# III.) The MSSM:



Gauge Sector includes the SM gauge fields and their SUSY partner gaugino's

| Group: | (Real) Vector Superfield: | SM field | SUSY Partner | |
|---|---|---|---|---|
| SU(3) | $G^i$ | $G^i_\mu$ | $\tilde{G}^i_\alpha ; \bar{\tilde{G}}^i_{\dot\alpha}$ | gluino |
| SU(2) | $A^a$ | $A^a_\mu$ | $\tilde{A}^a_\alpha , \bar{\tilde{A}}_{\dot\alpha}$ | |
| alternate notation | $W^a$ | $W^a_\mu$ | $\tilde{W}^a_\alpha , \bar{\tilde{W}}^a_{\dot\alpha}$ | |
| U(1) | $B$ | $B_\mu$ | $\tilde{B}_\alpha , \bar{\tilde{B}}_{\dot\alpha}$ | |

After Electroweak symmetry breaking $W^a, B$ become

| | | | |
|---|---|---|---|
| $W^\pm$ | $W^\pm_\mu$ | $\tilde{W}^\pm_\alpha , \bar{\tilde{W}}^\pm_{\dot\alpha}$ | wino's |
| $Z$ | $Z_\mu$ | $\tilde{Z}_\alpha , \bar{\tilde{Z}}_{\dot\alpha}$ | zino |
| $A$ | $A_\mu$ | $\tilde{A}_\alpha , \bar{\tilde{A}}_{\dot\alpha}$ | photino |

The gauge fields are in the adjoint representations of their associated global symmetry groups and the Y-M kinetic energy terms that are gauge invariant have their usual form

Introducing chiral field strength spinors for each field, first $SU(3)$

$$W_\alpha^{SU(3)} = \bar{D}\bar{D}\left[e^{-g_3 G^i T^i} D_\alpha e^{+g_3 G^i T^i}\right]$$

with $(T^i)_{jk} = i f_{jik}$ the adjoint representation of the $SU(3)$ generators with $f_{ijk}$ the $SU(3)$ structure constants and anti-chiral field strength

$$\bar{W}_{\dot\alpha}^{SU(3)} = DD\left[e^{+g_3 G\cdot T} \bar{D}_{\dot\alpha} e^{-g_3 G\cdot T}\right]$$

Then the electroweak fields

$$W_\alpha^{SU(2)} = \bar{D}\bar{D}\left[e^{-g_2 \vec{A}\cdot\vec{T}} D_\alpha e^{+g_2 \vec{A}\cdot\vec{T}}\right]$$

$$\bar{W}_{\dot\alpha}^{SU(2)} = DD\left[e^{+g_2 \vec{A}\cdot\vec{T}} \bar{D}_{\dot\alpha} e^{-g_2 \vec{A}\cdot\vec{T}}\right]$$

with $(T^a)_{bc} = i\epsilon_{bac}$ for $SU(2)$

and $$W_\alpha^{U(1)} = \bar{D}\bar{D}\left[e^{-g_1 B} D_\alpha e^{+g_1 B}\right] = g_1 \bar{D}\bar{D} D_\alpha B.$$

$$\bar{W}_{\dot\alpha}^{U(1)} = DD\left[e^{+g_1 B} \bar{D}_{\dot\alpha} e^{-g_1 B}\right] = -g_1 DD\bar{D}_{\dot\alpha} B.$$

The 3-2-1 & SUSY invariant Y-M action is given by

$$\boxed{\begin{aligned}\Gamma_{YM} = {} & \frac{Z_{SU(3)}}{g_3^2}\int dS\, \mathrm{Tr}\left[W^{SU(3)\alpha} W^{SU(3)}_{\alpha}\right] \\ & + \frac{Z_{SU(2)}}{g_2^2}\int dS\, \mathrm{Tr}\left[W^{SU(2)\alpha} W^{SU(2)}_{\alpha}\right] \\ & + \frac{Z_{U(1)}}{g_1^2}\int dS\, \mathrm{Tr}\left[W^{U(1)\alpha} W^{U(1)}_{\alpha}\right] \\ & + \frac{Z_{SU(3)}}{g_3^2}\int d\bar{S}\, \mathrm{Tr}\left[\bar{W}^{SU(3)}_{\dot{\alpha}} \bar{W}^{SU(3)\dot{\alpha}}\right] \\ & + \frac{Z_{SU(2)}}{g_2^2}\int d\bar{S}\, \mathrm{Tr}\left[\bar{W}^{SU(2)}_{\dot{\alpha}} \bar{W}^{SU(2)\dot{\alpha}}\right] \\ & + \frac{Z_{U(1)}}{g_1^2}\int d\bar{S}\, \mathrm{Tr}\left[\bar{W}^{U(1)}_{\dot{\alpha}} \bar{W}^{U(1)\dot{\alpha}}\right]\end{aligned}}$$

where the traces are over their respective SU(3) & SU(2) adjoint representation matrices.

Next are the matter fields. As we have seen they will be components of chiral superfields so it is convenient to recall p. 182 where we converted the SM fields from left & right to all left handed fields — using the charge conjugate fields for the right handed fermions. In left-right notation we had

| Field | $(SU(3), SU(2), U(1))$ | Family Electroweak Multiplets |
|---|---|---|
| $l_{mL} = \begin{pmatrix} \nu_{mL} \\ e_{mL} \end{pmatrix}$ | $(1, 2, -\frac{1}{2})$ | $\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$ |
| $q^a_{mL} = \begin{pmatrix} u^a_{mL} \\ d^a_{mL} \end{pmatrix}$ | $(3, 2, +\frac{1}{6})$ | $\begin{pmatrix} u^a_L \\ d^a_L \end{pmatrix}, \begin{pmatrix} c^a_L \\ s^a_L \end{pmatrix}, \begin{pmatrix} t^a_L \\ b^a_L \end{pmatrix}$ |
| $e_{mR}$ | $(1, 1, -1)$ | $e_R, \mu_R, \tau_R$ |
| $u^a_{mR}$ | $(3, 1, +\frac{2}{3})$ | $u^a_R, c^a_R, t^a_R$ |
| $d^a_{mR}$ | $(3, 1, -\frac{1}{3})$ | $d^a_R, s^a_R, b^a_R$ |
| $\phi$ | $(1, 2, +\frac{1}{2})$ | $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ |
| (or $\tilde{\phi} = i\sigma^2 \phi^*$) | $(1, 2, -\frac{1}{2})$ | $\tilde{\phi} = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$ |

We then introduced the charge conjugate fields for the righthanded fermions and expressed the SM in terms of these fields instead of the L-R fields

| Field | $(SU(3), SU(2), U(1))$ | Family Multiplets |
|---|---|---|
| $l_{mL} = \begin{pmatrix} \nu_{mL} \\ e_{mL} \end{pmatrix}$ | $(1, 2, -\frac{1}{2})$ | $\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$ |
| $q^a_{mL} = \begin{pmatrix} u^a_{mL} \\ d^a_{mL} \end{pmatrix}$ | $(3, 2, +\frac{1}{6})$ | $\begin{pmatrix} u^a_L \\ d^a_L \end{pmatrix}, \begin{pmatrix} c^a_L \\ s^a_L \end{pmatrix}, \begin{pmatrix} t^a_L \\ b^a_L \end{pmatrix}$ |
| $e^+_{mL} = e^c_{mL}$ | $(1, 1, +1)$ | $e^c_L, \mu^c_L, \tau^c_L$ |
| $u^{ca}_{mL}$ | $(\bar{3}, 1, -\frac{2}{3})$ | $u^{ca}_L, c^{ca}_L, t^{ca}_L$ |
| $d^{ca}_{mL}$ | $(\bar{3}, 1, +\frac{1}{3})$ | $d^{ca}_L, s^{ca}_L, b^{ca}_L$ |
| $\phi$ | $(1, 2, +\frac{1}{2})$ | $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ |
| $(\tilde{\phi} = i\sigma^2 \phi^*)$ | $(1, 2, -\frac{1}{2})$ | $\tilde{\phi} = \begin{pmatrix} \phi^{0+} \\ -\phi^- \end{pmatrix}$ |

These left-handed SM fermions can be put into chiral superfields of the MSSM and we can introduce 2 Higgs multiplets (to render the model anomaly free)

| Chiral Superfield | (SU(3), SU(2), U(1)) | Chiral Family Multiplets |
|---|---|---|
| $L_m = \begin{pmatrix} \nu_m \\ e_m \end{pmatrix}$ | $(1, 2, -\frac{1}{2})$ | $\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$ |
| $Q^a_m = \begin{pmatrix} u^a_m \\ d^a_m \end{pmatrix}$ | $(3, 2, +\frac{1}{6})$ | $\begin{pmatrix} u^a \\ d^a \end{pmatrix}, \begin{pmatrix} c^a \\ s^a \end{pmatrix}, \begin{pmatrix} t^a \\ b^a \end{pmatrix}$ |
| $E^c_m$ | $(1, 1, +1)$ | $e^c, \mu^c, \tau^c$ |
| $U^{ca}_m$ | $(\overline{3}, 1, -\frac{2}{3})$ | $u^{ca}, c^{ca}, t^{ca}$ |
| $D^{ca}_m$ | $(\overline{3}, 1, +\frac{1}{3})$ | $d^{ca}, s^{ca}, b^{ca}$ |
| $H_u$ | $(1, 2, +\frac{1}{2})$ | $H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$ |
| $H_d$ | $(1, 2, -\frac{1}{2})$ | $H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$ |

We can now make the SUSY and gauge invariant kinetic energy Kähler potential terms using the gauge fields already introduced:

The kinetic energy terms for the quarks are

$$\bar{Q}^a_{mi}\left(e^{g_3\vec{G}\cdot\vec{T}^{SU(3)}}\right)_{ab}\left(e^{g_2\vec{A}\cdot\vec{T}^{SU(2)}}\right)_{ij}\left(e^{\frac{1}{6}g_1 B}\right)Q^b_{mj}$$

$$= \bar{Q}_m\, e^{\left[g_3\vec{G}\cdot\vec{T}^{SU(3)}+g_2\vec{A}\cdot\vec{T}^{SU(2)}+\frac{1}{6}g_1 B\right]}Q_m$$

(Recall $m$ is the family index)

Likewise the lepton kinetic energy is

$$\bar{L}_{mi}\left(e^{g_2\vec{A}\cdot\vec{T}^{SU(2)}}\right)_{ij}\left(e^{-\frac{1}{2}g_1 B}\right)L_{mj}$$

$$= \bar{L}_m\, e^{\left[g_2\vec{A}\cdot\vec{T}^{SU(2)}-\frac{1}{2}g_1 B\right]}L_m$$

and of course all the charge conjugate SU(2) singlet fields

$$\bar{E}^c_m e^{g_1 B}E^c_m + U^{c\,a}_m\left(e^{-g_3\vec{G}\cdot\vec{T}^{SU(3)}}\right)_{ab}\left(e^{-\frac{2}{3}g_1 B}\right)\bar{U}^{c\,b}_m$$

$$+ D^{c\,a}_m\left(e^{-g_3\vec{G}\cdot\vec{T}^{SU(3)}}\right)_{ab}\left(e^{+\frac{1}{3}g_1 B}\right)\bar{D}^{c\,b}_m$$

and finally the Higgs fields' kinetic energy

$$\bar{H}_u\, e^{\left[g_2 \vec{A}\cdot\vec{T}^{SU(2)} + \frac{1}{2} g_1 B\right]} H_u$$

$$+ \bar{H}_d\, e^{\left[g_2 \vec{A}\cdot\vec{T}^{SU(2)} - \frac{1}{2} g_1 B\right]} H_d \, .$$

In all these expressions we have $T^a_{SU(2)} = \frac{1}{2}\sigma^a$ and $T^i_{SU(3)} = \frac{\lambda^i}{2}$ (and the indices in each expression run over their respective domains)

To check the invariance of these terms recall the transformation properties of the fields

(suppress family index $m$)

$$L'_i = \left(e^{ig_2 \vec{\Lambda}_{SU(2)}\cdot\frac{\vec{\sigma}}{2}}\right)_{ij} \left(e^{-\frac{i}{2} g_1 \Lambda_{U(1)}}\right) L_j$$

$$\bar{L}'_i = \bar{L}_j \left(e^{-ig_2 \vec{\Lambda}_{SU(2)}\cdot\frac{\vec{\sigma}}{2}}\right)_{ji} \left(e^{+\frac{i}{2} g_1 \Lambda_{U(1)}}\right)$$

And

$$e^{g_2 \vec{A}'\cdot\vec{T}_{SU(2)}} = e^{+ig_2 \vec{\Lambda}_{SU(2)}\cdot\vec{T}_{SU(2)}}\, e^{g_2 \vec{A}\cdot\vec{T}_{SU(2)}} \times e^{-ig_2 \vec{\Lambda}_{SU(2)}\cdot\vec{T}_{SU(2)}}$$

$$e^{-\frac{1}{2} g_1 B'} = e^{-\frac{1}{2} i g_1 \bar{\Lambda}_{U(1)}}\, e^{-\frac{1}{2} g_1 B}\, e^{+\frac{1}{2} i g_1 \Lambda_{U(1)}}$$

and so

$$\underline{\left(\bar{L}\, e^{[g_2 \vec{A}\cdot\vec{T}_{SU(2)} - \frac{1}{2}g_1 B]} L\right)'} = \left(\bar{L}\, e^{[g_2 \vec{A}\cdot\vec{T}_{SU(2)} - \frac{1}{2}g_1 B]} L\right)$$

Similarly

$$Q' = \left(e^{ig_2 \vec{\Lambda}_{SU(2)}\cdot\vec{T}_{SU(2)}}\right)\left(e^{ig_3 \vec{\Lambda}_{SU(3)}\cdot\vec{T}_{SU(3)}}\right)\times$$

$$\times\left(e^{+\frac{i}{6}g_1 \Lambda_{U(1)}}\right) Q$$

&

$$\bar{Q}' = \bar{Q}\left(e^{-\frac{i}{6}g_1 \Lambda_{U(1)}}\right)\left(e^{-ig_3 \vec{\Lambda}_{SU(3)}\cdot\vec{T}_{SU(3)}}\right)\left(e^{-ig_2 \vec{\Lambda}_{SU(2)}\cdot\vec{T}_{SU(2)}}\right)$$

and

$$e^{g_3 \vec{G}'\cdot\vec{T}_{SU(3)}} = e^{+ig_3 \vec{\Lambda}_{SU(3)}\cdot\vec{T}_{SU(3)}}\, e^{g_3 \vec{G}\cdot\vec{T}_{SU(3)}}\, e^{-ig_3 \vec{\Lambda}_{SU(3)}\cdot\vec{T}_{SU(3)}}$$

while $e^{+\frac{1}{6}g_1 B'} = e^{\frac{i}{6}g_1 \Lambda_{U(1)}}\, e^{\frac{1}{6}g_1 B}\, e^{-\frac{i}{6}g_1 \Lambda_{U(1)}}$

(i.e. in general $e^{\frac{q}{6}g_1 B'} = e^{i\frac{q}{6}g_1 \Lambda_{U(1)}}\, e^{\frac{q}{6}g_1 B}\, e^{-i\frac{q}{6}g_1 \Lambda_{U(1)}}$ )

and as previously

$$\left(\bar{Q}\, e^{[g_3 \vec{G}\cdot\vec{T}_{SU(3)} + g_2 \vec{A}\cdot\vec{T}_{SU(2)} + \frac{1}{6}g_1 B]} Q\right)'$$

$$= \left(\bar{Q}\, e^{[g_3 \vec{G}\cdot\vec{T}_{SU(3)} + g_2 \vec{A}\cdot\vec{T}_{SU(2)} + \frac{1}{6}g_1 B]} Q\right)$$

and for the Higgs fields

$$H_u' = \left(e^{ig_2 \vec{\lambda}_{SU(2)}\cdot\vec{T}_{SU(2)}}\, e^{+\frac{i}{2}g_1\lambda_{U(1)}}\right) H_u$$

$$\bar{H}_u' = \bar{H}_u \left(e^{-ig_2 \vec{\lambda}_{SU(2)}\cdot\vec{T}_{SU(2)}}\, e^{-\frac{i}{2}g_1\lambda_{U(1)}}\right)$$

$$H_d' = \left(e^{ig_2 \vec{\lambda}_{SU(2)}\cdot\vec{T}_{SU(2)}}\, e^{-\frac{i}{2}g_1\lambda_{U(1)}}\right) H_d$$

$$\bar{H}_d' = \bar{H}_d \left(e^{-ig_2 \vec{\lambda}_{SU(2)}\cdot\vec{T}_{SU(2)}}\, e^{+\frac{i}{2}g_1\lambda_{U(1)}}\right)$$

So as before

$$\left(\bar{H}_u e^{[g_2 \vec{A}\cdot\vec{T}_{SU(2)} + \frac{1}{2}g_1 B]} H_u\right)'$$
$$= \left(\bar{H}_u e^{[g_2 \vec{A}\cdot\vec{T}_{SU(2)} + \frac{1}{2}g_1 B]} H_u\right)$$

&

$$\left(\bar{H}_d e^{[g_2 \vec{A}\cdot\vec{T}_{SU(2)} - \frac{1}{2}g_1 B]} H_d\right)'$$
$$= \left(\bar{H}_d e^{[g_2 \vec{A}\cdot\vec{T}_{SU(2)} - \frac{1}{2}g_1 B]} H_d\right)$$

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Finally consider the charge conjugate fields

$$E^{c\prime} = e^{ig_1\Lambda_{U(1)}} E^c \quad ; \quad \overline{E^c}' = e^{-ig_1\overline{\Lambda}_{U(1)}} \overline{E^c}$$

So indeed $(\overline{E^c} e^{g_1 B} E^c)' = (\overline{E^c} e^{g_1 B} E^c)$

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Now $U^c$ is in the $(\overline{3}, 1, -\frac{2}{3})$ representation

So

$$U^{c\prime} = U^c e^{-ig_3\vec{\Lambda}_{SU(3)}\cdot\vec{T}_{SU(3)}} e^{-\frac{2i}{3}g_1\Lambda_{U(1)}}$$

($U^c$ is on the left of the SU(3) transformation matrix since it is a $\overline{3}$ not a 3)

while $\overline{U^c}' = e^{+ig_3\overline{\vec{\Lambda}}_{SU(3)}\cdot\vec{T}_{SU(3)}} e^{+\frac{2i}{3}g_1\overline{\Lambda}_{U(1)}} \overline{U^c}$

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Recall

$$e^{g_3\vec{G}'\cdot\vec{T}_{SU(3)}} = e^{+ig_3\overline{\vec{\Lambda}}_{SU(3)}\cdot\vec{T}_{SU(3)}} e^{g_3\vec{G}\cdot\vec{T}_{SU(3)}} e^{-ig_3\vec{\Lambda}_{SU(3)}\cdot\vec{T}_{SU(3)}}$$

but $e^{-g_3\vec{G}'\cdot\vec{T}_{SU(3)}} e^{+g_3\vec{G}'\cdot\vec{T}_{SU(3)}} = 1$

$$\Rightarrow \boxed{e^{-g_3\vec{G}'\cdot\vec{T}_{SU(3)}} = e^{+ig_3\vec{\Lambda}_{SU(3)}\cdot\vec{T}_{SU(3)}} e^{-g_3\vec{G}\cdot\vec{T}_{SU(3)}} e^{-ig_3\overline{\vec{\Lambda}}_{SU(3)}\cdot\vec{T}_{SU(3)}}}$$

So

$$\left(U^c e^{[-g_3 \vec{G}\cdot\vec{T}_{SU(3)} - \frac{2}{3}g_1 B]}\,\overline{U^c}\right)'$$

$$= \left(U^c e^{[-g_3 \vec{G}\cdot\vec{T}_{SU(3)} - \frac{2}{3}g_1 B]}\,\overline{U^c}\right)$$

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and finally

$$D^{c\prime} = D^c e^{-ig_3 \vec{\Lambda}_{SU(3)}\cdot\vec{T}_{SU(3)}} e^{+\frac{i}{3}g_1 \Lambda_{U(1)}}$$

$$\overline{D^c}' = e^{+ig_3 \vec{\Lambda}_{SU(3)}\cdot\vec{T}_{SU(3)}} e^{-\frac{i}{3}g_1 \overline{\Lambda}_{U(1)}}\,\overline{D^c}$$

hence

$$\left(D^c e^{[-g_3 \vec{G}\cdot\vec{T}_{SU(3)} + \frac{1}{3}g_1 B]}\,\overline{D^c}\right)'$$

$$= \left(D^c e^{[-g_3 \vec{G}\cdot\vec{T}_{SU(3)} + \frac{1}{3}g_1 B]}\,\overline{D^c}\right)$$

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So we have the complete set of kinetic energy Kähler potential $SU(3)\times SU(2)\times U(1)$ SUSY invariant terms

$$\Gamma_K = \frac{1}{16}\int dV\, K$$

$$K = Z_Q \overline{Q} e^{[g_3 G + g_2 A + \frac{1}{6} g_1 B]} Q$$
$$+ Z_L \overline{L} e^{[g_2 A - \frac{1}{2} g_1 B]} L$$
$$+ Z_{E^c} \overline{E^c}\, e^{g_1 B} E^c$$
$$+ Z_{U^c} U^c e^{[-g_3 G - \frac{2}{3} g_1 B]} \overline{U^c}$$
$$+ Z_{D^c} D^c e^{[-g_3 G + \frac{1}{3} g_1 B]} \overline{D^c}$$
$$+ Z_{H_u} \overline{H}_u e^{[g_2 A + \frac{1}{2} g_1 B]} H_u$$
$$+ Z_{H_d} \overline{H}_d e^{[g_2 A - \frac{1}{2} g_1 B]} H_d$$

where we defined $G \equiv \vec{G}\cdot\vec{T}_{SU(3)} = \vec{G}\cdot\frac{\vec{\lambda}}{2}$

$$A \equiv \vec{A}\cdot\vec{T}_{SU(2)} = \vec{A}\cdot\frac{\vec{\sigma}}{2}$$

and each Z factor will be chosen for normalization later.

Next the matter fields can interact via a superpotential — let's list the chiral superfields again & their 3-2-1 representations

$L$ $(1, 2, -\frac{1}{2})$
$Q$ $(3, 2, +\frac{1}{6})$
$E^c$ $(1, 1, +1)$
$U^c$ $(\bar{3}, 1, -\frac{2}{3})$
$D^c$ $(\bar{3}, 1, +\frac{1}{3})$
$H_u$ $(1, 2, +\frac{1}{2})$
$H_d$ $(1, 2, -\frac{1}{2})$

We must look at 3-2-1 invariant products of 1, 2, 3 fields.

1) There are no single field total invariants

2) Products of 2 : $H_u^i H_d^j \epsilon_{ij}$ $\left(\epsilon_{ij} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right)$
$\equiv H_u H_d$

$H_u H_d$ is $U(1)$ invariant since they have opposite $U(1)$ charges $\pm\frac{1}{2}$.
The $\epsilon_{ij}$ symbol converts the $2$ to a $\bar{2}$ so that the product is also $SU(2)$ invariant

$$H_u' = e^{ig_2 \Lambda_{SU(2)}} H_u \quad \text{under } SU(2) \text{ with } \Lambda_{SU(2)} \equiv \vec{\Lambda}_{SU(2)} \cdot \vec{T}_{SU(2)}$$

So

$$\epsilon_{ij} H'_{uj} = \epsilon_{ij} \left(e^{ig_2 \vec{\Lambda}_{SU(2)} \cdot \frac{\vec{\sigma}}{2}}\right)_{jk} H_{uk}$$

Recall $\epsilon_{ij} = i\sigma^2_{ij}$ & $(i\sigma^2)^\dagger \sigma^k (i\sigma^2) = -\sigma^{k\,T}$

So

$$\epsilon_{ij} H'_{u_j} = \left[ \left( i\sigma^2 e^{ig_2 \vec{\Lambda}_{SU(2)} \cdot \frac{\vec{\sigma}}{2}} (-i\sigma^2 i\sigma^2) \right) H_u \right]_i$$

$$= \left( \left[ (i\sigma^2)^\dagger e^{ig_2 \vec{\Lambda}_{SU(2)} \cdot \frac{\vec{\sigma}}{2}} (i\sigma^2) \right]_{ij} \left( (i\sigma^2) H_u \right)_j \right.$$

$$= \left[ e^{-ig_2 \vec{\Lambda}_{SU(2)} \cdot \frac{\vec{\sigma}}{2}} \right]_{ji} \epsilon_{jk} H_{uk}$$

$$= \left( \epsilon_{jk} H_{u_k} \right) \left( e^{-ig_2 \vec{\Lambda}_{SU(2)} \cdot \frac{\vec{\sigma}}{2}} \right)_{ji}$$

So

$$H'_u \epsilon H'_d = - H'_{d_i} \epsilon_{ij} H'_{u_j}$$

$$= - \left( \epsilon_{jk} H_{uk} \right) \left( e^{-ig_2 \vec{\Lambda}_{SU(2)} \cdot \frac{\vec{\sigma}}{2}} \right)_{ji} \left( e^{+ig_2 \vec{\Lambda}_{SU(2)} \cdot \frac{\vec{\sigma}}{2}} \right)_{ik} H_{dk}$$

$$= - \epsilon_{jk} H_{uk} H_{d_j}$$

$$= - H_d \epsilon H_u = + H_u \epsilon H_d$$

$$(H_u H_d)' = (H_u H_d) \checkmark$$

2) Similarly we see a lepton number violating 3-2-1 invariant term as well

$H_u^i \epsilon_{ij} L^j_m$ this will need to be eliminated.

3) Products of 3:

$Q\vec{U}^c$ is $(1, 2, -\frac{1}{2})$ The only SU(3) invariants possible
$QD^c$ is $(1, 2, +\frac{1}{2})$

So $H_u Q U^c$ is invariant
$H_d Q D^c$ is invariant

but so is $LQD^c$ invariant another problematic term as is $(\underbrace{U^c \underbrace{D^c D^c}_{=\bar{3}\times\bar{3}=\bar{6}+3}}_{\bar{3}\times 3 = 1 \text{ for } SU(3)})$

Now
$LE^c$ is $(1, 2, +\frac{1}{2})$
$L_m L_n$ is $(1, 1, -1)$
$H_u L$ is $(1, 1, 0)$ (previous singlet)
$H_d L$ is $(1, 1, -1)$
$E^c E^c$ is $(1, 1, +2)$
$H_u E^c$ is $(1, 2, +3/2)$
$H_d E^c$ is $(1, 2, +\frac{1}{2})$
$H_u H_u = 0$ $H_u H_d$ is $(1, 1, 0)$ (previous singlet)
$H_d H_d = 0$

3) From these we only find 2 3-2-1 invariant

$H_d L E^c$

and the problematic one $L_m L_n E^c$.

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So we have as a superpotential 2 types of terms

$$\boxed{W = \mu H_u H_d + y_u H_u Q U^c + y_d H_d Q D^c + y_\ell H_d L E^c}$$

and the problematic terms

$$\boxed{W_p = m_m H_u L_m + g^Q_{mnp} L_m Q_n D^c_p + g^\ell_{mnp} L_m L_n E^c_p + g^u_{mnp} d_{abc} U^c_{ma} D^c_{nb} D^c_{pc}}$$

Note the Yukawa couplings above are 3x3 generation matrices

$$\boxed{W = \mu H_u H_d + y_{umn} H_u Q_m U^c_n + y_{dmn} H_d Q_m D^c_n + y_{\ell mn} H_d L_m E^c_n}$$

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Let's first use a discrete $Z_2$ symmetry known as R-parity to eliminate the problematic terms

$$W_p = m_m H_u L_m + g^Q_{mnp} L_m Q_n D^c_p$$
$$+ g^u_{mnp} d_{abc} U^c_{ma} D^c_{nb} D^c_{pc}$$
$$+ g^l_{mnp} L_m L_n E^c_p$$

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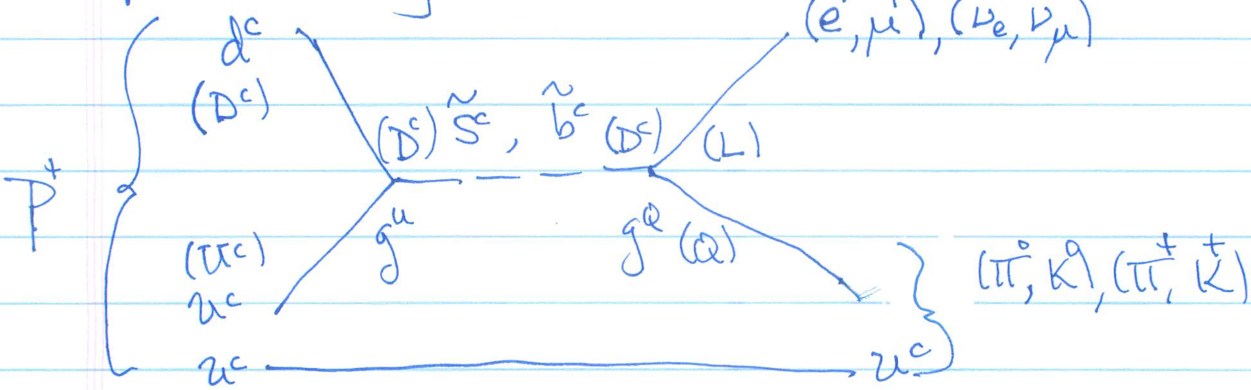
Why is this problematic — $W_p$ violates baryon and lepton number this can lead to, for example, proton decay!




Estimate the decay rate and hence lifetime of the proton

$$\Gamma_p \simeq \underbrace{\left| \frac{g^u g^Q}{m^2_{\tilde{s},\tilde{b}}} \right|^2}_{\text{Feynman diagram}} \left( \frac{m_p^5}{\underset{\text{phase space}}{8\pi}} \right)$$

Engineering dimensions

So

$$\tau_p = \frac{1}{\Gamma_p} \approx \frac{1}{|g^u g^Q|^2}\left(\frac{m_{\tilde{q}}}{m_p}\right)^4 \frac{8\pi}{m_p}\hbar$$ ← units of Energy x time

Now $m_p \approx 1\,\text{GeV} = 10^{-3}\,\text{TeV} = 10^3\,\text{MeV}$; $\hbar = 6.58\times10^{-22}\,\text{MeV sec}$

$$\tau_p = \frac{1}{|g^u g^Q|^2}\left(\frac{m_{\tilde{q}}}{1\,\text{TeV}}\right)^4\left(\frac{8\pi}{10^3 10^{-12}}6.58\times10^{-22}\right)\text{sec}$$

$$\tau_p \approx \frac{1}{|g^u g^Q|^2}\left(\frac{m_{\tilde{q}}}{1\,\text{TeV}}\right)^4 1.7\times10^{-11}\,\text{sec},$$

Experimentally $\tau_p > 10^{32}$ years $\approx 3\times10^{39}$ sec.

So for $m_{\tilde{q}} \sim 1\,\text{TeV} \Rightarrow |g^u g^Q| < 10^{-25}$ a very small coupling!

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So we would like to eliminate the $W_p$ terms from the Lagrangian (action) to begin with.

Note that $\int ds W_p$ will always involve an odd number of sparticle fields or auxiliary fields

$\int ds\, H_u L_m \sim \tilde{h}e, h\tilde{e}, hF_e, \ldots$

$\int ds\, LQD^c \sim \tilde{e}qd^c, eq\tilde{d}^c, e\tilde{q}d^c, F_e\tilde{q}\tilde{d}^c$, etc

$\int d s\, U^c D^c D^c \sim u^c d^c \tilde{d}^c$ etc.

$\int d s\, L L E^c \sim e e \tilde{e}^c$ etc.

---

So if we introduce a discrete $Z_2$ symmetry we can eliminate these terms. In more detail

$H_u L$, $L Q D^c$, $L L E^c$ terms violate lepton # $\Delta L = 1$

$U^c D^c D^c$ violates baryon # $\Delta B = 1$

That is superfields $Q$ have baryon # $= B = \frac{1}{3}$
i.e. $Q' = (-1)^B Q$
while $U^c, D^c$ have $B = -\frac{1}{3}$ and $L$ has

lepton # $L = +1$ while $E^c$ has $L = -1$.

So $L = 0 = B$ for the Higgs fields $H_{u,d}$ so

$$\left.\begin{aligned}(H_u L)' &= (-1)^1 (H_u L)\\ (L Q D^c)' &= (-1)^1 (L Q D^c)\\ (L L E^c)' &= (-1)^1 (L L E^c)\end{aligned}\right\}$$ under lepton # $\underline{\Delta L = +1}$

$(U^c D^c D^c)' = (-1)^1 (U^c D^c D^c)$ under baryon # $\Delta B = 1$

Recall "good" superpotential terms
ex. $(H_u Q U^c)' = (-1)^0 (H_u Q U^c)$ etc.

respect the B & L number discrete symmetries.

So to eliminate $W_P$ we introduce a $(B-L)$ discrete symmetry ("matter" parity $P_M = (-1)^{3(B-L)}$)

$$R = (-1)^{3(B-L)+2s}$$

where $s$ = spin of particle

fermion = 1
scalar = 0

or $$R = (-1)^{3(B-L)+F}$$

Since every term in the action has an even number of fermions the $(-1)^F$ adds up to be +1
So R is equivalent to the "matter" parity discrete symmetry $P_M = (-1)^{3(B-L)}$.

We cannot impose separate B & L discrete symmetries since non-perturbative electroweak effects violate these symmetries and whose effects might be relevant in the early universe.

So we impose the discrete symmetry

$$R = (-1)^{3(B-L)+2s}$$

Note for "fermion" matter fields

$R^{-1}QR = (-1)Q$, $\theta' = -\theta$:
$$\left\{\begin{array}{l} R^{-1}\psi_Q R = (-1)^{3(\frac{1}{3})+1}\psi_Q = +\psi_Q \\ R^{-1}A_Q R = (-1)^{3(\frac{1}{3})}A_Q = -A_Q \end{array}\right.$$

$R^{-1}U^c R = (-1)U^c$, $(\theta' = -\theta)$:
$$\left\{\begin{array}{l} R^{-1}\psi_{U^c} R = (-1)^{3(-\frac{1}{3})+1}\psi_{U^c} = +\psi_{U^c} \\ R^{-1}A_{U^c} R = (-1)^{3(-\frac{1}{3})}A_{U^c} = -A_{U^c} \end{array}\right.$$

$R^{-1}E^c R = (-1)E^c$, $(\theta' = -\theta)$:
$$\left\{\begin{array}{l} R^{-1}\psi_{E^c} R = (-1)^{-3(-1)+1}\psi_{E^c} = +\psi_{E^c} \\ R^{-1}A_{E^c} R = (-1)^{-3(-1)}A_{E^c} = -A_{E^c} \end{array}\right.$$

etc. $R^{-1}H_{u,d}R = H_{u,d}$ $(\theta' = -\theta)$: $A'_{H_u, H_d} = +A_{H_u, H_d}$, $\psi'_{H_u, H_d} = -\psi_{H_u, H_d}$

particles' = + particles

sparticles' = − sparticles

(likewise for F fields) $\tilde{F}' = -\tilde{F}$

$F_H' = +F_H$

So all the problem terms will involve partner fields and hence not conserve R-parity

i.e.
$$R^{-1}Q(x,\theta,\bar\theta)R = (-1)\,Q(x,-\theta,-\bar\theta), \text{ etc.}$$

$$\text{So } R^{-1}\left(\int dS\, L Q D^c\right)R = \int dS\,(-1)\,L(-\theta)\,Q(-\theta)\,D^c(-\theta)$$
$$\underset{(\theta\to-\theta)}{=} -\int dS\, L(\theta)\,Q(\theta)\,D^c(\theta)$$

But for the superpotential

ex.

$$R^{\dagger}\left(\int dS\, H_d Q D^c\right) R = \int dS\, H_d(-\theta)\, Q(-\theta)\, D^c(-\theta)$$

$$(\theta \to -\theta) = \int dS\, H_d(\theta)\, Q(\theta)\, D^c(\theta)$$

So we impose R-parity to eliminate $W_p$ but keep $W$.

R-parity has consequences — every interaction vertex has an even number of $R=-1$ sparticles $\Rightarrow$

1) In colliders, sparticles are only produced in pairs

2) The lightest supersymmetric partner (LSP) is stable, and if electrically neutral, can be a dark matter candidate

3) The sparticles (other than the LSP) eventually decay into an odd number of LSP's.

(Aside: R-symmetry - continuous symmetry

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu$$

is invariant under phase transformation of $Q$

$U(\alpha)_R = R(\alpha)$:

$$Q'_\alpha = R Q_\alpha R^{-1} = e^{i\alpha} Q_\alpha \qquad R P_\mu R^{-1} = P_\mu = P'_\mu$$

$$\bar{Q}'_{\dot\alpha} = R \bar{Q}_{\dot\alpha} R^{-1} = e^{-i\alpha} \bar{Q}_{\dot\alpha}$$

$\Rightarrow$

Anti-commutator unchanged

$$\{Q'_\alpha, \bar{Q}'_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu .$$

$R M^{\mu\nu} R^{-1} = M'^{\mu\nu}$ & $\{Q'_\alpha, Q'_\beta\} = 0$ etc. are all unchanged.

So we have R-symmetry it can be realized in superspace by

$$R(\alpha)\phi(x,\theta,\bar\theta)R^{-1}(\alpha) = e^{i\alpha R}\phi(x,\theta,\bar\theta)e^{-i\alpha R}$$

$$= e^{i\alpha n}\phi(x, e^{i\alpha}\theta, e^{-i\alpha}\bar\theta)$$

$n$ is called the R-weight of the field

Now for $\alpha = \pi$ $R(\pi) = R$ parity

$$R(\pi)\phi(x,\theta,\bar\theta)R^{-1}(\pi) = e^{i\pi n}\phi(x,-\theta,-\bar\theta)$$

and $e^{i\pi n} = (-1)^n$ so let $n = 3(B-L)$

Hence R-parity is a discrete $Z_2$ subgroup of the continuous R-symmetry.)

---

So finally we have the MSSM invariant terms

$$\boxed{\begin{aligned}\Gamma_{MSSM} &= \Gamma_{YM} + \Gamma_K + \Gamma_W \\ \Gamma_W &= \int dS\, W + \int d\bar{S}\, \bar{W}\end{aligned}}$$

---

Finally we want to make the MSSM realistic hence SUSY must be broken. The exact mechanism for the breaking we leave unspecified for the present & we retain only its desired effects — the SUSY masses must split ~~so~~ so as to make all partners heavier and we must also have electroweak symmetry breaking. Also we want to maintain the good UV behavior of the Theory in that scalar masses should not receive quadratic radiative corrections only logarithmic in the large masses & scales

So it has been shown that a set of very general "soft" SUSY breaking terms parameterize the SUSY breaking but yet maintain the good UV behavior.

These terms are gaugino mass terms, smatter mass terms, tri-linear terms. Generically these have the form:

$\lambda\lambda, \bar{\lambda}\bar{\lambda}$ Gaugino masses

$\left.\begin{array}{l}\bar{A}A \\ AA, \bar{A}\bar{A}\end{array}\right\}$ smatter masses

$AAA, \bar{A}\bar{A}\bar{A}$ tri-linear terms

( $A$, $\bar{A}AA$ only under special circumstances if $A$ is a gauge singlet.)

So the MSSM we have the soft susy breaking terms:

(Note: mass terms $\Psi\Psi$ $\bar{\Psi}\bar{\Psi}$ are not included — they would only redefine the susy parameters of the theory)

$$
\begin{aligned}
\mathcal{L}^{MSSM}_{soft} = & -\tfrac{1}{2}\left(M_3 \tilde{G}\tilde{G} + M_2 \tilde{W}\tilde{W} + M_1 \tilde{B}\tilde{B} + h.c.\right) \\
& -\Big[H_u \tilde{Q} A_{u^c} \tilde{U}^c \quad H_d \tilde{Q} A_{D^c} \tilde{D}^c - H_d \tilde{L} A_{E^c} \tilde{E}^c \\
& \qquad + h.c.\Big] \\
& - \bar{\tilde{Q}}\, m_Q^2\, \tilde{Q} - \bar{\tilde{L}}\, m_L^2\, \tilde{L} - \bar{\tilde{U}}^c\, m_{u^c}^2\, \tilde{U}^c \\
& - \tilde{D}^c\, m_{D^c}^2\, \bar{\tilde{D}}^c - \bar{\tilde{E}}^c\, m_{E^c}^2\, \tilde{E}^c \\
& - m_{H_u}^2\, \bar{\tilde{H}}_u \tilde{H}_u - m_{H_d}^2\, \bar{\tilde{H}}_d \tilde{H}_d \\
& - \mu B\left(\tilde{H}_u \tilde{H}_d + h.c.\right)
\end{aligned}
$$

This plus the MSSM action has 105 extra parameters above the SM !.

The $A_{u^c}$, $A_{D^c}$, $A_{E^c}$, $m_Q^2$, $m_L^2$, $m_{u^c}^2$, $m_{D^c}^2$, $m_{E^c}^2$ are all $3\times3$ matrices in family space and can have complex entries. The $m^2$'s must also be hermitian.

---

We can obtain these "soft" Susy breaking terms by introducing constant but $\theta$-dependent spurion fields in the MSSM Susy invariant action.

So for $\Gamma_{YM}$ let $Z \to Z(1-\frac{1}{8}M\,\theta^2\bar\theta^2) \equiv Z(\theta,\bar\theta)$

$\Gamma_K$ let $Z \to Z(1-m^2\theta^2\bar\theta^2) \equiv Z(\theta,\bar\theta)$

$\Gamma_W$ let $\mu \to \mu(1+\frac{1}{4}B\theta^2) \equiv \mu(\theta)$

$y \to \frac{1}{4}A\theta^2 \equiv y(\theta)$

---

So we finally can write the MSSM with soft-Susy breaking terms in the compact form:

$$\Gamma = \Gamma_{MSSM} + \Gamma_{MSSM}^{soft}$$

$$= \Gamma_{SYM} + \Gamma_{SK} + \Gamma_{SW}$$

$$\Gamma_{SYM} = \int dS \frac{Z_3(\theta,\bar\theta)}{g_3^2} Tr[W_3 W_3] + h.c.$$
$$+ \int dS \frac{Z_2(\theta,\bar\theta)}{g_2^2} Tr[W_2 W_2] + h.c.$$
$$+ \int dS \frac{Z_1(\theta,\bar\theta)}{g_1^2} Tr[W_1 W_1] + h.c.$$

$$\Gamma_{SK} = \frac{1}{16}\int dV K_S$$

$$K_S = Z_Q(\theta,\bar\theta)\,\bar{Q}\, e^{[g_3 G + g_2 A + \frac{1}{6} g_1 B]} Q$$
$$+ Z_L(\theta,\bar\theta)\,\bar{L}\, e^{[g_2 A - \frac{1}{2} g_1 B]} L + Z_{E^c}(\theta,\bar\theta)\,\bar{E}^c e^{g_1 B} E^c$$
$$+ Z_{U^c}(\theta,\bar\theta)\, U^c e^{[-g_3 G - \frac{2}{3} g_1 B]} \bar{U}^c$$
$$+ Z_{D^c}(\theta,\bar\theta)\, D^c e^{[-g_3 G + \frac{1}{3} g_1 B]} \bar{D}^c$$
$$+ Z_{H_u}(\theta,\bar\theta)\,\bar{H}_u e^{[g_2 A + \frac{1}{2} g_1 B]} H_u$$
$$+ Z_{H_d}(\theta,\bar\theta)\,\bar{H}_d\, e^{[g_2 A - \frac{1}{2} g_1 B]} H_d .$$

$$\boxed{\begin{aligned}\Gamma_{SW} = \int dS \Big[ &\mu(\theta) H_u H_d \\ &+ H_u Q y_u(\theta) U^c + H_d Q y_d(\theta) D^c \\ &+ H_d L y_e(\theta) E^c \Big] + h.c.\end{aligned}}$$

We must now analyze the electroweak symmetry breaking, i.e. the ground state of this model, and then the spectrum of particles & sparticles; and the interactions of ordinary matter with smatter & gaugino's.