MSSM

Recall the gauge invariant but soft SUSY broken MSSM action:

$$\Gamma = \Gamma_{MSSM} + \Gamma_{MSSM}^{soft}$$

$$= \Gamma_{SYM} + \Gamma_{SK} + \Gamma_{SW}$$

with

$$\Gamma_{SYM} = \int dS \frac{Z_3(\theta)}{g_3^2} Tr[W_3 W_3] + h.c.$$

$$+ \int dS \frac{Z_2(\theta)}{g_2^2} Tr[W_2 W_2] + h.c.$$

$$+ \int dS \frac{Z_1(\theta)}{g_1^2} [W_1 W_1] + h.c.$$

$$\Gamma_{SW} = \int dS [\mu(\theta) H_u H_d$$

$$+ H_u Q y_u(\theta) U^c + H_d Q y_d(\theta) D^c$$

$$+ H_d L y_l(\theta) E^c] + h.c.$$

$$\Gamma_{SK} = \frac{1}{16}\int dV\, K_S \quad \text{where}$$

$$K_S = Z_Q(\theta,\bar\theta)\,\bar{Q}\, e^{[g_3 G + g_2 A + \frac{1}{6} g_1 B]} Q$$
$$+ Z_L(\theta,\bar\theta)\,\bar{L}\, e^{[g_2 A - \frac{1}{2} g_1 B]} L + Z_{E^c}(\theta,\bar\theta)\,\bar{E}^c e^{g_1 B} E^c$$
$$+ Z_{U^c}(\theta,\bar\theta)\, U^c e^{[-g_3 G - \frac{2}{3} g_1 B]} \bar{U}^c$$
$$+ Z_{D^c}(\theta,\bar\theta)\, D^c e^{[-g_3 G + \frac{1}{3} g_1 B]} \bar{D}^c$$
$$+ Z_{H_u}(\theta,\bar\theta)\,\bar{H}_u e^{[g_2 A + \frac{1}{2} g_1 B]} H_u$$
$$+ Z_{H_d}(\theta,\bar\theta)\,\bar{H}_d e^{[g_2 A - \frac{1}{2} g_1 B]} H_d .$$

A model with 123 parameters (plus neutrino masses & mixing angles and CP phase) that is 105 more than the SM.

Let's recall the different fields and superspace manipulations & in the process investigate the component details of the MSSM:

Let's begin by analyzing the Yang-Mills or gauge field sector: $\mathcal{L}_{sym}$

Group	Real Vector Superfield	SM field	Susy Partner
$SU(3)$	G^m	G^m_μ	$\tilde{G}^m_\alpha, \bar{\tilde{G}}^m_{\dot\alpha}$ gluino
$SU(2)$	A^i	A^i_μ	$\tilde{A}^i_\alpha, \bar{\tilde{A}}^i_{\dot\alpha}$ -inos
(Alternate Notation	W^i $(W^\pm, W^0)$	W^i_μ	$\tilde{W}^i_\alpha, \bar{\tilde{W}}^i_{\dot\alpha}$)
$U(1)$	B	B_μ	$\tilde{B}_\alpha, \bar{\tilde{B}}_{\dot\alpha}$

After Electroweak symmetry breaking W^a, B become

	$W^\pm$	$W^\pm_\mu$	$\tilde{W}^\pm_\alpha, \bar{\tilde{W}}^\pm_{\dot\alpha}$ wino's
	Z	Z_μ	$\tilde{Z}_\alpha, \bar{\tilde{Z}}_{\dot\alpha}$ zino
	A	A_μ	$\tilde{A}_\alpha, \bar{\tilde{A}}_{\dot\alpha}$ photino

The gauge fields are in the adjoint representations of their associated global symmetry groups. The gauge invariant kinetic energy terms are made using the chiral field strength spinors for each field for SU(3): the chiral field strength

$$W_\alpha^{SU(3)} = \bar{D}\bar{D}\left[e^{-g_3 G^m T^m_{SU(3)}} D_\alpha \, e^{+g_3 G^m T^m_{SU(3)}}\right]$$

with $(T^l_{SU(3)})_{mn} \equiv i f_{mln}$, the adjoint representation of the SU(3) generators with f_{mln} the SU(3) structure constants; the anti-chiral field strength

$$\bar{W}_{\dot\alpha}^{SU(3)} = DD\left[e^{+g_3 \vec{G}\cdot\vec{T}^{SU(3)}} \bar{D}_{\dot\alpha} \, e^{-g_3 \vec{G}\cdot\vec{T}^{SU(3)}}\right].$$

The electroweak fields have for SU(2)

$$W_\alpha^{SU(2)} = \bar{D}\bar{D}\left[e^{-g_2 \vec{A}\cdot\vec{T}^{SU(2)}} D_\alpha \, e^{+g_2 \vec{A}\cdot\vec{T}^{SU(2)}}\right]$$

$$\bar{W}_{\dot\alpha}^{SU(2)} = DD\left[e^{+g_2 \vec{A}\cdot\vec{T}^{SU(2)}} \bar{D}_{\dot\alpha} \, e^{-g_2 \vec{A}\cdot\vec{T}^{SU(2)}}\right]$$

with $(T^i_{SU(2)})_{jk} \equiv i\epsilon_{jik}$ for SU(2)

and for U(1) hypercharge:

$$W_\alpha^{U(1)} = \bar{D}\bar{D}\left[e^{-g_1 B} D_\alpha \, e^{+g_1 B}\right] = g_1 \bar{D}\bar{D} D_\alpha B$$

$$\bar{W}_{\dot\alpha}^{U(1)} = DD\left[e^{+g_1 B} \bar{D}_{\dot\alpha} \, e^{-g_1 B}\right] = -g_1 DD\bar{D}_{\dot\alpha} B.$$

Since the kinetic energies are gauge invariant we might as well evaluate them in the Wess-Zumino gauge:
In general the real or vector superfield had the component expansion: Grekevic group

$$V^i(x,\theta,\bar\theta) = C^i(x) + \theta\chi^i(x) + \bar\theta\bar\chi^i + \tfrac{1}{2}\theta^2 M^i$$

$$+ \tfrac{1}{2}\bar\theta^2 M^{i\dagger} + \theta\sigma^\mu\bar\theta A^i_\mu + \tfrac{1}{2}\theta^2\bar\theta_{\dot\alpha}\bar\lambda^{i\dot\alpha}$$

$$+ \tfrac{1}{2}\bar\theta^2\theta^\alpha\lambda^i_\alpha + \tfrac{1}{4}\theta^2\bar\theta^2 D^i$$

Gauge Transformations

$$e^{gV'^iT^i} = e^{+ig\bar\Lambda\cdot T}\, e^{gV\cdot T}\, e^{-ig\Lambda\cdot T}$$

$\Rightarrow$

$$V'^i = V^i + \underbrace{i(\bar\Lambda^i - \Lambda^i)}_{\text{inhomogeneous piece}} - \frac{ig}{2}(\Lambda^j + \bar\Lambda^j) f_{ijk} V^k + \cdots$$

W-Z gauge: use $i(\bar\Lambda^i - \Lambda^i)$ to eliminate $C^i, \chi^i, \bar\chi^i, M^i, M^{i\dagger}$

$\Rightarrow$ W-Z gauge

$$\boxed{V^i = \theta\sigma^\mu\bar\theta A^i_\mu + \tfrac{1}{2}\theta^2\bar\theta_{\dot\alpha}\bar\lambda^{i\dot\alpha} + \tfrac{1}{2}\bar\theta^2\theta^\alpha\lambda^i_\alpha + \tfrac{1}{4}\theta^2\bar\theta^2 D^i}$$

In the W-Z gauge we are still left with the gauge transformations $\Lambda^i = e^{-i\theta\sigma\bar\theta}\,\omega^i(x)$

&

$$\bar\Lambda^i = e^{+i\theta\sigma\bar\theta}\,\underset{\substack{\| \\ \omega^i(x)}}{\omega^{i\dagger}(x)} \qquad \text{i.e. } \omega^i = \omega^{i\dagger}$$

So $\bar\Lambda^i - \Lambda^i = i\,\theta\sigma^\mu\bar\theta\,\partial_\mu(\omega^{i\dagger} + \omega^i)$

So that for infinitesimal transformations

$$V'^i_{WZ} = V^i_{WZ} - \frac{i}{2} g\,(\Lambda^j + \bar\Lambda^j) f_{ijk} V^k_{WZ}$$
$$- i(-i)\,\theta\sigma^\mu\bar\theta\,\partial_\mu(\omega^{i\dagger} + \omega^i)$$

$\Rightarrow$

$$\boxed{\begin{aligned} A'^i_\mu &= A^i_\mu - i g\,\omega^j f_{ijk} A^k_\mu - 2\partial_\mu \omega^i \\ \lambda'^i_\alpha &= \lambda^i_\alpha - i g\,\omega^j f_{ijk} \lambda^k_\alpha \\ \bar\lambda'^{i\dot\alpha} &= \bar\lambda^{i\dot\alpha} - i g\,\omega^j f_{ijk} \bar\lambda^{k\dot\alpha} \\ D'^i &= D^i - i g\,\omega^j f_{ijk} D^k \end{aligned}}$$

we see that λ^i_α, $\bar\lambda^i_{\dot\alpha}$, D^i are in the adjoint representation of the gauge group and A^i_μ is the associated YM field!

Now in the W-Z gauge we have

$$e^{\pm g V\cdot T} = 1 \pm g\,\theta\sigma^\mu\bar\theta A_\mu\cdot T \pm \frac{g}{2}\theta^2\bar\theta_{\dot\alpha}\bar\lambda^{\dot\alpha}\cdot T$$

$$\pm \frac{g}{2}\bar\theta^2\theta^\alpha\lambda_\alpha\cdot T \pm \frac{g}{4}\theta^2\bar\theta^2 D\cdot T$$

$$+ \frac{g^2}{4}\theta^2\bar\theta^2(A_\mu\cdot T)(A^\mu\cdot T)$$

So we can determine the field strength spinors in the W-Z gauge:

$$W_\alpha = \bar D\bar D\left[e^{-gV\cdot T} D_\alpha e^{+gV\cdot T}\right]$$

$$= \bar D\bar D\left[e^{-gV\cdot T}\left(g D_\alpha V\cdot T + \frac{g^2}{4}D_\alpha\left(\theta^2\bar\theta^2(A_\mu\cdot T)(A^\mu\cdot T)\right)\right)\right]$$

$$= \bar D\bar D\left[\left(1 - gV\cdot T + \frac{g^2}{4}\theta^2\bar\theta^2(A_\mu\cdot T)^2\right)\times\right.$$

$$\left.\times\left(g D_\alpha V\cdot T + \frac{g^2}{2}\theta_\alpha\bar\theta^2(A_\mu\cdot T)^2\right)\right]$$

$$= \bar D\bar D\left[g D_\alpha V\cdot T + \frac{g^2}{2}\theta_\alpha\bar\theta^2(A_\mu\cdot T)^2\right.$$

$$\left. - gV\cdot T\, g\left[(\sigma^\mu\bar\theta)_\alpha A_\mu\cdot T + \theta_\alpha\bar\theta_{\dot\alpha}\bar\lambda^{\dot\alpha}\cdot T\right]\right]$$

$$= g\bar D\bar D D_\alpha V\cdot T + \frac{g^2}{2}\bar D\bar D\,\theta_\alpha\bar\theta^2(A_\mu\cdot T)^2$$

$$-g^2\bar D\bar D\left[\left(\theta\sigma^\nu\bar\theta A_\nu\cdot T + \frac{1}{2}\theta^2\bar\theta_{\dot\alpha}\bar\lambda^{\dot\alpha}\cdot T\right)\left((\sigma^\mu\bar\theta)_\alpha A_\mu\cdot T\right.\right.$$

$$\left.\left. + \theta_\alpha\bar\theta_{\dot\alpha}\bar\lambda^{\dot\alpha}\cdot T\right)\right]$$

$$W_\alpha = g\,\bar{D}\bar{D}D_\alpha V\cdot T + \frac{g^2}{2}\bar{D}\bar{D}\,\theta_\alpha\bar{\theta}^2(A_\mu\cdot T)^2$$

$$-g^2\bar{D}\bar{D}\Big(\theta\sigma^\nu\bar{\theta}\,(\sigma^\mu\bar{\theta})_\alpha(A_\nu\cdot T)(A_\mu\cdot T)$$

$$-\cancel{\alpha t}\,\frac{1}{2}\theta^2\bar{\theta}_{\dot\alpha}(\sigma^\mu\bar{\theta})_\alpha(\bar{\lambda}^{\dot\alpha}\cdot T)(A_\mu\cdot T)$$

$$+\theta\sigma^\nu\bar{\theta}\,\theta_\alpha\bar{\theta}_{\dot\alpha}(A_\nu\cdot T)(\bar{\lambda}^{\dot\alpha}\cdot T)\Big)$$

Now as usual

$$\theta\sigma^\nu\bar{\theta}\,\sigma^\mu_{\alpha\dot\alpha}\bar{\theta}^{\dot\alpha} = \theta^\beta\sigma^\nu_{\beta\dot\beta}\sigma^\mu_{\alpha\dot\alpha}\bar{\theta}^{\dot\beta}\bar{\theta}^{\dot\alpha}$$

$$= \frac{1}{2}\bar{\theta}^2\theta^\beta\,\epsilon^{\dot\beta\dot\alpha}\,\sigma^\nu_{\beta\dot\beta}\sigma^\mu_{\alpha\dot\alpha}$$

$$= \frac{1}{2}\bar{\theta}^2\theta^\beta\,\sigma^\nu_{\beta\dot\beta}\,\bar{\sigma}^{\mu\dot\beta}{}_{\alpha}$$

$$= \frac{1}{2}\bar{\theta}^2\theta^\beta\left[\eta^{\mu\nu}\,\epsilon_{\alpha\beta} + i\sigma^{\mu\nu}_{\alpha\beta}\right]$$

$$\bar{\theta}_{\dot\alpha}(\sigma^\mu\bar{\theta})_\alpha = \epsilon_{\dot\alpha\dot\beta}\bar{\theta}^{\dot\beta}\sigma^\mu_{\alpha\dot\gamma}\bar{\theta}^{\dot\gamma} = \frac{1}{2}\bar{\theta}^2\,\sigma^\mu_{\alpha\dot\gamma}\,\epsilon_{\dot\alpha\dot\beta}\epsilon^{\dot\beta\dot\gamma}$$

$$= \frac{1}{2}\bar{\theta}^2\,\sigma^\mu_{\alpha\dot\alpha}$$

$$\theta\sigma^\nu\bar{\theta}\,\theta_\alpha\bar{\theta}_{\dot\alpha} = \theta_\alpha\theta^\beta\sigma^\nu_{\beta\dot\beta}\bar{\theta}^{\dot\beta}\bar{\theta}_{\dot\alpha}$$

$$= \epsilon_{\alpha\gamma}\theta^\gamma\theta^\beta\sigma^\nu_{\beta\dot\beta}\epsilon_{\dot\alpha\dot\gamma}\bar{\theta}^{\dot\beta}\bar{\theta}^{\dot\gamma}$$

$$= -\frac{1}{4}\theta^2\bar{\theta}^2\,\epsilon_{\alpha\gamma}\epsilon^{\gamma\beta}\sigma^\nu_{\beta\dot\beta}\,\epsilon_{\dot\alpha\dot\gamma}\epsilon^{\dot\beta\dot\gamma}$$

$$\theta\sigma^\nu\bar\theta\,\theta_\alpha\bar\theta_{\dot\alpha} = +\tfrac{1}{4}\theta^2\bar\theta^2\,\delta_\alpha{}^\beta\,\sigma^\nu_{\beta\dot\beta}\,\delta_{\dot\alpha}{}^{\dot\beta}$$

$$= +\tfrac{1}{4}\theta^2\bar\theta^2\,\sigma^\nu_{\alpha\dot\alpha}$$

So we have

$$W_\alpha = g\bar D\bar D D_\alpha V\cdot T$$

$$+\frac{g^2}{2}\bar D\bar D\Big[\cancel{\theta_\alpha\bar\theta^2(A_\mu\cdot T)^2 - \bar\theta^2\theta_\alpha(A_\mu\cdot T)^2}$$

$$+ i\bar\theta^2(\sigma^{\mu\nu}\theta)_\alpha(A_\nu\cdot T)(A_\mu\cdot T)\Big]$$

$$-g^2\bar D\bar D\Big[-\tfrac{1}{4}\theta^2\bar\theta^2\sigma^\mu_{\alpha\dot\alpha}(\bar\lambda^{\dot\alpha}\cdot T)(A_\mu\cdot T)$$

$$+\tfrac{1}{4}\theta^2\bar\theta^2\sigma^\nu_{\alpha\dot\alpha}(A_\nu\cdot T)(\bar\lambda^{\dot\alpha}\cdot T)\Big]$$

$$\boxed{W_\alpha = g\bar D\bar D\Big[D_\alpha V\cdot T + \bar\theta^2\frac{ig}{4}(\sigma^{\mu\nu}\theta)_\alpha[A_\nu\cdot T, A_\mu\cdot T] - \frac{g}{4}\theta^2\bar\theta^2\sigma^\mu_{\alpha\dot\alpha}[A_\mu\cdot T, \bar\lambda^{\dot\alpha}\cdot T]\Big]}$$

Now recall that $[T^i, T^j] = i f_{ijk} T^k$
and from p. -425- last semester

$$D_\alpha V^i = (\sigma^\mu\bar\theta)_\alpha A^i_\mu + \theta_\alpha\bar\theta\bar\lambda^i + \tfrac{1}{2}\bar\theta^2\lambda^i_\alpha + \tfrac{1}{2}\theta_\alpha\bar\theta^2 D^i$$

$$-\tfrac{i}{2}\bar\theta^2(\theta\sigma^\mu\bar\sigma^\nu)_\alpha\partial_\nu A^i_\mu + \tfrac{i}{4}\theta^2\bar\theta^2(\not\partial\bar\lambda^i)_\alpha$$

$$\boxed{\begin{aligned} W_\alpha = g\bar{D}\bar{D}\Big[&(\sigma^\mu\bar{\theta})_\alpha(A_\mu\cdot T) + \theta_\alpha\bar{\theta}_{\dot{\alpha}}(\bar{\lambda}^{\dot{\alpha}}\cdot T) \\ &+\tfrac{1}{2}\bar{\theta}^2(\lambda_\alpha\cdot T) + \tfrac{1}{2}\theta_\alpha\bar{\theta}^2(D\cdot T) \\ &-\tfrac{i}{2}\bar{\theta}^2(\theta\sigma^\mu\bar{\sigma}^\nu)_\alpha\partial_\nu(A_\mu\cdot T) \\ &-\tfrac{g}{4}\bar{\theta}^2(\sigma^{\mu\nu}\theta)_\alpha f_{ijk}A^i_\nu A^j_\mu T^k \\ &+\tfrac{i}{4}\theta^2\bar{\theta}^2\sigma^\mu_{\alpha\dot{\alpha}}\left(\partial_\mu(\bar{\lambda}^{\dot{\alpha}}\cdot T) + g f_{ijk}A^i_\mu\bar{\lambda}^{j\dot{\alpha}}T^k\right)\Big] \end{aligned}}$$

Now recall

$$\bar{D}\bar{D} = \frac{\partial^2}{\partial\bar{\theta}^2} - 2i\theta\partial\frac{\partial}{\partial\bar{\theta}} + \theta^2\partial^2$$

$\Rightarrow$

$$\begin{aligned} W_\alpha = g\Big[&-2(\lambda_\alpha\cdot T) - 2\theta_\alpha(D\cdot T) \\ &+2i(\theta\sigma^\mu\bar{\sigma}^\nu)_\alpha\partial_\nu(A_\mu\cdot T) \\ &+g(\sigma^{\mu\nu}\theta)_\alpha f_{ijk}A^i_\nu A^j_\mu T^k \\ &-i\theta^2\sigma^\mu_{\alpha\dot{\alpha}}\left[\partial_\mu\bar{\lambda}^{k\dot{\alpha}} - g f_{ijk}A^i_\mu\bar{\lambda}^{j\dot{\alpha}}\right]T^k \end{aligned}$$

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$$+\theta^2(\sigma^\mu\bar{\theta})_\alpha\partial^2(A_\mu\cdot T) + \tfrac{1}{2}\theta^2\bar{\theta}^2\partial^2(\lambda_\alpha\cdot T)$$

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$$+2i(\theta\not{\partial})^{\dot\alpha}\left(\sigma^{\mu}_{\alpha\dot\alpha}(A_\mu\cdot T)+\theta_\alpha(\bar\lambda_{\dot\alpha}\cdot T)\right.$$

$$-\bar\theta_{\dot\alpha}(\lambda_\alpha\cdot T)+\theta_\alpha\bar\theta_{\dot\alpha}(D\cdot T)$$

$$+i\bar\theta_{\dot\alpha}(\theta\sigma^\mu\bar\sigma^\nu)_\alpha\partial_\nu(A_\mu\cdot T)$$

$$\left.\left.+\frac{g}{2}\bar\theta_{\dot\alpha}(\sigma^{\mu\nu}\theta)_\alpha f_{ijk}A^i_\nu A^j_\mu T^k\right)\right]$$

Now we know W_α is a chiral superfield
So it has the form

$$W_\alpha=e^{-i\theta\not{\partial}\bar\theta}\left[W_{1\alpha}+\theta^\beta W_{2\beta\alpha}+\theta^2 W_{3\alpha}\right]$$

Also $W_\alpha=T^k W^k_\alpha$ so first factoring out
the generator T^k we have

$$W^k_\alpha = -2g\Big[\lambda^k_\alpha + \theta_\alpha D^k$$

$$-i(\theta\sigma^\mu\bar\sigma^\nu)_\alpha \partial_\nu A^k_\mu - \frac{g}{2}(\sigma^{\mu\nu}\theta)_\alpha f_{ijk} \times A^i_\nu A^j_\mu$$

$$+\frac{i}{2}\theta^2\sigma^\mu_{\alpha\dot\alpha}(\partial_\mu\bar\lambda^{k\dot\alpha} - g f_{ijk} A^i_\mu \bar\lambda^{j\dot\alpha})$$

$$-\frac{1}{4}\theta^2\bar\theta^2\partial^2\lambda^k_\alpha \quad -\frac{1}{2}\theta^2(\sigma^\mu\bar\theta)_\alpha\partial^2 A^k_\mu$$

$$-i(\theta\not\partial)^{\dot\alpha}\Big(\sigma^\mu_{\alpha\dot\alpha} A^k_\mu + \theta_\alpha\bar\lambda^k_{\dot\alpha} - \bar\theta_{\dot\alpha}\lambda^k_\alpha$$

$$-\bar\theta_{\dot\alpha}\theta_\alpha D^k$$

$$+i\bar\theta_{\dot\alpha}(\theta\sigma^\mu\bar\sigma^\nu)_\alpha\partial_\nu A^k_\mu$$

$$+\frac{g}{2}\bar\theta_{\dot\alpha}(\sigma^{\mu\nu}\theta)_\alpha f_{ijk} A^i_\nu A^j_\mu\Big)\Big]$$

$$= -2g\Big[e^{-i\theta\not\partial\bar\theta}\lambda^k_\alpha + e^{-i\theta\not\partial\bar\theta}\theta_\alpha D^k$$

$$+i(\theta\sigma^\mu\bar\sigma^\nu)_\alpha(\partial_\mu A^k_\nu - \partial_\nu A^k_\mu)$$

$$+i(\theta\sigma^\mu\bar\sigma^\nu)_\alpha\Big[\frac{g}{2} f_{kij} A^i_\nu A^j_\mu\Big]$$

$$+i\theta^2\sigma^\mu_{\alpha\dot\alpha}\Big[\partial_\mu\bar\lambda^{k\dot\alpha} - \frac{g}{2} f_{kij} A^i_\mu \bar\lambda^{j\dot\alpha}\Big]$$

$$-\frac{1}{2}\theta^2(\sigma^\mu\bar\theta)_\alpha\partial^2 A^k_\mu$$

$$-i(\theta\not\partial)^{\dot\alpha}\Big[i\bar\theta_{\dot\alpha}(\theta\sigma^\mu\bar\sigma^\nu)_\alpha\partial_\nu A^k_\mu$$
$$+\frac{g}{2}\bar\theta_{\dot\alpha}(\sigma^{\mu\nu}\theta)_\alpha f_{kij}A^i_\nu A^j_\mu\Big]\Big]$$

Now to analyze the last 2 terms!

$$\text{last term} = +i\frac{g}{2}\theta\not\partial\bar\theta\ (\sigma^{\mu\nu}\theta)_\alpha f_{kij}A^i_\nu A^j_\mu$$
$$= \frac{g}{2}\theta\not\partial\bar\theta\ (\theta\sigma^\mu\bar\sigma^\nu)_\alpha f_{kij}A^i_\nu A^j_\mu$$
$$= -i\theta\not\partial\bar\theta\Big[i\frac{g}{2}(\theta\sigma^\mu\bar\sigma^\nu)_\alpha f_{kij}A^i_\nu A^j_\mu\Big]$$

$$\text{penultimate term} = -(\theta\not\partial\bar\theta)(\theta\sigma^\mu\bar\sigma^\nu)_\alpha\partial_\nu A^k_\mu$$
$$= -i\theta\not\partial\bar\theta\Big[i(\theta\sigma^\mu\bar\sigma^\nu)_\alpha(\partial_\mu A^k_\nu-\partial_\nu A^k_\mu)\Big]$$
$$-\theta\not\partial\bar\theta(\theta\sigma^\mu\bar\sigma^\nu)_\alpha\partial_\mu A^k_\nu$$

but

$$-\theta\not\partial\bar\theta(\theta\sigma^\mu\bar\sigma^\nu)_\alpha\partial_\mu A^k_\nu$$
$$= -\theta^\beta(\not\partial\bar\theta)_\beta\,\theta^\gamma(\sigma^\mu\bar\sigma^\nu)_{\gamma\alpha}\partial_\mu A^k_\nu$$
$$= -\frac{1}{2}\theta^2\epsilon^{\beta\gamma}(\not\partial\bar\theta)_\beta(\sigma^\mu\bar\sigma^\nu)_{\gamma\alpha}\partial_\mu A^k_\nu$$
$$= +\frac{1}{2}\theta^2(\not\partial\bar\theta)^\beta(\sigma^\mu\bar\sigma^\nu)_{\beta\alpha}\partial_\mu A^k_\nu$$

$$= \frac{1}{2}\theta^2 \bar\theta^{\dot\beta} (\bar\sigma^\rho \sigma^\mu \bar\sigma^\nu)_{\dot\beta\alpha} \partial_\rho \partial_\mu A^k_\nu$$

(Recall $\bar\sigma^\rho \sigma^\mu \bar\sigma^\nu = \eta^{\rho\mu}\bar\sigma^\nu + \eta^{\mu\nu}\bar\sigma^\rho - \eta^{\rho\nu}\bar\sigma^\mu - i\epsilon^{\rho\mu\nu\sigma}\bar\sigma_\sigma$)

$\Rightarrow$

$$= \frac{1}{2}\theta^2 \bar\theta^{\dot\beta} \bar\sigma^\nu_{\dot\beta\alpha} \partial^2 A^k_\nu$$

$$= \frac{1}{2}\theta^2 (\sigma^\mu\bar\theta)_\alpha \partial^2 A^k_\mu$$

So the last two terms become

$$\boxed{\begin{aligned} &-i\theta\partial\!\!\!/\bar\theta \Big[i(\theta\sigma^\mu\bar\sigma^\nu)_\alpha \Big[\partial_\mu A^k_\nu - \partial_\nu A^k_\mu \\ &\qquad\qquad + \frac{g}{2} f_{kij} A^i_\nu A^j_\mu \Big]\Big] \\ &\qquad + \frac{1}{2}\theta^2 (\sigma^\mu\bar\theta)_\alpha \partial^2 A^k_\mu \end{aligned}}$$

So we finally obtain the expected form of the chiral spinor field strength!

$$W^i_\alpha = -2g\, e^{-i\theta\gamma\bar\theta}\Big[\lambda^i_\alpha + \theta_\alpha D^i$$

$$+ i(\theta\sigma^\mu\bar\sigma^\nu)_\alpha F^i_{\mu\nu}$$

$$+ i\theta^2 \sigma^\mu_{\alpha\dot\alpha}(D_\mu\bar\lambda^{\dot\alpha})^i\Big]$$

where the Y-M field strength tensor is

$$F^i_{\mu\nu} \equiv \partial_\mu A^i_\nu - \partial_\nu A^i_\mu - \frac{g}{2} f_{ijk} A^j_\mu A^k_\nu$$

and the gauge covariant derivative of the gaugino adjoint representation field is

$$(D_\mu\bar\lambda_{\dot\alpha})^i = \partial_\mu\bar\lambda^i_{\dot\alpha} - \frac{g}{2} f_{ijk} A^j_\mu \bar\lambda^k_{\dot\alpha}$$

To make the invariant action consider

$$\int dS\, Tr[WW] = \int dS\, W^{i\alpha} W^j{}_\alpha \underbrace{Tr[T^i T^j]}_{=\frac{1}{2}\delta^{ij}}$$

$$= \frac{1}{2}\int dS\, W^{i\alpha} W^i{}_\alpha$$

(Note in U(1) Abelian case there is No Trace so multiply action by 2)

$$= 2g^2 \int dS \Big[\theta^2 D^i D^i + 2i\theta^2 \lambda^i (\not{D}\lambda)^i$$

$$- \cancel{2}(\theta\sigma^\mu\bar{\sigma}^\nu)^\alpha (\theta\sigma^\rho\bar{\sigma}^\sigma)_\alpha F^i_{\mu\nu} F^i_{\rho\sigma}$$

$$+ 2i \underbrace{(\theta\sigma^\mu\bar{\sigma}^\nu\theta) F^i_{\mu\nu}}_{= -i\theta\cancel{\sigma^{\mu\nu}}\theta F^i_{\mu\nu} \to 0} D^i + \underbrace{\ldots}_{\text{lower powers of } \theta} \Big]$$

(see p. -426- -427-)

$$= 2g^2 \int dS\, [\theta^2] \Big[D^i D^i + 2i\lambda^i (\not{D}\bar{\lambda})^i$$

$$- \overset{2}{\cancel{4}} F^i_{\mu\nu} F^{i\mu\nu} + \underset{2}{\cancel{4}} i F^i_{\mu\nu} \tilde{F}^{i\mu\nu} \Big]$$

$$\int dS\, Tr[WW] = 2g^2(-4) \int d^4x \Big[D^i D^i + 2i\lambda^i (\not{D}\bar{\lambda})^i$$

$$- \underset{2}{\cancel{4}} F^i_{\mu\nu} F^{i\mu\nu} + \underset{2}{\cancel{4}} i F^i_{\mu\nu} \tilde{F}^{i\mu\nu} \Big]$$

$$\int dS \, Tr[WW] = -\frac{(16)^2}{2} g^2 \int d^4x \Big[\frac{1}{16} D^i D^i + \frac{i}{8} \lambda^i (\not{D} \bar{\lambda})^i - \frac{1}{8} F^i_{\mu\nu} F^{i\mu\nu} + \frac{i}{8} F^i_{\mu\nu} \tilde{F}^{i\mu\nu} \Big]$$

where recall F-dual is defined as

$$\boxed{\tilde{F}^i_{\mu\nu} \equiv \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{i\rho\sigma}}$$

Likewise the anti-chiral field strength spinor terms can be determined

$$\bar{W}^i_{\dot{\alpha}} = -2g \, e^{+i\theta \not{\partial} \bar{\theta}} \Big[\bar{\lambda}^i_{\dot{\alpha}} + \bar{\theta}_{\dot{\alpha}} D^i + i(\bar{\theta} \bar{\sigma}^\mu \sigma^\nu)_{\dot{\alpha}} F^i_{\mu\nu} + i \bar{\theta}^2 \bar{\sigma}^{\mu}_{\dot{\alpha}}{}^{\alpha} (D_\mu \lambda_\alpha)^i \Big]$$

where $(D_\mu \lambda_\alpha)^i = \partial_\mu \lambda^i_\alpha - \frac{g}{2} f_{ijk} A^j_\mu \lambda^k_\alpha$

Hence we find

$$\boxed{\int d\bar{S}\, \mathrm{Tr}[\bar{W}\bar{W}] = -\frac{(16)^2}{2} g^2 \int d^4x \Big[\frac{1}{16} D^i D^i - \frac{i}{8} (D_\mu \lambda)^i \sigma^\mu \bar{\lambda}^i - \frac{1}{8} F^i_{\mu\nu} F^{i\mu\nu} - \frac{i}{8} F^i_{\mu\nu} \tilde{F}^{i\mu\nu} \Big]}$$

This yields the SYM kinetic terms

$$\int dS\, \mathrm{Tr}[WW] + \int d\bar{S}\, \mathrm{Tr}[\bar{W}\bar{W}]$$

$$= -\frac{(16)^2}{2} g^2 \int d^4x \Big[\frac{1}{8} D^i D^i + \frac{i}{8} \lambda^i \sigma^\mu (D_\mu \bar{\lambda})^i - \frac{i}{8} (D_\mu \lambda)^i \sigma^\mu \bar{\lambda}^i - \frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} \Big]$$

Further notice that

$$\boxed{\begin{aligned}&\int dS\, Tr[WW] - \int d\bar{S}\, Tr[\overline{W}\overline{W}] \\ &= -\frac{(16)^2}{2} g^2 \int d^4x \Big[\frac{i}{4} F^i_{\mu\nu} \tilde{F}^{i\mu\nu} \\ &\qquad + \frac{i}{8} \partial_\mu (\lambda^i \sigma^\mu \bar{\lambda}^i) \Big]\end{aligned}}$$

(total divergence integrand)
related to anomalies & supercurrent

Also recall we can include the soft SUSY breaking terms by means of a θ-dependent wavefunction renormalization factor — that is a spurion gauge coupling field

$$\int dS\, Z(\theta)\, Tr[WW] + \int d\bar{S}\, \bar{Z}(\bar{\theta})\, Tr[\overline{W}\overline{W}]$$

where
$$Z(\theta) = Z(1 + 2M\theta^2)$$
$$\bar{Z}(\bar{\theta}) = Z(1 + 2\bar{M}\bar{\theta}^2)$$

So the soft SUSY breaking just picks out the θ-independent terms in the $Tr[WW]$ or $Tr[\overline{W}\overline{W})$ terms:

$$\int dS\, Z(\theta)\,\mathrm{Tr}[WW] + \int d\bar{S}\, \bar{Z}(\bar{\theta})\,\mathrm{Tr}[\bar{W}\bar{W}]$$

$$= -\frac{(16)^2}{2} g^2 Z \int d^4x \Big[\frac{1}{8} D^i D^i - \frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + \frac{i}{8} \lambda \sigma^\mu \overleftrightarrow{D}_\mu \bar{\lambda} + \frac{1}{8}\left(M \lambda^i \lambda^i + \bar{M} \bar{\lambda}^i \bar{\lambda}^i \right) \Big]$$

(multiply by 2 in U(1) case)

So we can state the Y-M part of the MSSM action: it is just the Y-M terms of the SM with 3 types of gaugino fields: a 3-2-1 singlet hypercharge gaugino $\hat{B}_\alpha$, $\bar{\hat{B}}_{\dot\alpha}$ in (1,1,0) representation of 3-2-1, a weak isospin gaugino $\hat{A}^i_\alpha$, $\bar{\hat{A}}^i_{\dot\alpha}$ in the (1,3,0) representation and finally the gluino fields $\hat{G}^m_\alpha$, $\bar{G}^m_{\dot\alpha}$ in the (8,1,0) representation. There are the auxiliary field quadratic terms as well.

$$\Gamma_{SYM} = \int d^4x\, \mathcal{L}_{SYM} = \int d^4x \left(\mathcal{L}_{YM} + \mathcal{L}_{\not{S}YM} \right)$$

$$\mathcal{L}_{YM} = \Big\{ -\tfrac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} - \tfrac{1}{4} B_{\mu\nu} B^{\mu\nu}$$
$$- \tfrac{1}{4} G^m_{\mu\nu} G^{m\mu\nu} + \tfrac{i}{2} \tilde{A}\, \sigma^\mu \overleftrightarrow{D}_\mu \bar{\tilde{A}}$$
$$+ \tfrac{i}{2} \tilde{B} \overleftrightarrow{\not{\partial}} \bar{\tilde{B}} + \tfrac{i}{2} \tilde{G} \sigma^\mu \overleftrightarrow{D}_\mu \bar{\tilde{G}}$$
$$+ \tfrac{1}{2} D^i_A D^i_A + \tfrac{1}{2} D_B D_B + \tfrac{1}{2} D^m_G D^m_G \Big\}$$

$$\mathcal{L}_{\not{S}YM} = \Big\{ \tfrac{1}{2}\left(M_2\, \tilde{A}^i \tilde{A}^i + \bar{M}_2 \bar{\tilde{A}}^i \bar{\tilde{A}}^i \right)$$
$$+ \tfrac{1}{2}\left(M_1\, \tilde{B}\tilde{B} + \bar{M}_1 \bar{\tilde{B}} \bar{\tilde{B}} \right) + \tfrac{1}{2}\left(M_3\, \tilde{G}^m \tilde{G}^m + \bar{M}_3 \bar{\tilde{G}}^m \bar{\tilde{G}}^m \right) \Big\}$$

where we have re-scaled the gaugino and auxiliary fields by $\sqrt{4} = 2$

$$\lambda \to \sqrt{4}\, \lambda \qquad D \to \sqrt{4}\, D$$
$$\bar{\lambda} \to \sqrt{4}\, \bar{\lambda}$$

and each covariant derivative is SU(3) or SU(2) covariant derivative according to which field it is acting upon.

Also for each subgroup we have let the Z be $2/-(16)^2$ i.e. $Z = \frac{-2}{(16)^2}$ and for the $U(1)$ we let $Z_{U(1)} = -\frac{1}{(16)^2}$ for the overall canonical normalization.

Of course when we re-normalize the model we will re-scale each field by its own wavefunction factor as we did in the SM case. The SUSY will imply relations amongst the particle and sparticle factors as with any symmetry.

Further we have let each coupling constant be re-scaled by $g \to -2g$ so that they coincide with our choices in the SM — hence

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g_2 \epsilon_{ijk} A^j_\mu A^k_\nu$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$G^m_{\mu\nu} = \partial_\mu G^m_\nu - \partial_\nu G^m_\mu + g_3 f_{mnp} G^n_\mu G^p_\nu$$

and similarly in the covariant derivatives.

Next consider the form of the gauged Kähler potential action term. In general it has the form

$$\Gamma_{SK} = \frac{1}{16}\int dV\, K_S \quad \text{with}$$

$$K_S = Z_K(\theta,\bar\theta)\, \bar\Phi\, e^{g V \cdot T} \Phi$$

where recall that Φ is in a particular representation of the gauge group