As we saw to treat all components is tedious we can keep the bookkeeping simpler by using Supergraph techniques.

Perturbation theory in Superspace:

Let's begin by putting together the component propagators to form a superpropagator. Consider all the (non-zero) component propagators

$$\langle 0|T A(x) F(y)|0\rangle = \frac{-im}{\partial_x^2+m^2}\delta^4(x-y)$$

$$\langle 0|T F(x) A(y)|0\rangle = \frac{-im}{\partial_x^2+m^2}\delta^4(x-y)$$

$$\langle 0|T \bar{A}(x) \bar{F}(y)|0\rangle = \frac{-im}{\partial_x^2+m^2}\delta^4(x-y) = \langle 0|T \bar{F}(x) \bar{A}(y)|0\rangle$$

$$\langle 0|T A(x) \bar{A}(y)|0\rangle = \frac{-i}{\partial_x^2+m^2}\delta^4(x-y) = \langle 0|T \bar{A}(x) A(y)|0\rangle$$

$$\langle 0|T F(x) \bar{F}(y)|0\rangle = \frac{+i\partial_x^2}{\partial_x^2+m^2}\delta^4(x-y) = \langle 0|T \bar{F}(x) F(y)|0\rangle$$

$$\langle 0|T \psi_\alpha(x) \psi_\beta(y)|0\rangle = \frac{-2im\,\epsilon_{\alpha\beta}}{\partial_x^2+m^2}\delta^4(x-y)$$

$$\langle 0|T \bar{\psi}_{\dot\alpha}(x) \bar{\psi}_{\dot\beta}(y)|0\rangle = \frac{+2im\,\epsilon_{\dot\alpha\dot\beta}}{\partial_x^2+m^2}\delta^4(x-y)$$

$$\langle 0|T \psi_\alpha(x) \bar{\psi}_{\dot\alpha}(y)|0\rangle = \frac{2\,\partial\!\!\!/_{x\alpha\dot\alpha}}{\partial_x^2+m^2}\delta^4(x-y) = \langle 0|T \bar{\psi}_{\dot\alpha}(x) \psi_\alpha(y)|0\rangle$$

These can be economically desribed by 3 super-propagators — consider the pure chiral propagator first:

$$\langle 0|T\phi(1)\phi(2)|0\rangle = \langle 0|T\phi(x_1,\theta_1,\bar\theta_1)\phi(x_2,\theta_2,\bar\theta_2)|0\rangle$$

$$= e^{-i\theta_1\not\partial_1\bar\theta_1} e^{-i\theta_2\not\partial_2\bar\theta_2}\langle 0|T[A(x_1)+\theta_1^\alpha\psi_\alpha(x_1)+\theta_1^2F(x_1)]\times$$

$$\times[A(x_2)+\theta_2^\beta\psi_\beta(x_2)+\theta_2^2F(x_2)]|0\rangle$$

$$= e^{-i\theta_1\not\partial_1\bar\theta_1} e^{-i\theta_2\not\partial_2\bar\theta_2}\Big\{\langle 0|TA(x_1)F(x_2)|0\rangle\theta_2^2$$

$$+\theta_1^2\langle 0|TF(x_1)A(x_2)|0\rangle - \theta_1^\alpha\theta_2^\beta\langle 0|T\psi_\alpha(x_1)\psi_\beta(x_2)|0\rangle\Big\}$$

$$= e^{-i\theta_1\not\partial_1\bar\theta_1-i\theta_2\not\partial_2\bar\theta_2}\Big\{\frac{-im}{\partial_{x_1}^2+m^2}\delta^4(x_1-x_2)\theta_2^2$$

$$-\frac{im}{\partial_{x_1}^2+m^2}\delta^4(x_1-x_2)\theta_1^2$$

$$-\theta_1^\alpha\theta_2^\beta\frac{(-2im)\epsilon_{\alpha\beta}}{\partial_{x_1}^2+m^2}\delta^4(x_1-x_2)\Big\}$$

$$= e^{-i\theta_1\not\partial_1\bar\theta_1-i\theta_2\not\partial_2\bar\theta_2}\Big\{\frac{(\theta_1-\theta_2)^2(-im)}{\partial_{x_1}^2+m^2}\delta^4(x_1-x_2)\Big\}$$

$$= \left(\frac{-im}{\partial_{x_1}^2+m^2}\right)\theta_{12}^2\, e^{-i\theta_1\not\partial_1\bar\theta_{12}}\delta^4(x_1-x_2)$$

So

$$\boxed{\langle 0|T\phi(1)\phi(2)|0\rangle = \left(\frac{-im}{\partial_{x_1}^2+m^2}\right)\theta_{12}^2\, e^{-i\theta_1\not\partial_1\bar\theta_{12}}\,\delta^4(x_1-x_2)}$$

Recall $\delta_S(1,2) = -\frac{1}{4}\theta_{12}^2\, e^{-i\theta_1\not\partial_1\bar\theta_{12}}\,\delta^4(x_1-x_2)$ [p. -276-]

$\Rightarrow$

$$\boxed{\langle 0|T\phi(1)\phi(2)|0\rangle = \frac{4im}{\partial_{x_1}^2+m^2}\delta_S(1,2)} = \frac{4im}{\partial_{x_1}^2+m^2}\bar D_1\bar D_1\delta_V(1,2)$$

Fourier transforming

$$\boxed{\begin{aligned}\langle 0|T\tilde\phi(p,1)\phi(0,2)|0\rangle &= \frac{-4im}{p^2-m^2}\tilde\delta_S(p,1,2)\\ &= \frac{im}{p^2-m^2}\theta_{12}^2\, e^{-\theta_1\not p\bar\theta_{12}}\end{aligned}}$$

Likewise

$$\boxed{\langle 0|T\bar\phi(1)\bar\phi(2)|0\rangle = \frac{+4im}{\partial_{x_1}^2+m^2}\delta_{\bar S}(1,2) = \frac{4im}{\partial_{x_1}^2+m^2}D_1D_1\delta_V(1,2)}$$

F.T. $\Rightarrow$

$$\boxed{\begin{aligned}\langle 0|T\tilde{\bar\phi}(p,1)\bar\phi(0,2)|0\rangle &= \frac{-4im}{p^2-m^2}\tilde\delta_{\bar S}(p,1,2)\\ &= \frac{+im}{p^2-m^2}\bar\theta_{12}^2\, e^{+\theta_{12}\not p\bar\theta_1}\end{aligned}}$$

And finally the mixed propagator

$$\langle 0|T\,\phi(1)\,\bar{\phi}(2)|0\rangle = e^{-i\theta_1\not{\partial}_1\bar{\theta}_1}\, e^{+i\theta_2\not{\partial}_2\bar{\theta}_2} \times$$

$$\times \langle 0|T\,[A(x_1)+\theta_1^\alpha\psi_\alpha(x_1)+\theta_1^2F(x_1)][\bar{A}(x_2)+\bar{\theta}_{2\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}(x_2) + \bar{\theta}_2^2\bar{F}(x_2)]|0\rangle$$

$$= e^{-i\theta_1\not{\partial}_1\bar{\theta}_1+i\theta_2\not{\partial}_2\bar{\theta}_2}\Big\{\langle 0|T\,A(x_1)\bar{A}(x_2)|0\rangle$$

$$+\theta_1^2\bar{\theta}_2^2\langle 0|T\,F(x_1)\bar{F}(x_2)|0\rangle$$

$$-\theta_1^\alpha\bar{\theta}_{2\dot{\alpha}}\langle 0|T\,\psi_\alpha(x_1)\bar{\psi}^{\dot{\alpha}}(x_2)|0\rangle\Big\}$$

$$= e^{-i\theta_1\not{\partial}_1\bar{\theta}_1+i\theta_2\not{\partial}_2\bar{\theta}_2}\Big\{1-\partial_{x_1}^2\theta_1^2\bar{\theta}_2^2+2i\theta_1\not{\partial}_1\bar{\theta}_2\Big\}\times$$

$$\times\left[\frac{-i}{\partial_{x_1}^2+m^2}\,\delta^4(x_1-x_2)\right]$$

Now recall that

$$e^{\pm 2i\theta_1\not{\partial}_1\bar{\theta}_2} = 1\pm 2i\theta_1\not{\partial}_1\bar{\theta}_2+\frac{1}{2}(2i)^2(\theta_1\not{\partial}_1\bar{\theta}_2)^2$$

but $(\theta_1\not{\partial}_1\bar{\theta}_2)^2 = \theta_1^\alpha\not{\partial}_{1\alpha\dot{\alpha}}\bar{\theta}_2^{\dot{\alpha}}\,\theta_1^\beta\not{\partial}_{1\beta\dot{\beta}}\bar{\theta}_2^{\dot{\beta}}$

$$= -\theta_1^\alpha\theta_1^\beta\bar{\theta}_2^{\dot{\alpha}}\bar{\theta}_2^{\dot{\beta}}\not{\partial}_{1\alpha\dot{\alpha}}\not{\partial}_{1\beta\dot{\beta}}$$

$$= \frac{1}{4}\theta_1^2\bar{\theta}_2^2\not{\partial}_{1\alpha\dot{\alpha}}\bar{\not{\partial}}_1^{\dot{\alpha}\alpha} = \frac{1}{2}\theta_1^2\bar{\theta}_2^2\partial_1^2$$

So
$$\boxed{e^{\pm 2i\theta_1 \not{\partial}_1 \bar{\theta}_2} = 1 \pm 2i\theta_1\not{\partial}_1\bar{\theta}_2 - \theta_1^2\bar{\theta}_2^2\partial_1^2}$$

$\Rightarrow$

$$\langle 0|T\phi(1)\bar{\phi}(2)|0\rangle = e^{-i\theta_1\not{\partial}_1\bar{\theta}_1 + i\theta_2\not{\partial}_2\bar{\theta}_2} \times e^{+2i\theta_1\not{\partial}_1\bar{\theta}_2} \times$$
$$\times \left(\frac{-i}{\partial_{x_1}^2 + m^2}\right)\delta^4(x_1 - x_2)$$

$$= e^{-i\theta_1\not{\partial}_1\bar{\theta}_1 - i\theta_2\not{\partial}_1\bar{\theta}_2 + 2i\theta_1\not{\partial}_1\bar{\theta}_2}\left(\frac{-i}{\partial_{x_1}^2 + m^2}\delta^4(x_1 - x_2)\right)$$

$$\boxed{= e^{-i\theta_1\not{\partial}_1\bar{\theta}_{12} + i\theta_{12}\not{\partial}_1\bar{\theta}_2}\left(\frac{-i}{\partial_{x_1}^2 + m^2}\delta^4(x_1 - x_2)\right)}$$

$$\boxed{= \langle 0|T\phi(1)\bar{\phi}(2)|0\rangle}$$

From the expression for the δ-functions, we have that

$$\left(D_2 D_2 \delta_S(1,2)\right)\Big|_{2\,\text{antichiral}} = \frac{\partial}{\partial\theta_{2\alpha}}\frac{\partial}{\partial\theta_2^\alpha}\left[e^{-i\theta_2\not{\partial}_2\bar{\theta}_2}\left(-\frac{1}{4}\right)\theta_{12}^2 \times \right.$$
$$\left. \times e^{-i\theta_1\not{\partial}_1\bar{\theta}_{12}}\delta^4(x_1 - x_2)\right]$$

$$= \frac{\partial}{\partial\theta_{2\alpha}}\frac{\partial}{\partial\theta_2^\alpha}\left[-\frac{1}{4}\theta_{12}^2 e^{-i\theta_1\not{\partial}_1\bar{\theta}_1 + 2i\theta_1\not{\partial}_1\bar{\theta}_2}\delta^4(x_1 - x_2)\right]$$

$$= e^{-i\theta_1 \gamma_1 \bar{\theta}_1} e^{+2i\theta_1\gamma_1\bar{\theta}_2} \delta^4(x_1 - x_2)$$

Now we convert the term back to the real Rep

$$\boxed{D_2 D_2 \delta_S(1,2) = e^{-i\theta_2\gamma_1\bar{\theta}_2} e^{-i\theta_1\gamma_1\bar{\theta}_1} e^{+2i\theta_1\gamma_1\bar{\theta}_2} \delta^4(x_1-x_2) = e^{-i\theta_1\gamma_1\bar{\theta}_{12} + i\theta_{12}\gamma_1\bar{\theta}_2} \delta^4(x_1-x_2)}$$

$\Rightarrow$

$$\boxed{\langle 0 | T \phi(1) \bar{\phi}(2) | 0 \rangle = \frac{-i}{\partial_{x_1}^2 + m^2} D_2 D_2 \delta_S(1,2)}$$

Now $\delta_S(1,2) = \bar{D}_1 \bar{D}_1 \delta_V(1,2)$

$\Rightarrow$

$$\boxed{\langle 0 | T \phi(1) \bar{\phi}(2) | 0 \rangle = \frac{-i}{\partial_{x_1}^2 + m^2} \bar{D}_1 \bar{D}_1 D_2 D_2 \delta_V(1,2)}$$

Note $\bar{D}_1 \langle 0 | T \phi(1) \bar{\phi}(2) | 0 \rangle = 0 = D_2 \langle 0 | T \phi(1) \bar{\phi}(2) | 0 \rangle$ as it should since $\bar{D}_1 \phi(1) = 0 = D_2 \bar{\phi}(2)$, for chiral & anti-chiral fields.

Fourier Transforming $\Rightarrow$

$$\boxed{\langle 0|T\,\phi(p,1)\,\bar\phi(0,2)|0\rangle = \frac{i}{p^2-m^2}\, e^{-\theta_1 \not{p}\bar\theta_{12}+\theta_{12}\not{p}\bar\theta_2}}$$

$$= \frac{i}{p^2-m^2}\hat{\bar D}_2 \tilde D_2\, \tilde\delta_S(p,1,2)$$

Now we can derive these superpropagators by working directly with superfields in superspace. Since we also are interested in superfield Green's functions, as their $\theta,\bar\theta$ expansions give us all component Green's functions, we introduce the superfield generating functionals

$$Z[J,\bar J] \equiv \langle 0|T\, e^{i\int ds\, J\phi + i\int d\bar s\, \bar J\bar\phi}|0\rangle$$

where $J(x,\theta,\bar\theta)$ is a chiral supersource for the chiral superfield $\phi(x,\theta,\bar\theta)$; $\bar D_{\dot\alpha} J = 0$. Similarly $\bar J(x,\theta,\bar\theta)$ is the anti-chiral supersource for the anti-chiral superfield $\bar\phi(x,\theta,\bar\theta)$: $D_\alpha \bar J = 0$.

Explicitly

$$J(x,\theta,\bar\theta) \equiv e^{-i\theta\gamma\bar\theta}\left(-\frac{1}{4}\right)\left[K(x) - 2\theta^\alpha \eta_\alpha(x) + \theta^2 J(x)\right]$$

$$\bar J(x,\theta,\bar\theta) \equiv e^{+i\theta\gamma\bar\theta}\left(-\frac{1}{4}\right)\left[\bar K(x) - 2\bar\theta_{\dot\alpha}\bar\eta^{\dot\alpha}(x) + \bar\theta^2 \bar J(x)\right]$$

So that

$$\int dS\, J\phi = \int d^4x \left[J(x)A(x) + \eta^\alpha(x)\psi_\alpha(x) + K(x)F(x)\right]$$

and

$$\int d\bar S\, \bar J\bar\phi = \int d^4x \left[\bar J(x)\bar A(x) + \bar\eta_{\dot\alpha}(x)\bar\psi^{\dot\alpha}(x) + \bar K(x)\bar F(x)\right]$$

Thus $Z[J,\bar J]$ is just the same as the component field generating functional $Z[J,\bar J,\eta,\bar\eta,K,\bar K]$.

The action for the W-Z model is given by

$$\Gamma_0 = i\int dV\, K(\phi,\bar\phi) + i\int dS\, W(\phi) + i\int d\bar S\, \bar W(\bar\phi)$$

$$= i\int dV \frac{1}{16}\phi_0\bar\phi_0 + i\int dS\left[\frac{m_0}{8}\phi_0^2 + \frac{g_0}{12}\phi_0^3\right]$$

$$+ i\int d\bar S\left[\frac{m_0}{8}\bar\phi_0^2 + \frac{g_0}{12}\bar\phi_0^3\right]$$

So the generating functional is given by the path integral as usual

$$Z[J,\bar{J}] = \int [d\phi][d\bar{\phi}]\, e^{i\left[\int dV K + \int dS W + \int d\bar{S}\bar{W} + J\phi + \bar{J}\bar{\phi}\right]}$$

Introducing the connected functions

$$Z[J,\bar{J}] = e^{Z^c[J,\bar{J}]}$$

where $Z^c[J,\bar{J}] = \langle 0|T\, e^{i\int dS J\phi + i\int d\bar{S}\bar{J}\bar{\phi}}|0\rangle^{\text{connected}}$

and the Legendre transform to one-particle irreducible (vertex) functions

$$\Gamma[\varphi,\bar{\varphi}] = Z^c[J,\bar{J}] - i\int dS J\varphi - i\int d\bar{S}\bar{J}\bar{\varphi}$$

with chiral φ and anti-chiral $\bar{\varphi}$ classical sources for the vertex functions

$$\varphi = \frac{\delta Z^c}{i\delta J} \quad ; \quad \bar{\varphi} = \frac{\delta Z^c}{i\delta \bar{J}}$$

and likewise

$$Z^c[J,\bar{J}] = \Gamma[\varphi,\bar{\varphi}] + i\int dS J\varphi + i\int d\bar{S}\bar{J}\bar{\varphi}$$

with $J = i\dfrac{\delta\Gamma}{\delta\varphi}$; $\bar{J} = i\dfrac{\delta\Gamma}{\delta\bar{\varphi}}$.

and

$$\Gamma[\varphi,\bar\varphi] = \langle 0|T e^{\int dS\,\varphi\phi + \int d\bar S\,\bar\varphi\bar\phi}|0\rangle^{1PI}$$

As previously, the propagators will be given by the negative inverse of the 1PI 2-point functions. Let $\tilde\varphi, \tilde J, d\tilde S$ stand for either the chiral, φ, J, dS, or anti-chiral fields & measure $\bar\varphi, \bar J, d\bar S$. Then

$$\int dS_3 \frac{\delta^2\Gamma}{\delta\tilde\varphi(1)\,\delta\varphi(3)}\frac{\delta^2 Z^c}{i\delta J(3)\,i\delta\tilde J(2)}$$

$$+\int d\bar S_3 \frac{\delta^2\Gamma}{\delta\tilde\varphi(1)\,\delta\bar\varphi(3)}\frac{\delta^2 Z^c}{i\delta\bar J(3)\,i\delta\tilde J(2)}$$

$$= \int dS_3 \frac{\delta\tilde J(1)}{i\delta\varphi(3)}\frac{\delta\varphi(3)}{i\delta\tilde J(2)} + \int d\bar S_3 \frac{\delta\tilde J(1)}{i\delta\bar\varphi(3)}\frac{\delta\bar\varphi(3)}{i\delta\tilde J(2)}$$

(chain Rule)

$$= -\frac{\delta\tilde J(1)}{\delta\tilde J(2)} = \boxed{-\delta_{\tilde S}(1,2)}$$

$$\boxed{= \int dS_3 \langle 0|T\tilde\phi(1)\phi(3)|0\rangle^{1PI}\langle 0|T\phi(3)\tilde\phi(2)|0\rangle^c + \int d\bar S_3 \langle 0|T\tilde\phi(1)\bar\phi(3)|0\rangle^{1PI}\langle 0|T\bar\phi(3)\tilde\phi(2)|0\rangle^c}$$

We can now apply this to our perturbation theory in Superspace. Recall the bare action ~~and~~ on p. -359- we can rescale the superfields

$$\phi_0 = Z^{1/2}\phi \; ; \; \bar{\phi}_0 = Z^{1/2}\bar{\phi} \quad , \; Z \equiv 1+b$$

with $\phi, \bar{\phi}$ the renormalized fields, The mass & coupling constants can be renormalized as before as well

$$m_0 Z \equiv m + a$$
$$g_0 Z^{3/2} \equiv Z_g g$$

where m, g are the finite renormalized mass and coupling constant. "a" is the mass counter-term & $Z_g = 1 + c/g$ the coupling constant renormalization factor. As before we will show that $Z_g = 1$ & $a = 0$. So the Renormalized action becomes

$$\Gamma_0 = i\Big[\frac{Z}{16}\int dV\,\phi\bar{\phi} + \frac{(m+a)}{8}\int dS\,\phi^2 + \frac{Z_g g}{12}\int dS\,\phi^3 + \frac{(m+a)}{8}\int d\bar{S}\,\bar{\phi}^2 + \frac{Z_g g}{12}\int d\bar{S}\,\bar{\phi}^3\Big]$$

(As previously we are assuming that we are regulating the theory in a manifestly supersymmetric manner so that the renormalization counter-terms & wavefunction re-scaling are supersymmetric as well.)

The 3 normalization conditions are given by

1) "Pole position"

$$\langle 0|T\,\tilde{\Phi}(p,\theta_1,\bar{\theta}_1)\,D_2D_2\,\phi(0,\theta_2,\bar{\theta}_2)|0\rangle^{1PI}\Big|_{\substack{p=0\\ \theta_1=\theta_2=0=\bar{\theta}_1=\bar{\theta}_2}} \equiv 4im$$

2) "Residue"

$$\frac{\partial}{\partial p^2}\langle 0|T\,\tilde{\Phi}(p,1)\,\bar{\Phi}(0,2)|0\rangle^{1PI}\Big|_{\substack{p^2=-\mu^2\\ \theta_1=0=\theta_2\\ \bar{\theta}_1=0=\bar{\theta}_2}} \equiv i$$

3) "Coupling Constant"

$$\langle 0|T\,\tilde{\Phi}(p_1,1)\,\tilde{\Phi}(p_2,2)\,D_3D_3\,\phi(0,3)|0\rangle^{1PI}\Big|_{\substack{p_1=p_2=0\\ \theta_1=0=\theta_2=\theta_3\\ \bar{\theta}_1=\bar{\theta}_2=\bar{\theta}_3=0}} \equiv -ig$$

[Since we assumed SUSY is valid we could use more supersymmetric appearing normalization conditions — for instance the pole is the same location for all components so

$$\langle 0|T\tilde{\Phi}(p,1)\,\phi(0,2)|0\rangle^{\text{1PI}}\Big|_{p=0} = 4im\,\tilde{\delta}_s(p,1,2)\Big|_{p=0}$$

$$= -im\,\theta_{12}^2\Big)$$

As usual we separate the action into free and interacting pieces and express the Green's functions in terms of a Feynman diagram expansion

free part

$$\Gamma_0 = \frac{i}{16}\int dV\,\phi\bar{\phi} + \frac{im}{8}\int dS\,\phi^2 + \frac{im}{8}\int d\bar{S}\,\bar{\phi}^2$$

$$\Gamma_{int} = \frac{i}{16}b\int dV\,\phi\bar{\phi} + \frac{i}{8}a\int dS\,\phi^2 + \frac{i}{8}a\int d\bar{S}\,\bar{\phi}^2 + i\frac{Z_g g}{12}\int dS\,\phi^3 + i\frac{Z_g g}{12}\int d\bar{S}\,\bar{\phi}^3$$

The Green's functions will be given by the Gell-Mann-Low expansion

$$Z[J,\bar{J}] = e^{\Gamma_{int}\left[\frac{\delta}{i\delta J}, \frac{\delta}{i\delta \bar{J}}\right]} \underbrace{\int [d\phi][d\bar{\phi}]\, e^{\Gamma_0 + i\int dS J\phi + i\int d\bar{S} \bar{J}\bar{\phi}}}_{\equiv Z_0[J,\bar{J}]}$$

or in terms of "free" in-field operators

$$Z[J,\bar{J}] = \langle 0\, in | T e^{i\int dS J\phi_{in} + i\int d\bar{S}\bar{J}\bar{\phi}_{in}} \times$$
$$\times\, e^{\Gamma_{int}[\phi_{in}, \bar{\phi}_{in}]} | 0\, in\rangle$$

where $\phi_{in}, \bar{\phi}_{in}$ are the in-superfields with dynamics given by the free field action

$$\Gamma_0 = \frac{i}{16}\int dV\, \phi_{in}\bar{\phi}_{in} + \frac{im}{8}\left[\int dS\, \phi_{in}^2 + \int d\bar{S}\, \bar{\phi}_{in}^2\right]$$

---

where $Z[0,0] \equiv 1$.

Now in either way of viewing the Gell-Mann – Low expansion we must find the Feynman propagators. Given

$$\Gamma_0 = \frac{i}{16}\int dV\, \phi\bar{\phi} + \frac{im}{8}\left[\int dS\,\phi^2 + \int d\bar{S}\,\bar{\phi}^2\right]$$

we can use the Legendre transform relations to find the 2-pt. functions

$$\frac{\delta\Gamma_0}{\delta\phi(1)} = \frac{i}{16}\int dV_2\, \delta_S(2,1)\,\bar{\phi}(2) + \frac{im}{8}\cdot 2\int dS_2\,\delta_S(2,1)\,\phi(2)$$

$$= \frac{i}{16}\int dS_2\,\delta_S(2,1)\,\bar{D}_2\bar{D}_2\bar{\phi}(2) + \frac{im}{4}\int dS_2\,\delta_S(2,1)\,\phi(2)$$

$$\boxed{\frac{\delta\Gamma_0}{\delta\phi(1)} = \frac{i}{16}\bar{D}_1\bar{D}_1\bar{\phi}(1) + \frac{im}{4}\phi(1)}$$

Likewise

$$\boxed{\frac{\delta\Gamma_0}{\delta\bar{\phi}(1)} = \frac{i}{16}D_1D_1\phi(1) + \frac{im}{4}\bar{\phi}(1)}$$

But recall $\quad \dfrac{\delta\Gamma_0}{\delta\phi(1)} = -iJ(1)\,;\quad \dfrac{\delta\Gamma_0}{\delta\bar{\phi}(1)} = -i\bar{J}(1)$

& $\quad \phi(1) = \dfrac{\delta Z^c}{i\delta J(1)}\,;\quad \bar{\phi}(1) = \dfrac{\delta Z^c}{i\delta\bar{J}(1)}$

(as usual we now use $\phi(1) = \varphi(1)$ as well as the quantum field $\phi(1)$ — context makes its use clear)

So
$$\boxed{\begin{aligned} -i J(1) &= \frac{i}{16}\bar{D}_1\bar{D}_1 \frac{\delta Z^c}{i\delta \bar{J}(1)} + \frac{im}{4}\frac{\delta Z^c}{i\delta J(1)} \\ -i \bar{J}(1) &= \frac{i}{16} D_1 D_1 \frac{\delta Z^c}{i\delta J(1)} + \frac{im}{4}\frac{\delta Z^c}{i\delta \bar{J}(1)} \end{aligned}}$$

or for now

$$\boxed{\begin{aligned} &1) \quad -i J(1) = \frac{i}{16}\bar{D}_1\bar{D}_1 \bar{\phi}(1) + \frac{im}{4}\phi(1) \\ &2) \quad -i \bar{J}(1) = \frac{i}{16} D_1 D_1 \phi(1) + \frac{im}{4}\bar{\phi}(1) \end{aligned}}$$

Differentiating 2) wrt $\bar{D}_1\bar{D}_1$
$\Rightarrow$
$$-i\bar{D}_1\bar{D}_1\bar{J}(1) = \frac{i}{16}\bar{D}_1\bar{D}_1 D_1 D_1 \phi(1) + \frac{im}{4}\bar{D}_1\bar{D}_1\bar{\phi}(1)$$

Now $\phi(1)$ is chiral; $\bar{D}_1\phi(1) = 0 \Rightarrow$

$$\bar{D}_1\bar{D}_1 D_1 D_1 \phi(1) = [\bar{D}_1\bar{D}_1, D_1 D_1]\phi(1)$$

but $\{D_\alpha, \bar{D}_{\dot\alpha}\} = 2i \not{\partial}_{\alpha\dot\alpha}$, $\{D_\alpha, D_\beta\} = 0 = \{\bar{D}_{\dot\alpha}, \bar{D}_{\dot\beta}\}$

$$\Rightarrow \quad [D_\alpha, \bar{D}\bar{D}] = 4i(\not{\partial}\bar{D})_\alpha$$

$$[\bar{D}_{\dot\alpha}, DD] = -4i(D\not{\partial})_{\dot\alpha}$$

$$[DD, \bar{D}\bar{D}] = +16\partial^2 + 8i D \not{\partial} \bar{D}$$

$$= -16\partial^2 - 8i \bar{D} \not{\partial} D$$

$$D\bar{D}_\alpha D = -\tfrac{1}{2}\{\bar{D}_\alpha, DD\} \; ; \; \bar{D} D_\alpha \bar{D} = -\tfrac{1}{2}\{D_\alpha, \bar{D}\bar{D}\}$$

$$D\bar{D}\bar{D}D = \bar{D}DD\bar{D}$$

$$D\bar{D}\bar{D}D = 8\partial^2 + \tfrac{1}{2}\{DD, \bar{D}\bar{D}\}$$

So

$$\bar{D}_1\bar{D}_1 D_1 D_1 \phi(1) = [\bar{D}_1\bar{D}_1, D_1 D_1]\phi(1)$$

$$= -16\partial_1^2 \phi(1)$$

(2) $\Rightarrow$

$$-i\bar{D}_1\bar{D}_1 \bar{J}(1) = -i\partial_1^2\phi(1) + \frac{im}{4}\bar{D}_1\bar{D}_1\phi(1)$$

but 1) $\Rightarrow \frac{i}{16}\bar{D}_1\bar{D}_1\phi(1) = -iJ(1) - \frac{im}{4}\phi(1)$

$\Rightarrow$

$$-i\bar{D}_1\bar{D}_1\bar{J}(1) = -i\partial_1^2\phi(1) + 4m\left(-iJ(1) - \frac{im}{4}\phi(1)\right)$$

$$= -i(\partial_1^2 + m^2)\phi(1) - 4imJ(1)$$

Thus $\boxed{+(\partial_1^2 + m^2)\phi(1) = -4mJ(1) + \bar{D}_1\bar{D}_1\bar{J}(1)}$

Similarly

$$\boxed{(\partial_1^2+m^2)\bar\phi(1) = -4m\bar J(1) + D_1 D_1 J(1)}$$

$$\boxed{(\partial_1^2+m^2)\phi(1) = -4m J(1) + \bar D_1 \bar D_1 \bar J(1)}$$

So we have that

$$(\partial_1^2+m^2)\langle 0|T\phi(1)\phi(2)|0\rangle = 4im\,\delta_S(1,2)$$

$$(\partial_1^2+m^2)\langle 0|T\phi(1)\bar\phi(2)|0\rangle = -i\,\bar D_1\bar D_1\,\delta_{\bar S}(1,2)$$

$$(\partial_1^2+m^2)\langle 0|T\bar\phi(1)\bar\phi(2)|0\rangle = 4im\,\delta_{\bar S}(1,2)$$

$$(\partial_1^2+m^2)\langle 0|T\bar\phi(1)\phi(2)|0\rangle = -i\,D_1 D_1\,\delta_S(1,2)$$

$\Rightarrow$

$$\langle 0|T\phi(1)\phi(2)|0\rangle = \frac{+4im}{\partial_1^2+m^2}\,\delta_S(1,2)$$

$$\langle 0|T\bar\phi(1)\bar\phi(2)|0\rangle = \frac{+4im}{\partial_1^2+m^2}\,\delta_{\bar S}(1,2)$$

$$\langle 0|T\phi(1)\bar\phi(2)|0\rangle = \frac{-i}{\partial_1^2+m^2}\,\bar D_1\bar D_1\,\delta_{\bar S}(1,2)$$

$$= \frac{-i}{\partial_1^2+m^2}\,D_2 D_2\,\underbrace{\delta_S(2,1)}_{=\delta_S(1,2)}$$

Hence we obtained the superpropagators as before from the component build up.

Now we would like to convert the Gell-Mann–Low expansion into an expansion in terms of superspace Feynman diagrams. We have found the free field generating functional

$$Z_0\{J,\bar{J}\} = e^{-\frac{1}{2}\int dS_1 \int dS_2\, J(1)\Delta_{12}J(2)} \times$$
$$\times\, e^{-\frac{1}{2}\int d\bar{S}_1 \int d\bar{S}_2\, \bar{J}(1)\Delta_{\bar{1}\bar{2}}\bar{J}(2)} \times$$
$$\times\, e^{-\frac{1}{2}\int dS_1 \int d\bar{S}_2\, J(1)\Delta_{1\bar{2}}\bar{J}(2)}$$
$$\times\, e^{-\frac{1}{2}\int d\bar{S}_1 \int dS_2\, \bar{J}(1)\Delta_{\bar{1}2}J(2)}$$

where the Feynman propagators are given by the free 2-pt. functions we just found.

$$\Delta_{12} \equiv \langle 0|T\,\phi(1)\phi(2)|0\rangle$$

$$\Delta_{\bar{1}\bar{2}} \equiv \langle 0|T\,\bar{\phi}(1)\bar{\phi}(2)|0\rangle$$

$$\Delta_{1\bar{2}} \equiv \langle 0|T\,\phi(1)\bar{\phi}(2)|0\rangle$$

$$\Delta_{\bar{1}2} \equiv \langle 0|T\,\bar{\phi}(1)\phi(2)|0\rangle$$