Spontaneous SUSY Breaking

Before discussing the MSSM - let's first consider spontaneous breaking of SUSY as well as gauge invariance

In general $\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu$

So for $\alpha = 1$, $\dot\alpha = 1$ $\Rightarrow$ $\left(\overset{\text{Recall}}{\bar{Q}_{\dot\alpha} = Q^\dagger_{\dot\alpha}}\right)$

$$Q_1 Q_1^\dagger + Q_1^\dagger Q_1 = 2(P_0 + P_3)$$

& for $\alpha = 2$, $\dot\alpha = 2$

$$Q_2 Q_2^\dagger + Q_2^\dagger Q_2 = 2(P_0 - P_3)$$

$\Rightarrow$

The Hamiltonian H

$$H = P_0 = \frac{1}{4}\left[Q_1^\dagger Q_1 + Q_1 Q_1^\dagger + Q_2^\dagger Q_2 + Q_2 Q_2^\dagger\right].$$

H is the sum of squares and, as we saw with the potential, is a non-negative operator

$$\langle \psi | H | \psi \rangle \geq 0 \text{ for all states } |\psi\rangle.$$

The supersymmetric vacuum, being the lowest energy state, has zero energy $\langle 0|H|0\rangle = 0$. It is supersymmetric because $\langle 0|H|0\rangle = 0$ implies $Q|0\rangle = \bar{Q}|0\rangle = 0$. Thus vacuum states with positive energy must spontaneously break SUSY.

Consider an arbitrary or generalized W-Z chiral model with fields ϕ_i & $\bar{\phi}_i$

$$\Gamma = \int dV \sum_i \phi_i \bar{\phi}_i z_i + \int dS\, 4\, \phi_i m_{ij} \phi_j$$
$$+ \int d\bar{S}\, 4\, \bar{\phi}_i m_{ij} \bar{\phi}_j + \int dS\, g_{ijk} \phi_i \phi_j \phi_k$$
$$+ g^*_{ijk} \int d\bar{S}\, \bar{\phi}_i \bar{\phi}_j \bar{\phi}_k + f_i \int dS\, \phi_i + f^*_i \int d\bar{S}\, \bar{\phi}_i .$$

In terms of components we found that

$$V = \sum_i 16\, z_i F_i^\dagger F_i$$

where $\frac{\delta \Gamma}{\delta F_i} = 0 \Rightarrow 16 z_i F_i^\dagger = 4 f_i + 32 m_{ij} A_j + 12 g_{ijk} A_j A_k$

$(\cancel{\sum_i})$

The supersymmetric vacuum corresponds to $\langle F_i \rangle = 0 = \langle F_i^\dagger \rangle$ so that $V=0$. The vacuum values of the A_i fields $\langle 0|A_i|0\rangle \equiv a_i$ must satisfy the quadratic equation

$$\boxed{0 = f_i + 8 m_{ij} a_j + 3 g_{ijk} a_j a_k}\,,$$

given the m, g, f parameters, to find the supersymmetric ground state.

If we arrange the parameters m, g, f such that no solution exists then SUSY is spontaneously broken. Since the fields have vevs now, we must shift the values of the fields about the minimum value $A_i \to A_i + a_i$; $F_i \to F_i + v_i$ so that the action now becomes

$$\Gamma \to \Gamma - \int d^4x \Big[2 g_{ijk} v_i A_j A_k + g_{ijk} a_i \psi_j \psi_k - 4 g_{ijk} a_i A_j F_k + h.c. \Big].$$

The scalar and pseudoscalar fields $A_i = (\underset{\uparrow\ \text{scalar}}{\mathcal{A}_i} + i \underset{\uparrow\ \text{pseudoscalar}}{B_i})$ have a mass shifted from each other and from that of the fermions. The masses are no longer degenerate.

The simplest such example of a chiral model exhibiting spontaneous SUSY breaking is the O'Raifeartaigh model. It consists of 3 fields Φ, Ψ, Z, with the action

$$\overset{\shortparallel}{\Phi_0}\quad \overset{\shortparallel}{\Phi_1}\quad \overset{\shortparallel}{\Phi_2}$$

$$\Gamma = \frac{1}{16}\int dV\, \bar{\Phi}_i \Phi_i + \int dS\, W + \int d\bar{S}\, \bar{W}$$

where the superpotential is

$$W = \frac{\lambda}{4}\Phi_0 + \frac{m}{4}\Phi_1\Phi_2 + \frac{g}{32}\Phi_0\Phi_1^2$$

$$= \frac{\lambda}{4}\Phi + \frac{m}{4}\Psi Z + \frac{g}{32}\Phi\Psi^2$$

with m, g, λ real. The effective potential is given by

$$V = \bar{F}_i F_i = |\lambda + \frac{g}{8}A_1^2|^2 + |mA_2 + \frac{g}{4}A_0A_1|^2$$

$$+ m^2\bar{A}_1 A_1 \geq 0 .$$

Now the second term is minimized by

$$A_2 = -\frac{g}{4m}A_0A_1 \quad \text{and} \quad A_1 = 0$$

minimizes the first term and the last term as long as $m^2 > |\frac{1}{8}\lambda g|$

Recall $\frac{\partial V}{\partial F_i} = 4\frac{\partial W(A)}{\partial A_i} = \bar{F}_i$ i.e. $0 = \frac{\delta\Gamma}{\delta F_i} = \bar{F}_i - 4\frac{\partial W(A)}{\partial A_i}$

That is

$$\bar{F}_0 = \frac{\partial V}{\partial F_0} = \lambda + \frac{g}{8}A_1^2$$

$$\bar{F}_1 = \frac{\partial V}{\partial F_1} = mA_2 + \frac{g}{4}A_0A_1$$

$$\bar{F}_2 = \frac{\partial V}{\partial F_2} = mA_1$$

we desire these to be zero to have a good SUSY vacuum $\bar{F}_1 = 0 \Rightarrow$

$$A_2 = -\frac{g}{4m}A_0A_1$$

$\bar{F}_2 = 0$ requires $A_1 = 0$ ($\Rightarrow A_2 = 0$ and A_0 is arbitrary) and $V = |\lambda|^2$

But then $\bar{F}_0 = \lambda \neq 0$. Setting $\bar{F}_0 = 0$ requires $A_1 \neq 0 \Rightarrow \bar{F}_2 \neq 0$. There is no solution to the set of quadratic equations $\bar{F}_i = 0$. The minimum of the potential is for $A_1 = 0$ since if

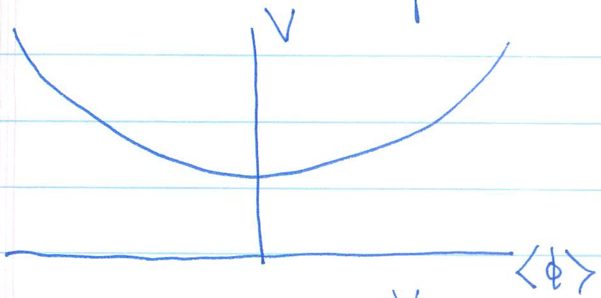
$\bar{F}_0 = 0 \Rightarrow \frac{g}{8}A_1^2 = -\lambda$, then $m^2\bar{A}_1A_1 = \left|m^2\left(-\frac{\lambda}{g/8}\right)\right|$

So the value of $V = \left|m^2 \lambda/g/8\right|$. Earlier we had for $A_1 = 0$ case $V = |\lambda|^2$. So if $\lambda^2 < m^2\lambda/g/8$

i.e. $m^2 > \left|\frac{\lambda g}{8}\right|$ $A_1 = 0$ gives the minimum of V.

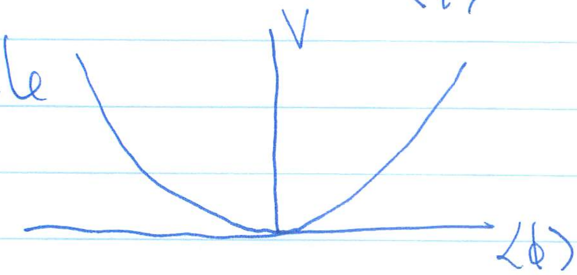
Thus we have broken the SUSY since $F_0 = \lambda \neq 0$ and $V = |\lambda|^2 > 0$

Note: The lack of solution to the quadratic eq. means the potential has the form



in the ~~SUSY~~ case

while



in the SUSY good case.

Now since A_0 is arbitrary we will choose the R-symmetry good value of $\langle A_0 \rangle = 0$. The $F_0, \bar{F}_0$ field will have non-zero vev's So we must shift the fields to be at the minimum of the potential $F_0 \to F_0 + \lambda$

$\bar{F}_0 \to \bar{F}_0 + \lambda$

The action then becomes

$$\Gamma \to \Gamma + \int d^4x \left(-\frac{g\lambda}{8}\right)\left(A_1^2 + \bar{A}_1^2\right)$$

This causes a mass splitting in the

X_1 supermultiplet one finds that the $\operatorname{Re} A_1 = \mathcal{A}_1$ and $\operatorname{Im} A_1 = \mathcal{B}_1$ fields split their masses above and below the fermion ψ_1 mass

$$m^2_{\mathcal{A}_1} = m^2 + \tfrac{1}{4}\lambda g$$

$$m^2_{\psi_1} = m^2$$

$$m^2_{\mathcal{B}_1} = m^2 - \tfrac{1}{4}\lambda g$$

We find a special case of the general mass sum rule for spontaneously broken SUSY

$$2\sum_{\text{fermions}}' m_f^2 - \sum_{\text{scalars}}' m_s^2 = 0$$

Also we note that

$$i\{Q_\alpha, \psi_{0\beta}\} = 2i\epsilon_{\alpha\beta} F_0 \longrightarrow 2i\epsilon_{\alpha\beta}\lambda + 2i\epsilon_{\alpha\beta} F_0$$

constant

Hence ψ_0 is identified as the Goldstone Fermion of the spontaneously broken SUSY called the Goldstino. A_0 remains massless by the choice of $\langle A_0 \rangle = 0$ R-preserving vev. It will obtain a mass when loops are taken into account (quantum corrections) it's a pseudo-Goldstone boson.

Besides the O'Raifeartaigh models, SUSY can be broken in gauge theories also. There we have the option to 1) break SUSY but not gauge invariance, 2) break both gauge invariance and SUSY, 3) break gauge invariance but not SUSY.

Let's study these possibilities in our SQED U(1) model. We can allow a term linear in V in the action (since the gauge group is abelian such a term is both gauge and SUSY invariant

$$\int dU\, V' = \int dV\left(V + i(\bar{\Lambda} - \Lambda)\right)$$

but $\int dV\, \Lambda = \int dS\, \bar{D}\bar{D}\Lambda = 0$ & $\int dV\, \bar{\Lambda} = \int d\bar{S}\, DD\bar{\Lambda} = 0$

So

$$\int dU\, V' = \int dU\, V, \text{ both gauge \& SUSY invariant.}$$

However $\int dUV$ is not parity invariant since V is a pseudo-scalar.)

So we add a Fayet-Illiopolous term to the SQED action

$$\Gamma = \Gamma_{SQED} + \frac{\xi}{4}\int dU\, V$$

$$= \Gamma_{SQED} + \xi \int d^4x\, D(x)$$

The potential is given by

$$\boxed{V = \tfrac{1}{2}D^2 + \bar{F}_+ F_+ + \bar{F}_- F_-}$$

where now the D, F - field equations become

$$\boxed{\begin{aligned} & D = \xi + \tfrac{g}{\sqrt{2}}(\bar{A}_+ A_+ - A_- \bar{A}_-) \\ & F_+ = m\bar{A}_- \quad , \quad \bar{F}_+ = m A_- \\ & F_- = m\bar{A}_+ \quad , \quad \bar{F}_- = m A_+ \end{aligned}}$$

SUSY remains unbroken only if $V = 0 \Rightarrow$ $\langle 0|D|0\rangle = 0 = \langle 0|F_\pm|0\rangle$.

Hence $\langle 0|D|0\rangle = 0 = \xi + \frac{g}{\sqrt{2}}(|A_+|^2 - |A_-|^2)$

BUT $\langle 0|F_\pm|0\rangle = 0 \Rightarrow \langle 0|A_\pm|0\rangle = 0$

So as long as $\xi \neq 0$, SUSY is spontaneously broken.

Writing out the potential

$$V = \tfrac{1}{2}D^2 + |F_+|^2 + |F_-|^2$$

$$= \tfrac{1}{2}\left(\xi + \tfrac{g}{\sqrt{2}}(|A_+|^2 - |A_-|^2)\right)^2 + m^2|A_-|^2 + m^2|A_+|^2$$

$$\boxed{V = \frac{1}{2}\xi^2 + \left(m^2 + \frac{g}{\sqrt{2}}\xi\right)|A_+|^2 + \left(m^2 - \frac{g}{\sqrt{2}}\xi\right)|A_-|^2 + \frac{g^2}{4}\left(|A_+|^2 - |A_-|^2\right)^2}$$

Now we have 2 cases

$$m^2 > \frac{g}{\sqrt{2}}\xi \quad \text{and} \quad m^2 < \frac{g}{\sqrt{2}}\xi$$

Case 1: $m^2 > \frac{g}{\sqrt{2}}\xi$: thus $\langle 0|A_\pm|0\rangle = 0$

minimizes the potential since it is the sum of squares and $\langle 0|D|0\rangle = \xi$.

Then again SUSY is broken and the $A_\pm$ have real masses with equal but opposite splitting while the $\psi_\pm$ remain at m^2!

$$\frac{m^2 + \frac{g}{\sqrt{2}}\xi}{} \quad A_+ = \frac{1}{2}(\mathcal{A}_+ + iB_-)$$

$$\frac{m^2}{} \quad \psi \to \psi_+, \psi_-$$

$$\frac{m^2 - \frac{g}{\sqrt{2}}\xi}{} \quad A_- = \frac{1}{2}(\mathcal{A}_- - iB_-)$$

$$\frac{0}{} \quad A^\mu, \lambda$$

As earlier $\underset{\text{fermions}}{2\sum_i m_f^2} - \underset{\text{scalars}}{\sum_i m_s^2} = 0$

The photon A^μ and photino λ remain massless — however for different reasons. A^μ still mediates the unbroken U(1) gauge interaction — hence it remains massless. λ becomes the Goldstino associated with the spontaneous breakdown of SUSY. Recall

$$i\{Q_\alpha, \lambda_\beta\} = \epsilon_{\alpha\beta} D \rightarrow \underbrace{\epsilon_{\alpha\beta}\xi}_{\text{constant.}} + \epsilon_{\alpha\beta} D$$

Thus
$$\langle 0| Q_\alpha \lambda_\beta + \lambda_\beta Q_\alpha |0\rangle = -i\epsilon_{\alpha\beta}\langle 0|D|0\rangle$$
$$= -i\epsilon_{\alpha\beta}\xi \neq 0$$

hence $Q_\alpha|0\rangle \neq 0$ and λ is the Goldstone fermion (it transforms inhomogenously when $\langle D\rangle \neq 0$).

Case 2: $m^2 < \frac{g}{\sqrt{2}}\xi$: The potential's minimum is found from

$$\frac{\partial V}{\partial A_+} = 0 = \left(m^2 + \frac{g}{\sqrt{2}}\xi\right)A_+ + \frac{g^2}{2}\left(|A_+|^2 - |A_-|^2\right)A_+$$

$$\frac{\partial V}{\partial A_-} = 0 = \left(m^2 - \frac{g}{\sqrt{2}}\xi\right)A_- - \frac{g^2}{2}\left(|A_+|^2 - |A_-|^2\right)A_-$$

So the first equation $\Rightarrow \langle 0|A_+|0\rangle = 0$ while the second $\Rightarrow \langle 0|A_-|0\rangle = v$ where

$$\boxed{\frac{g^2}{2}v^2 = -\left(m^2 - \frac{g}{\sqrt{2}}\xi\right)}$$ yields the minimum.

Now we expand the action about this minimum

$$A_+ \to A_+ \ , \ A_- \to A_- + v$$

$\Rightarrow$

$$F_+ \to F_+ + mv \ , \ F_- \to F_- \ , \ D \to D + \xi - \frac{gv^2}{\sqrt{2}}$$

The mass terms become

$$\Gamma_{mass} = \int d^4x \Big[m\left(\psi_+\psi_- + \bar{\psi}_+\bar{\psi}_-\right) + \frac{gv}{2}\left(\lambda\psi_- + \bar{\lambda}\bar{\psi}_-\right)$$

$$+ \frac{g^2v^2}{4}A_\mu A^\mu + \left(m^2 + \frac{g}{\sqrt{2}}\xi\right)|A_+|^2$$

$$+ \left(m^2 - \frac{g}{\sqrt{2}}\xi\right)|A_-|^2 + \frac{g^2v^2}{4}\Big[(A_-^\dagger + A_-)^2 + 2\left(|A_-|^2 - |A_+|^2\right)\Big]\Big]$$

So we see the photon has become massive. The $U(1)$ gauge symmetry is broken. In addition $V > 0$ so the SUSY is broken. The massless Goldstone fermion is a linear combination of ψ_- and λ while the orthogonal combination is massive as is ψ_+.

One scalar field will have mass degenerate with A_μ while one complex scalar field will have mass^2 m^2.

To summarize case 2: SUSY is broken when an auxiliary field gets a vacuum value. The gauge symmetry is broken when a non-singlet A-field also gets a vacuum value.

Finally we would like to consider a model in which SUSY remains unbroken while the gauge symmetry is broken. To do this we must add a neutral chiral field ϕ_0 to our SQED model with action

$$\Gamma = \Gamma_{SQED} + \frac{1}{16}\int dV\, \phi_0 \bar{\phi}_0 + \frac{\xi}{4}\int dV\, V$$

$$+ \int dS \left[\frac{\mu}{4}\phi_0^2 + \frac{1}{4} y\, \phi_0 \phi_+ \phi_- + \frac{\lambda}{4}\phi_0^3 + \frac{f}{4}\phi_0 \right]$$

$$+ \int d\bar{S} \left[\frac{\mu}{4}\bar{\phi}_0^2 + \frac{1}{4} y^* \bar{\phi}_0 \bar{\phi}_+ \bar{\phi}_- + \frac{\lambda^*}{4}\bar{\phi}_0^3 + \frac{f^*}{4}\bar{\phi}_0 \right]$$

The auxiliary D, F field equations become

$$D = \xi + \frac{g}{\sqrt{2}}\left[|A_+|^2 - |A_-|^2 \right]$$

$$\bar{F}_+ = mA_- + y A_0 A_-$$

$$\bar{F}_- = mA_+ + y A_0 A_+$$

$$\bar{F}_0 = f + yA_+A_- + 2\mu A_0 + 3\lambda A_0^2$$

For unbroken SUSY we must have $\langle D \rangle = 0$ $\langle F_\pm \rangle = 0$. Thus we have 4 equations for $A_\pm$ and A_0: There are 2 solutions

i) If $\xi = 0$, then $A_+ = A_- = 0$ and

$$\text{and } 3\lambda A_0^2 + 2\mu A_0 + f = 0$$

Since ϕ_0 is a $U(1)$ singlet, this solution breaks neither SUSY nor gauge invariance: we just shift A_0 to re-define the existing parameters.

2) If $\xi \neq 0$, then $A_0 = -\frac{m}{y}$

$$\text{and } A_+A_- = -\frac{1}{y}\left[f + 2\mu A_0 + 3\lambda A_0^2\right]$$

$$= -\frac{1}{y}\left[f - \frac{2m\mu}{y} + 3\frac{\lambda m^2}{y^2}\right].$$

This satisfies the last 3 equations But equation 1 can be satisfied since the last 3 equations are really invariant under a complex extension of $U(1)$

$A_\pm \to e^{\pm\alpha} A_\pm = A'_\pm$ where α is complex

The potential that determines the vacuum has a higher symmetry than just $U(1)$.

So if $A_+ A_-$ satisfies the $F_0 = 0$ equation

$$f + y A_+ A_- + 2\mu A_0 + 3\lambda A_0^2 = 0$$

so does $A'_+ A'_-$. Equation $D = 0$ then becomes

$$0 = \xi + \frac{g}{\sqrt{2}}\left[|A_+|^2 - |A_-|^2 \right]$$

$$= \xi + \frac{g}{\sqrt{2}}\left[|A'_+|^2 e^{-(\alpha+\alpha^*)} - |A'_-|^2 e^{+(\alpha+\alpha^*)} \right]$$

Now $(\alpha + \alpha^*)$ can be adjusted so that this equation is always satisfied! Hence this D-term can be transformed away and <u>does not</u> catalyze a SUSY breakdown.

As usual we expand the action about the minimum to find the new mass terms $A_\pm \to a_\pm + A_\pm$; $A_0 \to a_0 + A_0$ we find

i.e. $\phi_\pm \to a_\pm + \phi_\pm$; $\phi_0 \to a_0 + \phi_0$

In superfields (since SUSY is unbroken)

$$\Gamma = \frac{Z}{g^2}\int dS\, W^\alpha W_\alpha + \frac{Z}{g^2}\int d\bar{S}\, \bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}$$

$$+\int dV \frac{1}{16}\left[\bar{\phi}_+\phi_- e^{gV} + \bar{\phi}_-\phi_- e^{-gV} + \phi_0\bar{\phi}_0\right]$$

$$+\int dV \frac{1}{16}\Big[\bar{\phi}_+(e^{gV}-1)a_+ + a_+^*(e^{gV}-1)\phi_+$$

$$+\bar{\phi}_-(e^{-gV}-1)a_- + a_-^*(e^{-gV}-1)\phi_-$$

$$+a_+^*a_+(e^{gV}-1-gV) + a_-^*a_-(e^{-gV}-1+gV)\Big]$$

$$+\Big\{\int dS\Big[\left(\frac{m}{4}+\frac{y}{4}a_0\right)\phi_+\phi_- + \frac{1}{4}y\phi_0\phi_+\phi_- + \frac{\lambda}{4}\phi_0^3$$

$$+\left(\frac{\mu}{4}+3\lambda a_0\right)\phi_0^2 + \frac{y}{4}a_+\phi_0\phi_- + \frac{y}{4}a_-\phi_0\phi_+\Big]$$

$$+h.c.\Big\}$$

So the vector field gets a mass term

$\frac{1}{16}\int dV \frac{g^2}{2}\left(|a_+|^2+|a_-|^2\right)V^2$; The $U(1)$

gauge symmetry is broken.

The ϕ_0, ϕ_+, ϕ_- mass matrix has a zero eigenvalue corresponding to the Goldstone boson superfield. The scalar field is eaten to give A^μ a mass, while the pseudoscalar and fermion become the new massive degrees of freedom in V— This is the Super Higgs mechanism — a massive vector superfield has the C & χ fields becoming dynamic —

$$\int dV\, V^2 = 8\int d^4x \left[CD + \chi\lambda \quad \bar{\chi}\bar{\lambda} + MM^\dagger - A_\mu A^\mu \right]$$

A massive vector multiplet describes a massive vector A^μ, 2 spin $\frac{1}{2}$ fermions λ and χ and a scalar C.

For non-abelian gauge groups the situation is similar. The linear chiral terms must be made to vanish, $\langle 0|F_a|0\rangle = 0$. When the gauge group does not contain an invariant U(1) the linear V term must also vanish, $\langle 0|D_i|0\rangle = 0$; that is

$$A_a^\dagger T^i_{ab} A_b = 0 \text{ . These}$$

solutions for non-zero A_a determine the moduli space of the theory.