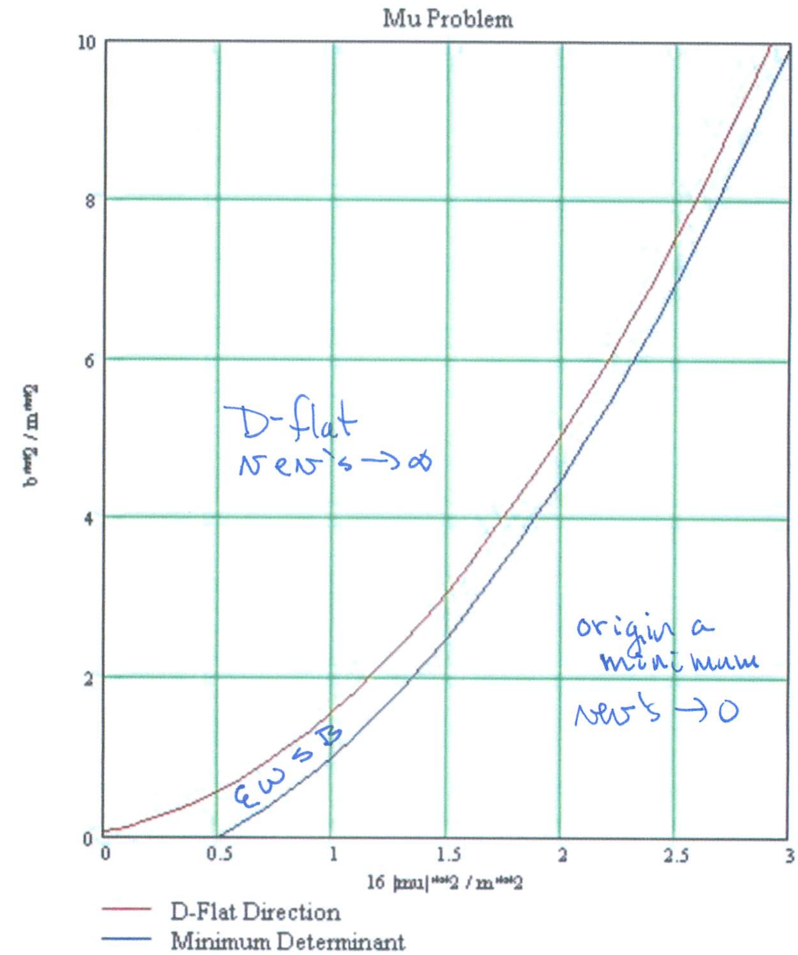
$$2|b| < (m^2 - \tfrac{1}{2}m^2 + 32|\mu|^2)$$

or $\cancel{2}|b| < \cancel{2}(16|\mu|^2 + \tfrac{1}{4}m^2)$

$\Rightarrow$ b.) $\boxed{\frac{b^2}{m^2} < \left(16\frac{|\mu|^2}{m^2} + \frac{1}{4}\right)^2}$

Notice in the graph — only the thin region between the 2 curves implies electroweak symmetry breaking. Above the top curve the D-flat directions the Higgs vev's run away to ∞ — the potential is not bounded below. Below the bottom curve the origin is the stable minimum and the Higgs vev's run to 0.

This required EWSB sliver of allowed μ-values is known as the μ-problem.



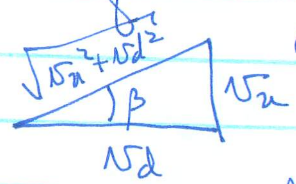
Supposing that these conditions between μ, b, $m^2_{H_u}$, $m^2_{H_d}$ are met — we have EWSB.

Then $\langle 0|H_u^0|0\rangle = v_u/\sqrt{2}$

$\langle 0|H_d^0|0\rangle = v_d/\sqrt{2}$

and we define the ratio of vev's

$$\boxed{\tan\beta \equiv \frac{v_u}{v_d}}$$



$v^2 \equiv v_u^2 + v_d^2$

where $0 < \beta < \pi/2$,

$\Rightarrow \sin\beta = \dfrac{v_u}{\sqrt{v_u^2+v_d^2}}$

$\cos\beta = \dfrac{v_d}{\sqrt{v_u^2+v_d^2}}$

The minimization conditions of the vanishing of the V_H derivatives becomes (p. -63-)

1) $(m^2_{H_u} + |\mu|^2)\tan\beta + b + \dfrac{M_Z^2}{2}\tan\beta\cos 2\beta = 0$

2) $(m^2_{H_d} + |\mu|^2)\cot\beta + b - \dfrac{M_Z^2}{2}\cot\beta\cos 2\beta = 0$

where $M_Z^2 \equiv \dfrac{(g_1^2+g_2^2)v^2}{4}$; $v^2 = v_u^2 + v_d^2$

$(v \simeq 246\ \text{GeV})$

Subtracting the equations $\Rightarrow$

tedious algebra + trig.

$$\boxed{\begin{aligned} &1')\quad \tfrac{1}{6}|\mu|^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} + \tfrac{1}{2}M_Z^2 \\ &\text{and adding the equations} \Rightarrow \\ &2')\quad -2b = (M_{H_u}^2 + M_{H_d}^2)\sin 2\beta \\ &\qquad\quad = (m_{H_u}^2 + m_{H_d}^2 + 2\cdot\tfrac{1}{6}|\mu|^2)\sin 2\beta \end{aligned}}$$

These are the minimization criteria for EWSB.

Equation 2') expresses parameter b in terms of β - so trade b for β.

Equation 1') gives the tuning equation for $|\mu|$ once $m_{H_d}^2, m_{H_u}^2, \beta$ are fixed to give the observed value of M_Z.

This is again another way to express the μ-problem since soft-SUSY breaking terms $m_{H_u}^2$, $m_{H_d}^2$ have nothing to do with the good SUSY μ-parameter.
