Hence we obtained the superpropagators as before from the component build up.

Now we would like to convert the Gell-Mann-Low expansion into an expansion in terms of superspace Feynman diagrams. We have found the free field generating functional

$$Z_0\{J,\bar{J}\} = e^{-\frac{1}{2}\int dS_1 \int dS_2 \, J(1)\Delta_{12} J(2)} \times e^{-\frac{1}{2}\int d\bar{S}_1 \int d\bar{S}_2 \, \bar{J}(1)\Delta_{\bar{1}\bar{2}} \bar{J}(2)} \times e^{-\frac{1}{2}\int dS_1 \int d\bar{S}_2 \, J(1)\Delta_{1\bar{2}} \bar{J}(2)} \times e^{-\frac{1}{2}\int d\bar{S}_1 \int dS_2 \, \bar{J}(1)\Delta_{\bar{1}2} J(2)}$$

where the Feynman propagators are given by the free 2-pt. functions we just found.

$$\Delta_{12} \equiv \langle 0 | T \phi(1) \phi(2) | 0 \rangle$$

$$\Delta_{\bar{1}\bar{2}} \equiv \langle 0 | T \bar{\phi}(1) \bar{\phi}(2) | 0 \rangle$$

$$\Delta_{1\bar{2}} \equiv \langle 0 | T \phi(1) \bar{\phi}(2) | 0 \rangle$$

$$\Delta_{\bar{1}2} \equiv \langle 0 | T \bar{\phi}(1) \phi(2) | 0 \rangle$$

The G-M-L expansion is then given by

$$Z[J,\bar{J}] = e^{\Gamma_{int}\left[\frac{\delta}{i\delta J}, \frac{\delta}{i\delta \bar{J}}\right]} Z_0[J,\bar{J}]$$

Or in terms of the interaction picture free fields the Heisenberg picture Green's function is given in terms of these free fields by the same G-M-L expansion

$$\boxed{\begin{aligned}&\langle 0|T\,\phi(1)\cdots\phi(n)\bar{\phi}(\bar{1})\cdots\bar{\phi}(\bar{n})|0\rangle\\ &= \langle 0\,in|T\,\phi_{in}(1)\cdots\bar{\phi}_{in}(\bar{n})\, e^{\Gamma_{int}[\phi_{in},\bar{\phi}_{in}]}|0\,in\rangle\end{aligned}}$$

(where the vacuum bubbles are already factored out)
(which in SUSY vanish anyway (check))

Now we can apply Wick's theorem to the G-M-L expansion to obtain a Feynman diagram expansion in coordinate superspace

For example consider

$$\langle 0|T\,\phi(1)\bar{\phi}(2)|0\rangle$$

$$= \langle 0_{in}|T\phi_{in}(1)\bar{\phi}_{in}(2)\Big[1 + \frac{ig}{12}\int dS_3\,\phi^3_{in}(3) + \frac{ig}{12}\int d\bar{S}_3\,\bar{\phi}^3_{in}(3) +$$

$$+ \frac{1}{2!}2\frac{ig}{12}\int dS_3\,\phi^3_{in}(3)\left(\frac{ig}{12}\right)\int d\bar{S}_4\,\bar{\phi}^3_{in}(4) + \cdots\Big]|0_{in}\rangle$$

So we have in particular the terms

$$\langle 0|T\,\phi(1)\,\phi(2)|0\rangle = \langle 0_{in}|T\,\phi_{in}(1)\,\phi_{in}(2)|0_{in}\rangle$$

$$+\ \frac{(ig)^2}{(12)^2}\int dS_3\int dS_4\,\langle 0_{in}|T\,\phi_{in}(1)\,\phi_{in}(2)\,\phi^3_{in}(3)\,\phi^3_{in}(4)|0_{in}\rangle$$

Applying Wick's theorem to evaluate the free field timeordered function we find all contractions of the free fields — in particular we have the non-zero contraction

$$\langle 0|T\,\phi(1)\,\phi(2)|0\rangle = \langle 0_{in}|T\,\phi_{in}(1)\,\phi_{in}(2)|0_{in}\rangle$$

$$+\left(\frac{ig}{12}\right)^2\int dS_3\int dS_4\,\langle 0|T\,\phi_{in}(1)\,\phi_{in}(3)\,\phi_{in}(3)\,\phi_{in}(3)\,\phi_{in}(4)\,\phi_{in}(4)\,\phi_{in}(4)\,\phi_{in}(2)|0_{in}\rangle$$

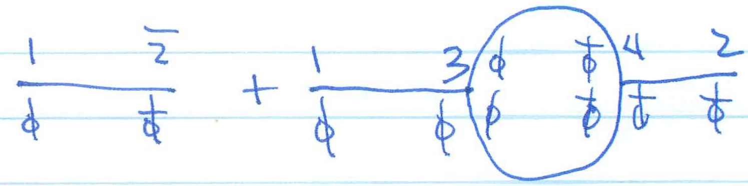
$3!\cdot 3$

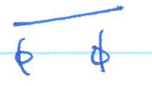
This can happen $3\cdot 3\cdot 2$ ways

$$= \Delta_{\phi\phi}(1,2) + \frac{1}{2}\left(\frac{ig}{2}\right)^2\int dS_3\int dS_4\,\Delta_{\phi\phi}(1,3)\,\Delta_{\phi\phi}(1,2)\cdot$$

$$\cdot\,\Delta_{\phi\phi}(3,4)\,\Delta_{\phi\phi}(3,4)$$

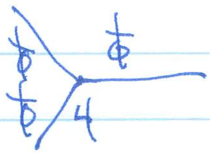
So we can associate a superspace Feynman diagram with this

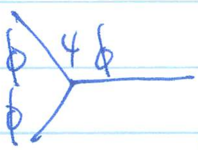
$\langle 0|T\,\phi(1)\bar\phi(2)|0\rangle =$ 

with the rules for propagators $\Delta_{\phi\phi} \longleftrightarrow$ 

$\Delta_{\bar\phi\bar\phi} \longleftrightarrow$ 

$\Delta_{\phi\bar\phi} \longleftrightarrow$ 

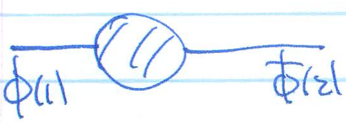
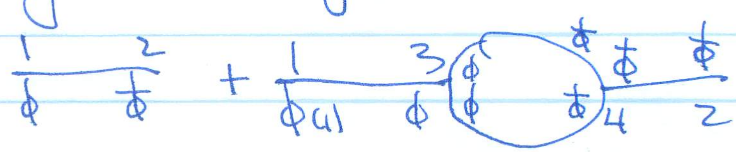
and vertices  $\longrightarrow \frac{ig}{2}\int d\bar{S}_4$

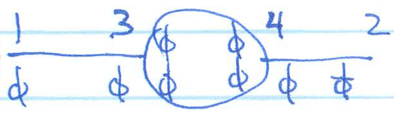
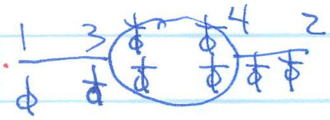
 $\longrightarrow \frac{ig}{2}\int dS_4$

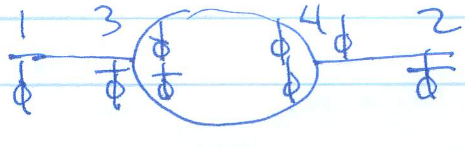
and a symmetry number — in this case $\left(\frac{1}{2}\right)$

So we must integrate over the superspace coordinate of each interaction vertex according to its nature, chiral, anti-chiral or vector.

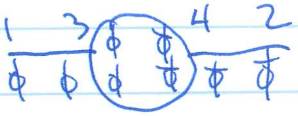
So we see we obtain in one-loop a sum over several Feynman diagrams

 $=$ 

$+2\cdot$  $+2\cdot$ 

$+$ 

Returning to the special case of graph



$$= \frac{1}{2}\left(\frac{ig}{2}\right)^2 \int dS_3\, d\bar S_4\, \Delta_{\phi\phi}(1,3)\, \Delta_{\phi\bar\phi}(4,2)\, \Delta^2_{\phi\bar\phi}(3,4)$$

$$= \frac{1}{2}\left(\frac{ig}{2}\right)^2 \int \frac{d^4p_1}{(2\pi)^4} \int \frac{d^4p_2}{(2\pi)^4} \int dS_3 \int d\bar S_4\, e^{-ip_1(x_1-x_3)}\, e^{-ip_2(x_2-x_4)} \times$$

$$\times\, \Delta_{\phi\phi}(p_1,1,3)\, \Delta_{\phi\bar\phi}(p_2,2,4) \int \frac{d^4k}{(2\pi)^4}\, \Delta_{\phi\bar\phi}(k,3,4)\, e^{-ik(x_3-x_4)}$$

$$\int \frac{d^4l}{(2\pi)^4}\, e^{-il(x_3-x_4)}\, \Delta_{\phi\bar\phi}(l,3,4)$$

$$= \frac{1}{2}\left(\frac{ig}{2}\right)^2 \int \frac{d^4p_1}{(2\pi)^4} \frac{d^4p_2}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \int d^4x_3 \int d^4x_4\, \frac{\partial^2}{\partial\theta_3^2} \frac{\partial^2}{\partial\bar\theta_4^2} \times$$

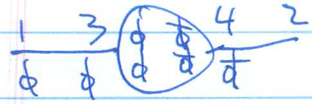
$$\times\, e^{-ip_1x_1}\, e^{-ip_2x_2}\, e^{-i(k-p_1+l)x_3}\, e^{+i(l+k+p_2)x_4} \times$$

$$\times\, \Delta_{\phi\phi}(p_1,1,3)\, \Delta_{\bar\phi\bar\phi}(p_2,2,4)\, \Delta_{\phi\bar\phi}(k,3,4)\, \Delta_{\phi\bar\phi}(l,3,4)$$

$$= \frac{1}{2}\left(\frac{ig}{2}\right)^2 \int \frac{d^4p_1}{(2\pi)^4} \frac{d^4p_2}{(2\pi)^4}\, e^{-ip_1x_1}\, e^{-ip_2x_2} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \frac{\partial^2}{\partial\theta_3^2} \frac{\partial^2}{\partial\bar\theta_4^2}$$

$$(2\pi)^4\delta^4(k+l-p_1)\, \underbrace{(2\pi)^4\delta^4(k+l+p_2)}_{=(2\pi)^4\delta^4(p_1+p_2)} \times$$

$$\times\, \Delta_{\phi\phi}(p_1,1,3)\, \Delta_{\bar\phi\bar\phi}(p_2,2,4)\, \Delta_{\phi\bar\phi}(k,3,4)\, \Delta_{\phi\bar\phi}(p_1-k,3,4)$$



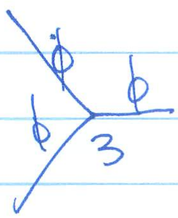
$$= \frac{1}{2}\left(\frac{ig}{2}\right)^2 \int \frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}\, e^{-ip_1x_1}\, e^{-ip_2x_2} \times$$

$$\times (2\pi)^4\delta^4(p_1+p_2)\int\frac{d^4k}{(2\pi)^4}\frac{\partial^2}{\partial\theta_3^2}\frac{\partial^2}{\partial\bar\theta_4^2}\Delta_{\phi\phi}(p_1,1,3)\times$$

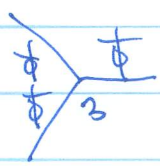
$$\times \Delta_{\bar\phi\bar\phi}(p_2,2,4)\,\Delta_{\phi\bar\phi}(k,3,4)\,\Delta_{\phi\bar\phi}(p_1-k,3,4)$$

So we can obtain the Feynman Rules in momentum space —

Vertices

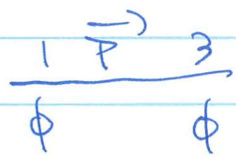


$$\longrightarrow \frac{\partial^2}{\partial\theta_3^2}\left(\frac{ig}{2}\right)$$

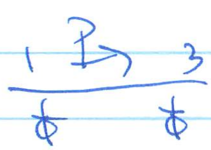


$$\longrightarrow \frac{ig}{2}\frac{\partial^2}{\partial\bar\theta_3^2}$$

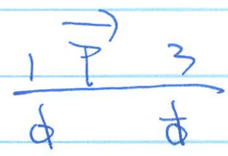
Lines: use momentum space propagators



$$\longrightarrow \frac{im}{p^2-m^2}\theta_{13}^2\, e^{-\theta_1 \not p\, \bar\theta_{13}}$$



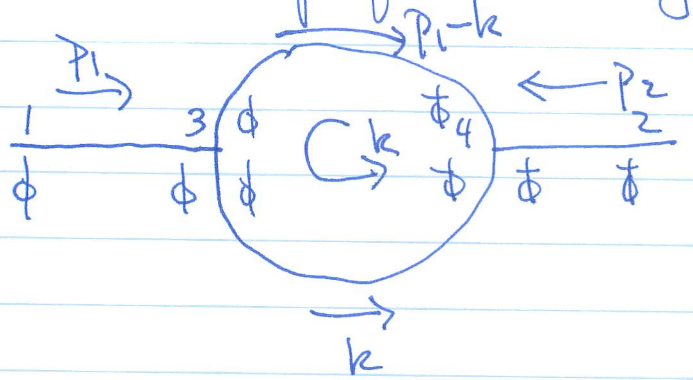
$$\longrightarrow \frac{im}{p^2-m^2}\bar\theta_{13}^2\, e^{+\theta_{13}\not p\,\bar\theta_1}$$



$$\longrightarrow \frac{i}{p^2-m^2}\, e^{-\theta_1\not p\,\bar\theta_{13}+\theta_{13}\not p\,\bar\theta_3}$$

$\int\frac{d^4k}{(2\pi)^4}$ for each loop & momentum conservation at each vertex, etc.

So the above Feynman integral corresponds to the superfield diagram

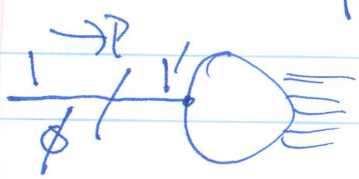


Now as usual connected Green's functions are built up from propagators connecting 1-PI diagrams. Recall that

$$\int dS_3\, \Gamma_{\phi\phi}(1,3)\, \Delta_{\phi\phi}(3,2) + \int d\bar S_3\, \Gamma_{\phi\bar\phi}(1,3)\, \Delta_{\bar\phi\phi}(3,2) = -\delta_S(1,2)$$

$$\int dS_3\, \Gamma_{\phi\phi}(1,3)\, \Delta_{\phi\bar\phi}(3,2) + \int d\bar S_3\, \Gamma_{\phi\bar\phi}(1,3)\, \Delta_{\bar\phi\bar\phi}(3,2) = 0$$

So when a chiral line 1-PI function is desired we must amputate the external "chiral field"



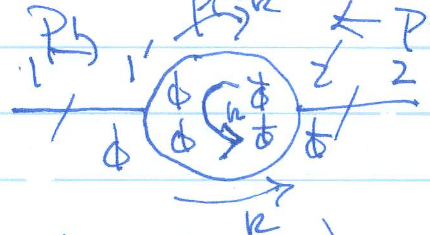
$\longrightarrow \tilde\delta_S(p,1,1')$

So for 1PI functions the Feynman rules include a super delta function for each amputated external line — a chiral δ-function for a chiral field, an anti-chiral δ-function for an anti-chiral field (and vector for a vector)

So we have

(free field)

$$\langle 0|T\,\phi(1)\,\bar\phi(2)|0\rangle^{1PI} = \overbrace{+i\bar D_1\bar D_1 D_2 D_2\,\delta_V(1,2)\left(\frac{Z}{16}\right)}^{=\Gamma_{\phi\bar\phi}(1,2)}$$

$+$ 

$$= +i\bar D_1\bar D_1 D_2 D_2\,\delta_V(1,2)\left(\frac{1+b}{16}\right)$$

$$+\int\frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}\,e^{-ip_1x_1}\,e^{-ip_2x_2}\,(2\pi)^4\delta^4(p_1+p_2)\cdot\frac{1}{2}\left(\frac{ig}{2}\right)^2\times$$

$$\frac{\partial^2}{\partial\theta_1'^2}\frac{\partial^2}{\partial\theta_2'^2}\,\tilde\delta_S(p_1,1,1')\,\tilde\delta_{\bar S}(p_2,2,2')\times\int\frac{d^4k}{(2\pi)^4}$$

$$\times\,\Delta_{\phi\bar\phi}(p_1-k,1',2')\,\Delta_{\phi\bar\phi}(+k,1',2')$$

The one-loop, 1PI function is

$$\boxed{\begin{aligned}\langle 0|T\,\phi(1)\,\bar\phi(2)|0\rangle^{1PI} &= +\frac{iZ}{16}\int\frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}\,e^{-ip_1x_1}\,e^{-ip_2x_2}\times\\ &\times(2\pi)^4\delta^4(p_1+p_2)\,e^{-\theta_1\not{p}_1\bar\theta_{12}}\,e^{-\theta_{12}\not{p}_2\bar\theta_2}\\ &+\frac{1}{2}\left(\frac{ig}{2}\right)^2\int\frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}\,e^{-ip_1x_1}\,e^{-ip_2x_2}(2\pi)^4\delta^4(p_1+p_2)\times\\ &\times\frac{\partial^2}{\partial\theta_1'^2}\frac{\partial^2}{\partial\bar\theta_2'^2}\,\tilde\delta_S(p_1,1,1')\,\tilde\delta_{\bar S}(p_2,2,2')\times\\ &\qquad\int\frac{d^4k}{(2\pi)^4}\Delta_{\phi\bar\phi}(p_1-k,1',2')\,\Delta_{\phi\bar\phi}(k,1',2')\end{aligned}}$$

$\Rightarrow$

$$\boxed{\begin{aligned}&\langle 0|T\,\tilde\phi(p_1,1)\,\bar\phi(0,2)|0\rangle^{1PI}\\ &= +\frac{iZ}{16}\,e^{-\theta_1\not{p}_1\bar\theta_{12}+\theta_{12}\not{p}_1\bar\theta_2}\\ &+\frac{1}{2}\left(\frac{ig}{2}\right)^2\frac{\partial^2}{\partial\theta_1'^2}\frac{\partial^2}{\partial\bar\theta_2'^2}\,\tilde\delta_S(p_1,1,1')\,\tilde\delta_{\bar S}(-p_1,2,2')\\ &\qquad\int\frac{d^4k}{(2\pi)^4}\Delta_{\phi\bar\phi}(p_1-k,1',2')\,\Delta_{\phi\bar\phi}(k,1',2')\end{aligned}}$$

Now turn to earlier work for propagators & δ-functions

p. -278-

$$\tilde{\delta}_5(p,1,1') = -\frac{1}{4}\theta_{11'}^2 e^{-\theta_1 \not{p} \bar{\theta}_{11'}}$$

$$\tilde{\delta}_{\bar{5}}(p,2,2') = -\frac{1}{4}\bar{\theta}_{22'}^2 e^{+\theta_{22'} \not{p} \bar{\theta}_2}$$

$\Rightarrow$

$$\langle 0 | T\, \Phi(p_1,1)\, \bar{\Phi}(0,2) | 0 \rangle^{1PI} = +\frac{iZ}{16} e^{-\theta_1 \not{p}_1 \bar{\theta}_{12} + \theta_{12} \not{p}_1 \bar{\theta}_2}$$

$$+ \frac{1}{2}\left(\frac{ig}{2}\right)^2 \frac{\partial^2}{\partial \theta_1'^2} \frac{\partial^2}{\partial \bar{\theta}_2'^2} \Bigg[\left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right) \theta_{11'}^2 \bar{\theta}_{22'}^2 e^{-\theta_1 \not{p}_1 \bar{\theta}_{11'}}$$

$$\int \frac{d^4k}{(2\pi)^4} e^{-\theta_{22'} \not{p}_1 \bar{\theta}_2} \left(\frac{i}{(p_1-k)^2 - m^2} e^{-\theta_1' (\not{p}_1 - \cancel{\not{k}}) \bar{\theta}_{1'2'}} \times e^{+\theta_{1'2'} (\not{p}_1 - \cancel{\not{k}}) \bar{\theta}_2'} \right) \times$$

$$\times \left(\frac{i}{k^2 - m^2} e^{\cancel{-\theta_1' \not{k} \bar{\theta}_{1'2'}} + \cancel{\theta_{1'2'} \not{k} \bar{\theta}_2'}} \right) \Bigg]$$

$$= +\frac{iZ}{16} e^{-\theta_1 \not{p}_1 \bar{\theta}_{12} + \theta_{12} \not{p}_1 \bar{\theta}_2}$$

$$+ \frac{1}{2}\left(\frac{ig}{2}\right)^2 \frac{\partial^2}{\partial \theta_1'^2} \frac{\partial^2}{\partial \bar{\theta}_2'^2} \Bigg[\left(-\frac{1}{4}\right)^2 \theta_{11'}^2 \bar{\theta}_{22'}^2 \times$$

$$\times e^{-\theta_1 \not{p}_1 \bar{\theta}_{11'} - \theta_{22'} \not{p}_1 \bar{\theta}_2} \times$$

$$\times e^{-\theta_1 \not{p}_1 \bar{\theta}_{1'2}} e^{\theta_{12'} \not{p}_1 \bar{\theta}_2} \int \frac{d^4k}{(2\pi)^4} \frac{i}{(p_1-k)^2 - m^2} \frac{i}{k^2 - m^2} \Bigg]$$

$\theta_1 = \theta_1'$

$\bar{\theta}_2 = \bar{\theta}_2'$

Now note
$$e^{-\theta_1 \not p_1 \bar\theta_{11'}}\, e^{-\theta_1 \not p_1 \bar\theta_{1'2}}$$
$$= e^{-\theta_1 \not p_1 [\bar\theta_1 \cancel{-\bar\theta_1'} \cancel{+\bar\theta_1'} - \bar\theta_2]}$$
$$= e^{-\theta_1 \not p_1 \bar\theta_{12}}$$
likewise $e^{-\theta_{22'} \not p_1 \bar\theta_2}\, e^{\theta_{12'} \not p_1 \bar\theta_2} = e^{\theta_{12} \not p_1 \bar\theta_2}$

So

$$\langle 0 | T\, \Phi(p_1, 1)\, \Phi(0, 2) | 0 \rangle^{1PI}$$
$$= e^{-\theta_1 \not p_1 \bar\theta_{12}}\, e^{\theta_{12} \not p_1 \bar\theta_2} \Big[\underbrace{+\frac{iZ}{16}}_{=1}$$
$$+ \frac{1}{2}\left(\frac{ig}{2}\right)^2 \frac{\partial^2}{\partial \theta_1'^2} \frac{\partial^2}{\partial \bar\theta_2'^2} \left[\left(-\frac{1}{4}\right)^2 \theta_{11'}^2\, \theta_{22'}^2 \right] \times$$
$$\times \int \frac{d^4k}{(2\pi)^4} \frac{i}{(p-k)^2 - m^2} \frac{i}{k^2 - m^2}$$

$$\boxed{\begin{aligned} &\langle 0 | T\, \Phi(p_1, 1)\, \Phi(0, 2) | 0 \rangle^{1PI} \\ &= e^{-\theta_1 \not p_1 \bar\theta_{12}}\, e^{\theta_{12} \not p_1 \bar\theta_2} \Big[+\frac{iZ}{16} \\ &\qquad + \frac{1}{2}\frac{g^2}{4} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m^2)[(p_1+k)^2 - m^2]} \Big] \end{aligned}}$$

$(k \to -k)$

So

$$\boxed{\begin{aligned}\langle 0|T\,\hat{\Phi}(p_1,1)\,\Phi(0,2)|0\rangle^{1PI} &= +\frac{i}{16}\,e^{-\theta_1 \not{p}_1 \bar{\theta}_{12}}\,e^{\theta_{12}\not{p}_1\bar{\theta}_2}\,\times \\ &\times\left[Z - 2ig^2\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m^2)[(p_1+k)^2-m^2]}\right]\end{aligned}}$$

Recall p.-346- & p.-347-

$$\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m^2)[(p_1+k)^2-m^2]} = \int_0^1 d\alpha\int\frac{d^4\ell}{(2\pi)^4}\frac{1}{[\ell^2+\alpha(1-\alpha)p^2-m^2]^2}$$

$$= \int_0^1 d\alpha\,\frac{i}{(4\pi)^{d/2}}\,\frac{\Gamma(2-d/2)}{\Gamma(2)}\left(\frac{1}{\underbrace{-\alpha(1-\alpha)p^2+m^2}_{\equiv\Delta}}\right)^{2-d/2}$$

$$= \int_0^1 d\alpha\,\frac{i}{(4\pi)^{d/2}}\,\Gamma(2-d/2)\,e^{-(2-d/2)\ln\Delta} \qquad (d=4-\epsilon)$$

$\Rightarrow$

$$\boxed{\begin{aligned}\langle 0|T\,\hat{\Phi}(p_1,1)\,\Phi(0,2)|0\rangle^{1PI} &= +\frac{i}{16}\,e^{-\theta_1 \not{p}_1 \bar{\theta}_{12}}\,e^{\theta_{12}\not{p}_1\bar{\theta}_2}\,\times \\ &\times\left[Z + 2g^2\int_0^1 d\alpha\,\frac{\Gamma(\epsilon/2)}{(4\pi)^{d/2}}\,e^{-\frac{\epsilon}{2}\ln\Delta}\right]\end{aligned}}$$

Now $\int_0^1 d\alpha\, f(\alpha(1-\alpha)) = \int_0^1 d\alpha\,[\alpha+(1-\alpha)]\,f(\alpha(1-\alpha))$

So this is $\alpha\leftrightarrow(1-\alpha)$ symmetric. in second term let $(1-\alpha)=\beta$; $\alpha=1-\beta \Rightarrow$

$$\int_0^1 d\alpha\, f(\alpha(1-\alpha)) = 2\int_0^1 d\alpha\, \alpha\, f((1-\alpha)\alpha)$$

$$\Rightarrow \boxed{\int_0^1 d\alpha\, \alpha\, f(\alpha(1-\alpha)) = \frac{1}{2}\int_0^1 d\alpha\, f(\alpha(1-\alpha))}$$

Now then

$$\boxed{\begin{aligned}\langle 0|T\,\Phi(p_1,1)\,\bar{\Phi}(0,2)|0\rangle^{1PI} &= \frac{+i}{16} e^{-\theta_1 \not{p}_1 \bar{\theta}_{12}}\, e^{\theta_{12}\not{p}_1\bar{\theta}_2} \times \\ &\times \left[Z + 4g^2 \int_0^1 d\alpha\,\alpha \frac{\Gamma(\epsilon/2)}{(4\pi)^{2-\epsilon/2}}\, e^{-\frac{\epsilon}{2}\ln[m^2-\alpha(1-\alpha)p^2]}\right]\end{aligned}}$$

Now using the pole "residue" normalization condition

$$\frac{\partial}{\partial p^2}\langle 0|T\,D_1D_1\Phi(p,1)\,\bar{D}_2\bar{D}_2\bar{\Phi}(0,2)|0\rangle^{1PI}\Big|_{\substack{p^2=-\mu^2\\ \theta_1=0=\theta_2\\ \bar{\theta}_1=0=\bar{\theta}_2}} = i$$

$$= iZ + i\frac{\partial}{\partial p^2}\left[p^2\, 4g^2\int_0^1 d\alpha\,\alpha\frac{\Gamma(\epsilon/2)}{(4\pi)^2}\, e^{-\frac{\epsilon}{2}\ln[m^2-\alpha(1-\alpha)p^2]}\right]\Bigg|_{p^2=-\mu^2}$$

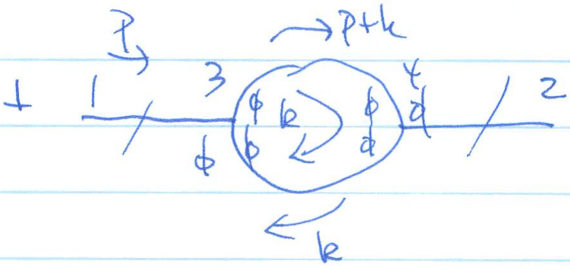
Recall $Z = 1+b \Rightarrow$

$$\boxed{\begin{aligned}Z = 1+b = 1 - \frac{g^2}{(2\pi)^2}\int_0^1 d\alpha\,\alpha\,\Gamma\left(\frac{\epsilon}{2}\right) e^{-\frac{\epsilon}{2}\ln[m^2+\alpha(1-\alpha)\mu^2]} \times \\ \times\left[1 - \frac{\epsilon}{2}\frac{\alpha(1-\alpha)\mu^2}{m^2+\alpha(1-\alpha)\mu^2}\right]\end{aligned}}$$

Exactly the results of p. -348- obtained in components.

Next, let's consider the pure chiral 2pt-function

$$\langle 0|T\,\Phi(p,1)\,\phi(0,2)|0\rangle^{1PI} = \frac{\delta^2\Gamma_0}{\delta\Phi(p,1)\,\delta\phi(0,2)} + \overset{1}{\underset{\phi}{/}}\overset{3}{\times}\overset{2}{\underset{\phi}{/}}$$

$+$ 

$$= \frac{i(m+a)}{4}\tilde{\delta}_S(p,1,2) + \int\frac{d^4k}{(2\pi)^4}\frac{\partial^2}{\partial\theta_3^2}\frac{\partial^2}{\partial\theta_4^2}\times\frac{1}{2}\left(\frac{ig}{2}\right)^2\times$$

$$\times\tilde{\delta}_S(p,1,3)\,\Delta_{\phi\phi}(p+k,3,4)\,\Delta_{\phi\phi}(-k,3,4)\,\tilde{\delta}_S(+p,4,2)$$

Now before we do anything we note that

$$\Delta_{\phi\phi}(p+k,3,4) = \frac{im}{(p+k)^2-m^2}\,\theta_{34}^2\,e^{-\theta_3(p+k)\bar{\theta}_{34}}$$

$$\Delta_{\phi\phi}(-k,3,4) = \frac{im}{k^2-m^2}\,\theta_{34}^2\,e^{+\theta_3 k\bar{\theta}_{34}}$$

we have $\theta_{34}^2\,\theta_{34}^2 = 0$ in the integrand!

The one-loop correction vanishes, hence to 1-loop

$$\boxed{\langle 0| T\, \tilde{\phi}(p,1)\,\phi(0,2)|0\rangle^{1PI} = \frac{i(m+a)}{4}\,\tilde{\delta}_S(p,1,2)} \quad \text{1-loop}$$

Likewise the pure anti-chiral 2pt. function has no 1-loop correction

$$\boxed{\langle 0| T\, \tilde{\bar{\phi}}(p,1)\,\bar{\phi}(0,2)|0\rangle^{1PI} = \frac{i(m+a)}{4}\,\tilde{\bar{\delta}}_S(p,1,2)} \quad \text{1-loop}$$

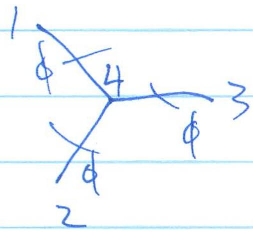
since

$\propto \bar{\theta}_{34}^2\, \theta_{34}^2 = 0.$

Further we have the pure chiral 3-pt. vertex

$$\langle 0| T\, \tilde{\phi}(p_1,1)\,\tilde{\phi}(p_2,2)\,\phi(0,3)|0\rangle^{1PI}$$

$=$



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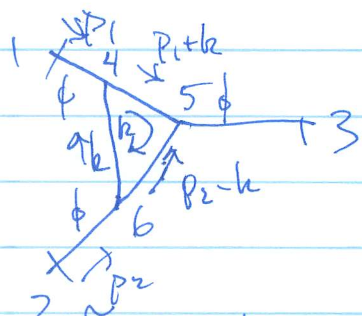

$$= Z_g \frac{ig}{2}\frac{\partial^2}{\partial\theta_4^2}\,\tilde{\delta}_S(\underset{p_1}{1},4)\,\tilde{\delta}_S(\underset{p_2}{2},4)\,\tilde{\delta}_S(\underset{p_3}{3},4)$$

$$+\frac{1}{2}\left(\frac{ig}{2}\right)^3\int\frac{d^4k}{(2\pi)^4}\frac{\partial^2}{\partial\theta_4^2}\frac{\partial^2}{\partial\theta_5^2}\frac{\partial^2}{\partial\theta_6^2}\,\tilde{\delta}_S(p_1,1,4)\,\tilde{\delta}_S(p_2,2,6)$$

$$\tilde{\delta}_S(-p_1-p_2,3,5)\,\Delta_{\phi\phi}(p_1+k,4,5)\,\Delta_{\phi\phi}(p_2-k,6,5)$$

$$\Delta_{\phi\phi}(k,6,4)$$

Once again the pure chiral propagators are proportional to θ-deltafunctions, so we have

$$\theta_{45}^2\,\theta_{56}^2\,\theta_{64}^2 = \theta_{45}^2\,\theta_{45}^2\,\theta_{46}^2 = 0\,!$$

The pure chiral vertex function has no one-loop corrections

$$\boxed{\langle 0|T\,\tilde{\Phi}(p_1,1)\,\tilde{\Phi}(p_2,2)\,\Phi(0,3)\,(0)\rangle^{1PI} = 2\frac{ig}{2}\,\tilde{\delta}_S(p_1,1,3)\,\tilde{\delta}_S(p_2,2,3)}$$

Likewise the pure anti-chiral vertex function has no radiative corrections in one-loop due to the loop of pure anti-chiral propagators

$$\Rightarrow\ \bar{\theta}_{45}^2\,\bar{\theta}_{56}^2\,\bar{\theta}_{64}^2 = \bar{\theta}_{45}^2\,\bar{\theta}_{45}^2\,\bar{\theta}_{46}^2 = 0.$$

$\Rightarrow$

$$\boxed{\langle 0|T\,\tilde{\Phi}(p_1,1)\,\tilde{\Phi}(p_2,2)\,\Phi(0,3)\,(0)\rangle^{1PI} = 2\frac{ig}{2}\,\tilde{\delta}_S(p_1,1,3)\,\tilde{\delta}_S(p_2,2,3)}$$

Before turning to a general result about no-radiative corrections to the superpotential — the "no-Renormalization theorem" let's return to the normalization conditions.

on p. -382- we found Z from the residue condition

The pole condition is

$$D_2 D_2 \langle 0 | T \bar{\phi}(p,1)\, \phi(0,2) | 0 \rangle^{1PI} \Big|_{\substack{p=0 \\ \forall \theta_s^i = 0}} \equiv \frac{im}{4}$$

we find $$= \frac{i(m+a)}{4} \Rightarrow \boxed{a=0}$$

Likewise the coupling constant normalization condition

$$D_1 D_1 D_2 D_2 \langle 0 | T \bar{\phi}(p_1,1)\, \bar{\phi}(p_2,2)\, \phi(0,3) | 0 \rangle^{1PI} \Big|_{\substack{p_1=p_2=0 \\ \forall \theta_s^i = 0}} \equiv \frac{ig}{2}$$

we find
$$= i\frac{Z_g g}{2} \Rightarrow \boxed{Z_g = 1}$$

We only need a wavefunction renormalization to render the theory finite (at one-loop)

Recall now the RGE for the 1-PI functions

$$\left[\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} + \gamma_m m\frac{\partial}{\partial m} - \gamma(N_\phi + N_{\bar\phi})\right]\Gamma[\phi,\bar\phi] = 0$$

Applying this to the normalization conditions $\Rightarrow$

1) Pole:

$$0 = \left[\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} + \gamma_m m\frac{\partial}{\partial m} - \gamma\cdot 2\right] \underbrace{D_2 D_2 \langle 0|T\,\tilde{\Phi}(p,1)\,\Phi(0,2)|0\rangle^{1PI}\Big|_{\substack{p=0\\ \forall\theta's=0}}}_{=\frac{im}{4}}$$

$$\Rightarrow \frac{im}{4}(\gamma_m - 2\gamma) = 0 \Rightarrow \boxed{\gamma_m = 2\gamma}$$

2) Coupling constant

$$0 = \left[\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} + \gamma_m m\frac{\partial}{\partial m} - 3\gamma\right] \overbrace{D_1 D_1 D_2 D_2 \langle 0|T\,\tilde{\Phi}(p_1,1)\,\tilde{\Phi}(p_2,2) \times \Phi(0,3)|0\rangle^{1PI}\Big|_{\substack{p_1=0=p_2\\ \forall\theta's=0}}}^{=\frac{ig}{2}}$$

$$\Rightarrow \frac{i}{2}(\beta - 3g\gamma) = 0 \Rightarrow \boxed{\beta = 3g\gamma}$$

3) Residue

$$D_1 D_1 \bar{D}_2 \bar{D}_2 \frac{\partial}{\partial p^2}\left[0 = \left[\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} + \gamma_m m\frac{\partial}{\partial m} - 2\gamma\right] \langle 0|T\,\tilde{\Phi}(p,1)\,\bar{\Phi}(0,2)|0\rangle^{1PI}\right]\Bigg|_{\substack{p^2=-\mu^2\\ \forall\theta's=0}}$$

$$\Rightarrow$$

$$\left[\mu\frac{\partial}{\partial\mu} D_1 D_1 \bar{D}_2 \bar{D}_2 \frac{\partial}{\partial p^2} \langle 0|T\,\tilde{\Phi}(p,1)\,\bar{\Phi}(0,2)|0\rangle^{1PI}\right]\Bigg|_{\substack{p^2=-\mu^2\\ \forall\theta's=0}}$$

$$+ \beta\frac{\partial}{\partial g}(i) + \gamma_m m\frac{\partial}{\partial m}(i) - 2\gamma(i) = 0$$

$$\Rightarrow \boxed{2\gamma = -i\left[\mu\frac{\partial}{\partial\mu} D_1 D_1 \bar{D}_2 \bar{D}_2 \frac{\partial}{\partial p^2} \langle 0 | T\, \Phi(p,1)\, \bar{\Phi}(0,2) | 0\rangle^{1PI}\right]_{\substack{p^2 = -\mu^2 \\ \forall \theta\text{'s} = 0}}}$$

Now in 1-loop

$$\langle 0 | T\, \Phi(p,1)\, \bar{\Phi}(0,2) | 0 \rangle^{1PI} = \frac{i}{16} e^{-\theta_1 \not{p} \bar{\theta}_{12}}\, e^{\theta_{12} \not{p} \bar{\theta}_2} \times$$

$$\times \left[Z + \underbrace{4g^2 \int_0^1 d\alpha\, \alpha \frac{\Gamma(\epsilon/2)}{(4\pi)^2} e^{-\frac{\epsilon}{2}\ln[\mu^2 \alpha(1-\alpha) p^2]}}_{\text{independent of } \mu \text{ in one-loop}} \right]$$

So

$$\mu\frac{\partial}{\partial\mu} \langle 0 | T\, \Phi(p,1)\, \bar{\Phi}(0,2) | 0 \rangle^{1PI} = \frac{i}{16} \underbrace{e^{-\theta_1 \not{p} \bar{\theta}_{12}}\, e^{\theta_{12} \not{p} \bar{\theta}_2}}_{= e^{2\theta_1 \not{p} \bar{\theta}_2}} \left(\mu\frac{\partial Z}{\partial\mu}\right)$$

$\Rightarrow$

$$\boxed{2\gamma = \mu\frac{\partial}{\partial\mu} Z}$$

The same result as we obtained in components on pages -349- to -351-.

Now let's return to the θ-structure of Green's functions implied by SUSY in general. SUSY invariance yields the Ward-identity for Green's functions

$$\langle 0|T\,\phi(1)\cdots\phi\cdots\bar{\phi}\quad\cdots\bar{\phi}(N)|0\rangle$$

insert the SUSY transformation unitary operator

$$U(\xi,\bar{\xi}) = e^{i(\xi Q + \bar{\xi}\bar{Q})}$$

$$\langle 0|T\,U^{-1}U\phi(1)U^{-1}U\phi(2)\cdots U^{-1}U\bar{\phi}(N)U^{-1}U|0\rangle$$

The vacuum is completely invariant $U|0\rangle = |0\rangle$ but recall

$$U(\xi,\bar{\xi})\,\phi(x,\theta,\bar{\theta})\,U^{-1}(\xi,\bar{\xi})$$

(p. -237-)

$$= \phi(x + i(\xi\sigma\bar{\theta} - \theta\sigma\bar{\xi}),\ \theta+\xi,\ \bar{\theta}+\bar{\xi})$$

So we find the SUSY WI:

$$\langle 0|T\,\phi(1)\cdots\bar{\phi}(N)|0\rangle$$

$$= \langle 0|T\,\phi(x_1 + i(\xi\sigma\bar{\theta}_1 - \theta_1\sigma\bar{\xi}),\theta_1+\xi,\bar{\theta}_1+\bar{\xi})$$

$$\cdots\bar{\phi}(x_N + i(\xi\sigma\bar{\theta}_N - \theta_N\sigma\bar{\xi}),\theta_N+\xi,\bar{\theta}_N+\bar{\xi})|0\rangle$$

Now we can choose $\xi = -\theta_N$; $\bar{\xi} = -\bar{\theta}_N$
So that

$\Phi(x_N, 0, 0)$ while all the other fields have dependence of the form

$$\phi(x_i - i(\theta_N \sigma \bar{\theta}_i - \theta_i \sigma \bar{\theta}_N), \underbrace{\theta_i - \theta_N}_{=\theta_{iN}}, \underbrace{\bar{\theta}_i - \bar{\theta}_N}_{=\bar{\theta}_{iN}})$$

So

$$\langle 0 | T\, \Phi(1) \cdots \Phi(N) | 0 \rangle = \langle 0 | T\, \phi(x_1 - i(\theta_N \sigma \bar{\theta}_1 - \theta_1 \sigma \bar{\theta}_N), \theta_{1N}, \bar{\theta}_{1N})$$

$$\ldots \Phi(x_N, 0, 0) | 0 \rangle$$

$$= e^{-i \sum_{i=1}^{N-1} (\theta_N \gamma_i \bar{\theta}_i - \theta_i \gamma_i \bar{\theta}_N)} \underbrace{\langle 0 | T\, \phi(x_1, \theta_{1N}, \bar{\theta}_{1N}) \cdots \Phi(x_N, 0, 0) | 0 \rangle}$$

only a function of θ_{iN}, $\bar{\theta}_{iN}$ differences i.e. only $(N-1)$ θ_{iN} differences & $(N-1)$ $\bar{\theta}_{iN}$ differences

So if we Fourier transform ⇒

$$\langle 0 | T\, \Phi(p_1, 1) \cdots \Phi(0, N) | 0 \rangle$$

$$= e^{E(p,\theta)} \langle 0 | T\, \Phi(p_1, \theta_{1N}, \bar{\theta}_{1N}) \cdots \Phi(0,0,0) | 0 \rangle$$

with
$$E(p,\theta) = \sum_{i=1}^{N-1} (\theta_N \not{p}_i \bar{\theta}_i - \theta_i \not{p}_i \bar{\theta}_N)$$

This susy form is exactly what we found for the propagators

$$\Delta_{\phi\phi}(p_1,1,2) = \langle 0 | T\, \tilde{\phi}(p_1,1)\, \phi(0,2) | 0 \rangle$$
$$= \frac{im}{p_1^2 - m^2}\, \theta_{12}^2\, e^{-\theta_1 \not{p}_1 \bar{\theta}_{12}}$$
$$= e^{-(\theta_2 \not{p}_1 \bar{\theta}_1 - \theta_1 \not{p}_1 \bar{\theta}_2)} \left[\frac{im\,\theta_{12}^2}{p_1^2 - m^2} \right]$$

$$\Delta_{\bar{\phi}\bar{\phi}}(p_1,1,2) = \langle 0 | T\, \tilde{\bar{\phi}}(p_1,1), \bar{\phi}(0,2) | 0 \rangle$$
$$= \frac{im}{p_1^2 - m^2}\, \bar{\theta}_{12}^2\, e^{+\theta_{12} \not{p}_1 \bar{\theta}_1}$$

$$= e^{-(\theta_2 \not{p}_1 \bar{\theta}_1 - \theta_1 \not{p}_1 \bar{\theta}_2)} \left[\frac{im\,\bar{\theta}_{12}^2}{p_1^2 - m^2} \right]$$

$$\Delta_{\phi\bar{\phi}}(p_1,1,2) = \langle 0 | T\, \tilde{\phi}(p_1,1)\, \bar{\phi}(0,2) | 0 \rangle$$
$$= \frac{i}{p^2 - m^2}\, e^{-\theta_1 \not{p}_1 \bar{\theta}_{12} + \theta_{12} \not{p}_1 \bar{\theta}_2}$$
$$= e^{-(\theta_2 \not{p}_1 \bar{\theta}_1 - \theta_1 \not{p}_1 \bar{\theta}_2)} \left[\frac{i\, e^{-\theta_{12} \not{p}_1 \bar{\theta}_{12}}}{p_1^2 - m^2} \right]$$

As required by SUSY the δ-functions have the same structure

$$\tilde{\delta}_S(p_1,1,2) = -\frac{1}{4}\theta_{12}^2\, e^{-\theta_1 \not{p}_1 \bar{\theta}_{12}}$$

$$= e^{-(\theta_2 \not{p}_1 \bar{\theta}_1 - \theta_1 \not{p}_1 \bar{\theta}_2)}\left[-\frac{1}{4}\theta_{12}^2\right]$$

$$\tilde{\delta}_{\bar{S}}(p_1,1,2) = -\frac{1}{4}\bar{\theta}_{12}^2\, e^{+\theta_{12}\not{p}_1\bar{\theta}_1}$$

$$= e^{-(\theta_2 \not{p}_1 \bar{\theta}_1 - \theta_1 \not{p}_1 \bar{\theta}_2)}\left[-\frac{1}{4}\bar{\theta}_{12}^2\right]$$

$$\tilde{\delta}_V(p_1,1,2) = \frac{1}{16}\theta_{12}^2\bar{\theta}_{12}^2$$

$$= e^{-(\theta_2 \not{p}_1 \bar{\theta}_1 - \theta_1 \not{p}_1 \bar{\theta}_2)}\left[\frac{1}{16}\theta_{12}^2\bar{\theta}_{12}^2\right]$$

Hence 1-PI functions have the same θ-function structure as the Green's functions as required by the SUSY

$$\langle 0|T\,\hat{\Phi}(p_1,1)\cdots\Phi(0,N)|0\rangle^{1PI}$$

$$= e^{E(p,\theta)}\langle 0|T\,\Phi(p_1,\theta_{1N},\bar{\theta}_{1N})\cdots\Phi(0,0,0)|0\rangle^{1PI}$$

Now we can apply this same reasoning to our Gell-Mann-Low formula in order to obtain information about radiative corrections and the no renormalization theorem.

$$\langle 0|T\,\phi(1)\cdots\phi(N)|0\rangle^{1PI} = {}_{(0)}\langle 0|T\,\phi_{(0)}(1)\cdots\phi_{(0)}(N)\,e^{\Gamma^{(0)}_{int}}|0\rangle^{1PI}_{(0)}$$

Now each interaction vertex is integrated over space-time — hence no external momentum flows into that vertex (i.e. $\int d^4x \Rightarrow p=0$ into x) So hence all the non-differences in $\theta, \bar\theta$ come from the external line $E(p,\theta)$ factors.
More simply put — in the GML expansion use the SUSY transformations to shift all $\theta, \bar\theta$ by $\theta_N, \bar\theta_N$. Then even in the interaction vertices we have, for example,

$$\int dS\, \phi^3(x,\theta,\bar\theta) \longrightarrow \int dS\, e^{-i(\theta_N\sigma_x\bar\theta - \theta\sigma_x\bar\theta_N)}\phi^3(x,\theta-\theta_N,\bar\theta-\bar\theta_N)$$

since x is integrated over the total derivatives vanish at the infinity

$$\longrightarrow \int dS\, \phi^3(x,\theta-\theta_N,\bar\theta-\bar\theta_N)$$

The only fields left without integrals (having non-zero momentum flow into them) are the external fields $\phi_{(0)}(1)\cdots\phi_{(0)}(N)$

and so this tells us that the Feynman integrand has only θ differences except the final external line E.
We saw this happen explicitly in the 1-loop graphs we calculated - all the $\theta, \bar{\theta}$ interaction vertex factors in the exponents cancelled!! So we find that SUSY implies for the Feynman integrand that

$$\langle 0 | T\, \phi(1) \cdots \bar{\phi}(N) | 0 \rangle^{1PI}$$

$$= \int \frac{d^4 p_1}{(2\pi)^4} \cdots \frac{d^4 p_N}{(2\pi)^4}\, e^{-i \sum_{i=1}^{N} p_i x_i} (2\pi)^4 \delta^4(p_1 + \cdots + p_N) \times$$

$$\times \sum_{\Gamma \in G^{(N)}_{1PI}} \alpha(\Gamma) \int \frac{d^4 k_1}{(2\pi)^4} \cdots \frac{d^4 k_{m(\Gamma)}}{(2\pi)^4}\, I_\Gamma(p, k, \theta, \bar{\theta})$$

with $I_\Gamma(p,k,\theta,\bar{\theta}) = e^{E(p,\theta)}\, \bar{I}_\Gamma(p,k,\theta_{\cdot\cdot},\bar{\theta}_{\cdot\cdot})$

that is $\bar{I}_\Gamma$ is a function of the $\theta, \bar{\theta}$ differences only!

Now suppose we are interested in the superpotential. Consider the pure chiral field effective action

$$\Gamma[\phi] = \Gamma[\phi, \bar{\phi} = 0]$$

Now the superpotential is given by the local approximate for the 1-PI functions that is no derivatives (momenta)

$$\Gamma[\phi]\Big| \equiv i\int dS\, W(\phi)$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!}\int dS_1 \cdots \int dS_n\, \phi(1)\cdots\phi(n) \langle 0|T\phi(1)\cdots\phi(n)|0\rangle^{1PI}\Big|$$

where we expand $\langle 0|T\phi(1)\cdots\phi(n)|0\rangle^{1PI}$ in derivatives

$$\langle 0|T\phi(1)\cdots\phi(n)|0\rangle^{1PI} \equiv i\Gamma_n \int dS_m\, \delta_S(1,m)\,\delta_S(2,m)\cdots\delta_S(n,m)$$

$$+ i\Lambda_n \int dS_m\, \delta_S(1,m)\cdots\partial_n^2\delta_S(n,m)$$

etc.

$$\boxed{\text{So} \quad \langle 0|T\phi(1)\cdots\phi(n)|0\rangle^{1PI}\Big|_{p=0} = i\Gamma_n \int dS_m\, \delta_S(1,m)\cdots\delta_S(n,m)}$$

$$\text{So } i\int dS\, W(\phi) = \sum_{n=0}^{\infty}\frac{i}{n!}\Gamma_n \int dS_1\cdots\int dS_n \int dS_m\, \phi(1)\cdots\phi(n)\,\delta_S(1,m)\cdots\delta_S(n,m)$$

$$= i\int dS_m \sum_{n=0}^{\infty}\frac{1}{n!}\Gamma_n\, \phi^n(m)$$

$$\Rightarrow \quad \boxed{W(\phi) = \sum_{n=0}^{\infty}\frac{1}{n!}\Gamma_n\,\phi^n}$$

Recall that in the tree approximation we had

$$\Gamma[\phi] = i\int dS\, W_{\text{classical}}(\phi) \qquad \text{just the given superpotential or the classical superpotential}$$

in the W-Z model this is just

$$\Gamma_{\text{tree}}[\phi] = i\int dS\left(\frac{m}{8}\phi^2 + \frac{g}{12}\phi^3\right)$$

and $\langle 0|T\phi(1)\phi(2)|0\rangle^{1PI}\Big|_{\text{tree}} = i\frac{m}{4}\int dS_4\, \delta_S(1,4)\delta_S(2,4)$

&

$$\langle 0|T\phi(1)\phi(2)\phi(3)|0\rangle^{1PI}\Big| = +i\frac{g}{2}\int dS_4\, \delta_S(1,4)\delta_S(2,4)\delta_S(3,4)$$

$\Rightarrow \Gamma_2 = \frac{m}{4}, \Gamma_3 = \frac{g}{2}$ tree

Now whether there are quantum corrections to the superpotential depends on whether there are radiative or loop corrections to the Γ_n.

Now Γ_n is the $p=0$ or local part of the 1-PI function. So we are interested in

$$\langle 0|T\phi(1)\cdots\phi(n)|0\rangle^{1PI}\Big|_{p=0} = i\Gamma_n\int dS_m\, \delta_S(1,m)\cdots\delta_S(n,m)$$

So

$$i\Gamma_n = (D_1 D_1)\cdots(D_{n-1}D_{n-1})\langle 0|T\tilde{\phi}(p_1,1)\cdots\tilde{\phi}(p_{n-1},n-1)\phi(0,n)|0\rangle^{1PI}\Big|_{\forall p_i=0}$$

So in short we must consider $\langle 0|T\,\phi(p_1,1)\cdots\phi(0,n)|0\rangle^{1PI}$, the pure chiral 1PI functions.

Now

$$\langle 0|T\,\phi(p_1,1)\,\phi(p_2,2)\cdots\phi(0,n)|0\rangle^{1PI}$$

$$= \sum_{\Gamma\in G^{(n)}_{1PI\,chiral}}{}' \alpha(\Gamma)\int\frac{d^4k_1}{(2\pi)^4}\cdots\frac{d^4k_{m(\Gamma)}}{(2\pi)^4}\, I_\Gamma(p,k,\theta)$$

but $I_\Gamma(p,k,\theta) = e^{E(p,\theta)}\, \overline{I}_\Gamma(p,k,\theta_{\circ\circ})$

Since all fields are chiral there is no $\overline{\theta}$ dependence at $\forall p_i=0$ — the 1-PI functions only depend on differences in θ's no $\overline{\theta}$'s.

Hence all the $\overline{\theta}$'s in the Feynman integrand must be integrated over, but how many $\overline{\theta}$'s are there

2 for each vector interaction vertex (n_V)
2 for each anti-chiral " " $(n_{\overline{S}})$

but only differences come into play so -2 $\overline{\theta}$'s.

$$\#\text{ of }\overline{\theta}\text{'s in }\Gamma = 2(n_V + n_{\overline{S}} - 1)$$

However each vertex has an integral over

2 $\bar{\theta}$s

$$\# \text{ of } \bar{\theta} \text{ integrals in } \Gamma = 2(n_V + n_{\bar{S}})$$

There are 2 more integrals than $\bar{\theta}$'s $\Longrightarrow$ the $I_\Gamma = 0$!! So we find

$$\boxed{\left.\langle 0 | T\, \tilde{\phi}(p_1,1)\, \tilde{\phi}(p_2,2) \cdots \phi(0,n) | 0 \rangle^{1PI}\right|_{\substack{\forall p_i = 0 \\ \text{quantum} \\ \text{corrections } (= \text{loops})}} = 0}$$

Hence $W(\phi)^{\text{quantum corrections}} = 0$

$$W(\phi) = W(\phi)^{\text{tree or classical}} \qquad (\text{wysiwyg})$$

(i.e. $\Gamma_2 = \frac{m}{4}$, $\Gamma_3 = \frac{g}{2}$, all others are 0)

The superpotential is not renormalized.

Recall this means $a = 0$, $c = 0$ ($Z_g = 1$), we still had to re-scale the fields by $Z^{1/2}$.

On to SUSY gauge theories!