Assuming $K_{ij}(x,y) = -[Z_{ij}\partial_x^2 + m_{ij}^2]\delta^4(x-y)$

$$\Rightarrow \quad -(Z_{ij}\partial_x^2 + m_{ij}^2)\langle 0|T\phi_j(x)\phi_k(z)|0\rangle = -\delta^i{}_k\delta^4(x-z)$$

Fourier Transforming $\Rightarrow$

$$(Z_{ij}p^2 - m_{ij}^2)\tilde{G}^{(2)}_{jk}(p) = -\delta_{ik}$$

$$\Rightarrow \quad \tilde{G}^{(2)}_{ij}(p) = -(Z_{ij}p^2 - m_{ij}^2)^{-1}$$

Now let's return to the W-Z model to determine the Feynman rules in components & then superspace! Back to p. -295-

$$\Gamma = i\int d^4x\Big[16Z(\partial_\mu A\partial^\mu\bar{A} + \tfrac{i}{4}\psi\overleftrightarrow{\not\partial}\bar{\psi} + F\bar{F})$$

$$-16m\left[2AF + 2\bar{A}\bar{F} - \tfrac{1}{2}\psi\psi - \tfrac{1}{2}\bar{\psi}\bar{\psi}\right]$$

$$-12g\left[AAF + \bar{A}\bar{A}\bar{F} - \tfrac{1}{2}A\psi\psi - \tfrac{1}{2}\bar{A}\bar{\psi}\bar{\psi}\right]\Big]$$

let $Z = \frac{1}{16}$, $m \to \frac{1}{32}m$, $g \to \frac{1}{12}g$

So the bare action is (adding subscript "0" to denote bare quantities)

$$\Gamma = i\int d^4x \Big\{ \left(\partial_\mu A_0 \partial^\mu \bar{A}_0 + \tfrac{i}{4} \psi_0 \overleftrightarrow{\not\partial} \bar{\psi}_0 + F_0 \bar{F}_0\right)$$

$$- m_0\left[A_0 F_0 + \bar{A}_0 \bar{F}_0 - \tfrac{1}{4}\psi_0\psi_0 - \tfrac{1}{4}\bar{\psi}_0\bar{\psi}_0\right]$$

$$- g_0\left[A_0 A_0 F_0 + \bar{A}_0\bar{A}_0\bar{F}_0 - \tfrac{1}{2}A_0\psi_0\psi_0 - \tfrac{1}{2}\bar{A}_0\bar{\psi}_0\bar{\psi}_0\right]\Big\}$$

Now we can re-scale the superfield $\phi_0 = Z^{1/2}\phi$ so that each component field is rescaled by the same wavefunction renormalization factor $Z \equiv 1+b$, $b = O(\hbar)$ comes from loops then we can renormalize the parameters

$$m_0 Z \equiv m + a \qquad \left(\begin{array}{ll} A_0 = Z^{1/2} A & \bar{\psi}_0 = Z^{1/2}\bar{\psi} \\ \psi_0 = Z^{1/2}\psi & F_0 = Z^{1/2} F \\ \bar{A}_0 = Z^{1/2}\bar{A}, & \bar{F}_0 = Z^{1/2}\bar{F} \end{array}\right)$$

$$g_0 Z^{3/2} \equiv Z_g\, g$$

So the renormalized action is

$$\Gamma = i\int d^4x \Big\{ Z\left(\partial_\mu A \partial^\mu \bar{A} + \tfrac{i}{4}\psi \overleftrightarrow{\not\partial} \bar{\psi} + F\bar{F}\right)$$

$$- (m+a)\left[AF + \bar{A}\bar{F} - \tfrac{1}{4}\psi\psi - \tfrac{1}{4}\bar{\psi}\bar{\psi}\right]$$

$$- Z_g\, g\left(AAF + \bar{A}\bar{A}\bar{F} - \tfrac{1}{2}A\psi\psi - \tfrac{1}{2}\bar{A}\bar{\psi}\bar{\psi}\right)\Big\}$$

(We have assume a SUSY covariant regulation and renormalization scheme — not a trivial assumption!)

Now we need 3 Super normalization conditions — let's choose

"Pole" 1) $\langle 0|T\tilde{A}(p)F(0)|0\rangle^{1PI}\Big|_{p=0} \equiv -im$

(The SUSY $\Rightarrow$ $\langle 0|T\tilde{\psi}(p)\psi(0)|0\rangle^{1PI}\Big|_{p=0} = \frac{i}{2}m$ etc.)

"Residue" 2) $\frac{\partial}{\partial p^2}\langle 0|T\tilde{A}(p)\bar{A}(0)|0\rangle^{1PI}\Big|_{p^2=\mu^2} \equiv i$

"Coupling Constant" 3) $\langle 0|T\tilde{A}(p_1)\tilde{A}(p_2)F(0)|0\rangle^{1PI}\Big|_{p_1=p_2=0} \equiv -ig$

So we can divide the action into free and interaction pieces and develop Feynman rules

$$\Gamma_0 = i\int d^4x\Big\{\partial_\mu A\partial^\mu\bar{A} + \frac{i}{4}\psi\overleftrightarrow{\not\partial}\bar{\psi} + F\bar{F}$$

free part

$$-m\Big(AF + \bar{A}\bar{F} - \frac{1}{4}\psi\psi - \frac{1}{4}\bar{\psi}\bar{\psi}\Big)\Big\}$$

$$\Gamma_{int} = i\int d^4x\Big\{-Z_g g\Big(AAF + \bar{A}\bar{A}\bar{F} - \frac{1}{2}A\psi\psi - \frac{1}{2}\bar{A}\bar{\psi}\bar{\psi}\Big)$$

$$-a\Big(AF + \bar{A}\bar{F} - \frac{1}{4}\psi\psi - \frac{1}{4}\bar{\psi}\bar{\psi}\Big)$$

$$+b\Big(\partial_\mu A\partial^\mu\bar{A} + \frac{i}{4}\psi\overleftrightarrow{\not\partial}\bar{\psi} + F\bar{F}\Big)\Big\}$$

The propagators are determined from Γ_0

$$\Gamma_0 = i\int d^4x \Big[\partial_\mu A \partial^\mu \bar{A} + \frac{i}{4} \bar{\psi} \overleftrightarrow{\not\partial} \psi + \bar{F} F$$
$$- m\left(A F + \bar{A}\bar{F} - \frac{1}{4}\psi\psi - \frac{1}{4}\bar{\psi}\bar{\psi} \right) \Big]$$

So as previously we differentiate the action Γ_0

$$\frac{\delta \Gamma_0}{\delta A(x)} = -i \partial^2 \bar{A}(x) - i m F$$

Now recall $\frac{\delta \Gamma_0}{\delta A(x)} = -i J(x)$ we have the Legendre transform

$$Z^c[J, \bar{J}, \eta, \bar{\eta}, K, \bar{K}] = \Gamma[A, \bar{A}, \psi, \bar{\psi}, F, \bar{F}]$$
$$+ i\int d^4x \left[J A + \bar{J}\bar{A} + \eta^\alpha \psi_\alpha + \bar{\eta}_{\dot\alpha} \bar{\psi}^{\dot\alpha} + K F + \bar{K}\bar{F} \right]$$

So $\frac{\delta \Gamma}{\delta A} = -iJ \qquad \frac{\delta Z^c}{i\delta J} = A$

$$\frac{\delta \Gamma}{\delta \bar{A}} = -i\bar{J} \qquad \frac{\delta Z^c}{i\delta \bar{J}} = \bar{A}$$

$$\frac{\delta \Gamma}{\delta \psi^\alpha} = -i\eta_\alpha \qquad \frac{\delta Z^c}{i\delta \eta^\alpha} = \psi_\alpha$$

$$\frac{\delta \Gamma}{\delta \bar{\psi}^{\dot\alpha}} = +i\bar{\eta}_{\dot\alpha} \qquad \frac{\delta Z^c}{i\delta \bar{\eta}^{\dot\alpha}} = -\bar{\psi}_{\dot\alpha}$$

$$\frac{\delta\Gamma}{\delta F} = -iK \qquad \frac{\delta Z^c}{i\delta K} = F$$

$$\frac{\delta\Gamma}{\delta \bar{F}} = -i\bar{K} \qquad \frac{\delta Z^c}{i\delta \bar{K}} = \bar{F}$$

So our first equation is

$$\frac{\delta\Gamma_0}{\delta A(x)} = -i\partial^2 \bar{A}(x) - imF(x) = -iJ(x)$$

$$\frac{\delta\Gamma_0}{\delta \bar{A}(x)} = -i\partial^2 A(x) - im\bar{F}(x) = -i\bar{J}(x)$$

$$\frac{\delta\Gamma_0}{\delta F(x)} = i\bar{F}(x) - im\bar{A}(x) = -i\bar{K}(x)$$

$$\frac{\delta\Gamma_0}{\delta \bar{F}(x)} = iF(x) - imA(x) = -iK(x)$$

$$\frac{\delta\Gamma_0}{\delta\psi^\alpha(x)} = -\frac{1}{2}(\not{\partial}\bar{\psi})_\alpha(x) + i\frac{m}{2}\psi_\alpha(x) = -i\eta_\alpha(x)$$

$$\frac{\delta\Gamma_0}{\delta\bar{\psi}^{\dot\alpha}(x)} = -\frac{1}{2}(\partial_\mu\psi(x)\sigma^\mu)_{\dot\alpha} - \frac{i}{2}m\bar{\psi}_{\dot\alpha}(x) = +i\bar{\eta}_{\dot\alpha}(x)$$

So we have the coupled equations for the propagators

Solving the auxiliary field equations

$$F = m\bar{A} - \bar{K}$$

$$\bar{F} = mA - K$$

Substitute this into the scalar field equations

$$\partial^2 \bar{A} + m(m\bar{A} - \bar{K}) = J \Rightarrow \boxed{(\partial^2 + m^2)\bar{A} = J + m\bar{K}}$$

$$\partial^2 A + m(mA - K) = \bar{J} \Rightarrow \boxed{(\partial^2 + m^2)A = \bar{J} + mK}$$

Now we have that $A = \frac{\delta Z^c}{i\delta J}$; $\bar{A} = \frac{\delta Z^c}{i\delta \bar{J}}$

$$\Rightarrow \boxed{\begin{aligned} (\partial^2 + m^2)\frac{\delta Z^c}{i\delta \bar{J}(x)} &= J(x) + m\bar{K}(x) \\ (\partial^2 + m^2)\frac{\delta Z^c}{i\delta J(x)} &= \bar{J}(x) + mK(x) \end{aligned}}$$

So we find the various 2-point functions by differentiating

$$(\partial^2 + m^2)\underbrace{\frac{\delta^2 Z^c}{i\delta J(y)\, i\delta \bar{J}(x)}}_{\langle 0|T\,\bar{A}(x)A(y)|0\rangle} = -i\delta^4(x-y)$$

Fourier Transforming ⇒

$$\boxed{\langle 0 | T \tilde{A}(p)\, A(0) | 0 \rangle = \frac{i}{p^2 - m^2}}$$

Note also.

$$(\partial^2 + m^2) \underbrace{\frac{\delta^2 Z^c}{i\delta \bar{J}(x)\, i\delta \bar{K}(y)}} = -im\, \delta^4(x-y)$$

$$= \langle 0 | T \bar{A}(x) \bar{F}(y) | 0 \rangle$$

Fourier Transforming

$$\boxed{\langle 0 | T \tilde{\bar{A}}(p)\, \bar{F}(0) | 0 \rangle = \frac{im}{p^2 - m^2}}$$

Continuing we see $\boxed{\begin{aligned} \langle 0 | T \bar{A}(x) \bar{A}(y) | 0 \rangle = 0 \\ \langle 0 | T \bar{A}(x) \bar{F}(y) | 0 \rangle = 0 \end{aligned}}$

Turning to the next equation of motion

$$(\partial^2 + m^2) \underbrace{\frac{\delta^2 Z^c}{i\delta J(x)\, i\delta \bar{J}(y)}} = -i\delta^4(x-y)$$

$$= \langle 0 | T A(x) \bar{A}(y) | 0 \rangle$$

⇒

$$\boxed{\langle 0 | T \tilde{A}(p)\, \bar{A}(0) | 0 \rangle = \frac{i}{p^2 - m^2}}$$

$$(\partial^2+m^2)\underbrace{\frac{\delta^2 Z^c}{i\delta J(x)\, i\delta K(y)}} = -im\delta^4(x-y)$$

$$= \langle 0|T A(x) F(y)|0\rangle$$

$$\Rightarrow \boxed{\langle 0|T\tilde{A}(p) F(0)|0\rangle = \frac{im}{p^2-m^2}}$$

oops! insert p. -334'

and likewise $\boxed{\begin{aligned}\langle 0|T A(x) A(y)|0\rangle = 0\\ \langle 0|T A(x) \bar{F}(y)|0\rangle = 0\end{aligned}}$

Finally consider the fermion equations of motion

$$+\frac{i}{2}(\not{\partial}\bar{\psi}(x))_\alpha + \frac{m}{2}\psi_\alpha(x) = -\eta_\alpha(x)$$

$$\frac{i}{2}(\partial_\mu\psi(x)\sigma^\mu)_{\dot{\alpha}} - \frac{m}{2}\bar{\psi}_{\dot{\alpha}}(x) = \bar{\eta}_{\dot{\alpha}}(x)$$

Applying $-i\bar{\not{\partial}}^{\dot{\alpha}\alpha}$ to the first

$$\Rightarrow \frac{1}{2}(\bar{\not{\partial}}\not{\partial}\bar{\psi})^{\dot{\alpha}} - m\frac{i}{2}(\bar{\not{\partial}}\psi)^{\dot{\alpha}} = +i(\bar{\not{\partial}}\eta)^{\dot{\alpha}}$$

but $(\bar{\not{\partial}}\not{\partial})^{\dot{\alpha}}{}_{\dot{\beta}} = \partial^2\delta^{\dot{\alpha}}{}_{\dot{\beta}}$

$$\Rightarrow \partial^2\bar{\psi}^{\dot{\alpha}} - 2m\frac{i}{2}(\bar{\not{\partial}}\psi)^{\dot{\alpha}} = 2i(\bar{\not{\partial}}\eta)^{\dot{\alpha}}$$

Now back to the $F, \bar{F}$ equations

$$F = m\bar{A} - \bar{K} \quad ; \quad \bar{F} = mA - K$$

$$\Rightarrow \frac{\delta Z^c}{i\delta K} = m\frac{\delta Z^c}{i\delta \bar{J}} - \bar{K} \quad ; \quad \frac{\delta Z^c}{i\delta \bar{K}} = m\frac{\delta Z^c}{i\delta J} - K$$

$$\Rightarrow \frac{\delta^2 Z^c}{i\delta K(x)\, i\delta \bar{K}(y)} = m\frac{\delta^2 Z^c}{i\delta \bar{J}(x)\, i\delta \bar{K}(y)} + i\delta^4(x-y)$$

$\Rightarrow$

$$\boxed{\langle 0|T\, F(x)\bar{F}(y)|0\rangle = m\langle 0|T\, \bar{A}(x)\bar{F}(y)|0\rangle + i\delta^4(x-y)}$$

F.T. $\Rightarrow$

$$\langle 0|T\, \tilde{F}(p)\bar{F}(0)|0\rangle = m\underbrace{\langle 0|T\, \tilde{\bar{A}}(p)\bar{F}(0)|0\rangle}_{=\frac{im}{p^2-m^2}\ (\text{p. -333-})} + i$$

$$= \frac{im^2 + i(p^2-m^2)}{p^2-m^2} = \frac{ip^2}{p^2-m^2}$$

$$\boxed{\langle 0|T\, \tilde{F}(p)\bar{F}(0)|0\rangle = \frac{ip^2}{p^2-m^2}}$$

Also

$$\frac{\delta^2 Z^c}{i\delta K(x)\, i\delta J(y)} = m\frac{\delta^2 Z^c}{i\delta \bar{J}(x)\, i\delta J(y)}$$

So

$$\boxed{\langle 0|T\, F(x)\, A(y)|0\rangle = m\langle 0|T\, \bar{A}(x)\, A(y)|0\rangle}$$

F.T. $\Rightarrow$

$$\langle 0|T\,\tilde{F}(p)\,A(0)|0\rangle = m\underbrace{\langle 0|T\,\tilde{\bar{A}}(p)\,A(0)|0\rangle}_{=\frac{i}{p^2-m^2}\quad p.\,-333-}$$

$\Rightarrow$

$$\boxed{\langle 0|T\,\tilde{F}(p)\,A(0)|0\rangle = \frac{im}{p^2-m^2}}$$

Likewise $\boxed{\langle 0|T\, F(x)\, F(y)|0\rangle = 0 = \langle 0|T\, F(x)\,\bar{A}(y)|0\rangle}$

Similarly using $\dfrac{\delta Z^c}{i\delta K} = m\dfrac{\delta Z^c}{i\delta J} - K$

$\Rightarrow$

$$\frac{\delta^2 Z^c}{i\delta K(x)\, i\delta K(y)} = m\frac{\delta^2 Z^c}{i\delta J(x)\, i\delta K(y)} + i\delta^4(x-y)$$

$\Rightarrow$

$$\langle 0|T\,\bar{F}(x)\,F(y)|0\rangle = m\underbrace{\langle 0|T\, A(x)\, F(y)|0\rangle}_{\left(\text{F.T.}\ = \frac{im}{p^2-m^2}\ (p.\,-334-)\right)} + i\delta^4(x-y)$$

$$\langle 0|T\,\tilde{\bar{F}}(p)\,F(0)|0\rangle = \frac{im^2 + i(p^2-m^2)}{p^2-m^2} = \frac{ip^2}{p^2-m^2}$$

So $\boxed{\langle 0|T\,\tilde{\bar{F}}(p)F(0)|0\rangle = \frac{ip^2}{p^2-m^2}}$ —334''—

$$\frac{\delta^2 Z^c}{i\delta\bar{K}(x)\,i\delta\bar{\bar{J}}(y)} = m\,\frac{\delta^2 Z^c}{i\delta J(x)\,i\delta\bar{\bar{J}}(y)}$$

$\Rightarrow$

$$\boxed{\langle 0|T\,\bar{F}(x)\bar{A}(y)|0\rangle = m\langle 0|T\,A(x)\bar{A}(y)|0\rangle}$$

F.T. $\Rightarrow$

$$\langle 0|T\,\tilde{\bar{F}}(p)\bar{A}(0)|0\rangle = m\underbrace{\langle 0|T\,\tilde{A}(p)\bar{A}(0)|0\rangle}_{=\frac{i}{p^2-m^2}\ (\text{p.}-333-)}$$

So

$$\boxed{\langle 0|T\,\tilde{\bar{F}}(p)\bar{A}(0)|0\rangle = \frac{im}{p^2-m^2}}$$

Likewise $\boxed{\langle 0|T\,\bar{F}(x)A(y)|0\rangle = 0 = \langle 0|T\,F(x)F(y)|0\rangle}$

but the second equation of motion is

$$\frac{i}{2}\partial_\mu \psi^\alpha \sigma^\mu_\alpha{}^{\dot\alpha} - \frac{m}{2}\bar\psi^{\dot\alpha} = \bar\eta^{\dot\alpha}$$

$$\parallel$$

$$-\frac{i}{2}\bar\sigma^{\mu\dot\alpha\alpha}\partial_\mu\psi_\alpha$$

$\Rightarrow$

$$-\frac{i}{2}(\bar{\partial\!\!\!/}\psi)^{\dot\alpha} - \frac{m}{2}\bar\psi^{\dot\alpha} = \bar\eta^{\dot\alpha}$$

plugging this into the first $\Rightarrow$

$$\partial^2\bar\psi^{\dot\alpha} + 2m\left(\frac{m}{2}\bar\psi^{\dot\alpha} + \bar\eta^{\dot\alpha}\right) = 2i(\bar{\partial\!\!\!/}\eta)^{\dot\alpha}$$

$$\Rightarrow \boxed{(\partial^2 + m^2)\bar\psi^{\dot\alpha} = -2i\,\partial_\mu\eta^\alpha\sigma^\mu_\alpha{}^{\dot\alpha} - 2m\bar\eta^{\dot\alpha}}$$

Likewise differentiating the second equation $\Rightarrow$

$$\frac{i}{2}(\psi\overleftarrow{\partial\!\!\!/}\overleftarrow{\bar{\partial\!\!\!/}})^\alpha - \frac{m}{2}(\bar\psi\overleftarrow{\bar{\partial\!\!\!/}})^\alpha = (\bar\eta\overleftarrow{\bar{\partial\!\!\!/}})^\alpha$$

also $(\partial\!\!\!/\bar{\partial\!\!\!/})_\beta{}^\alpha = \partial^2\delta_\beta{}^\alpha \Rightarrow$

$$\frac{i}{2}\partial^2\psi^\alpha - \frac{m}{2}(\bar\psi\overleftarrow{\bar{\partial\!\!\!/}})^\alpha = (\bar\eta\overleftarrow{\bar{\partial\!\!\!/}})^\alpha$$

but the first eq. is $-\frac{i}{2}(\bar\psi\overleftarrow{\bar{\partial\!\!\!/}})^\alpha = -\frac{m}{2}\psi^\alpha - \eta^\alpha$

$$\partial^2\psi^\alpha + 2m\left(\frac{m}{2}\psi^\alpha + \eta^\alpha\right) = -2i(\bar\eta\overleftarrow{\bar{\partial\!\!\!/}})^\alpha$$

Substituting $\Rightarrow$

$$\boxed{(\partial^2+m^2)\psi_\alpha = -2i(\partial_\mu\bar\eta\,\bar\sigma^\mu)_\alpha - 2m\eta_\alpha}$$

$$\Rightarrow \boxed{(\partial_x^2+m^2)\frac{\delta Z^c}{i\delta\eta^\alpha(x)} = +2i(\partial_\mu\bar\eta_{\dot\alpha}(x)\,\bar\sigma^{\mu\,\dot\alpha}{}_{\alpha}) - 2m\eta_\alpha(x)}$$

So differentiating wrt η & $\bar\eta$ $\Rightarrow$

$$-(\partial_x^2+m^2)\underbrace{\frac{\delta^2 Z^c}{i\delta\eta^\alpha(x)\,i\delta\eta^\beta(y)}}_{} = +2mi\,\epsilon_{\alpha\beta}\,\delta^4(x-y)$$

$$= \langle 0|T\,\psi_\alpha(x)\,\psi_\beta(y)|0\rangle$$

$$\Rightarrow \boxed{(-\partial_x^2-m^2)\langle 0|T\,\psi_\alpha(x)\,\psi_\beta(y)|0\rangle = 2im\,\epsilon_{\alpha\beta}\,\delta^4(x-y)}$$

Fourier Transform $\Rightarrow$

$$\boxed{\langle 0|T\,\tilde\psi_\alpha(p)\,\psi_\beta(0)|0\rangle = \frac{2im\,\epsilon_{\alpha\beta}}{p^2-m^2}}$$

Also we obtain

$$-(\partial_x^2+m^2)\frac{\delta^2 Z^c}{i\delta\eta^\alpha(x)\, i\delta\bar\eta^{\dot\alpha}(y)} = 2\,\bar\sigma^\mu_{\dot\alpha\alpha}\,\partial^x_\mu\,\delta^4(x-y)$$

$$= \langle 0|T\psi_\alpha(x)\left(-\bar\psi_{\dot\alpha}(y)\right)|0\rangle$$

$\Rightarrow$

$$\boxed{(-\partial_x^2-m^2)\langle 0|T\psi_\alpha(x)\bar\psi_{\dot\alpha}(y)|0\rangle = -2\,\sigma^\mu_{\alpha\dot\alpha}\,\partial^x_\mu\,\delta^4(x-y)}$$

Fourier Transform

$$(-\partial_x^2-m^2)\int\frac{d^4p}{(2\pi)^4}e^{-ip(x-y)}\langle 0|T\tilde\psi_\alpha(p)\bar\psi_{\dot\alpha}(0)|0\rangle$$

$$= -2\,\sigma^\mu_{\alpha\dot\alpha}\,\partial^x_\mu\int\frac{d^4p}{(2\pi)^4}e^{-ip(x-y)}$$

$\Rightarrow$

$$\boxed{\langle 0|T\tilde\psi_\alpha(p)\bar\psi_{\dot\alpha}(0)|0\rangle = \frac{+2i\,p\!\!\!/_{\alpha\dot\alpha}}{p^2-m^2}}$$

Similarly we could use the other field equation

$$(\partial_x^2+m^2)\,\bar{\psi}_{\dot{\alpha}}(x) = -2i\,\partial^x_\mu\,\eta^\beta(x)\,\sigma^\mu_{\beta\dot{\alpha}} - 2m\,\bar{\eta}_{\dot{\alpha}}(x)$$

$\Rightarrow$

$$\boxed{(\partial_x^2+m^2)\frac{\delta Z^c}{i\delta\bar{\eta}^{\dot{\alpha}}(x)} = +2i\,\partial^x_\mu\,\eta^\beta(x)\,\sigma^\mu_{\beta\dot{\alpha}} + 2m\,\bar{\eta}_{\dot{\alpha}}(x)}$$

Differentiating wrt $\bar{\eta}^{\dot{\beta}}$ $\Rightarrow$

$$-(\partial_x^2+m^2)\frac{\delta^2 Z^c}{i\delta\bar{\eta}^{\dot{\alpha}}(x)\,i\delta\bar{\eta}^{\dot{\beta}}(y)} = -2im\,\epsilon_{\dot{\alpha}\dot{\beta}}\,\delta^4(x-y)$$

$\Rightarrow$

$$\boxed{(-\partial_x^2-m^2)\langle 0|T\,\bar{\psi}_{\dot{\alpha}}(x)\,\bar{\psi}_{\dot{\beta}}(y)|0\rangle = -2im\,\epsilon_{\dot{\alpha}\dot{\beta}}\,\delta^4(x-y)}$$

Fourier Transform $\Rightarrow$

$$\boxed{\langle 0|T\,\tilde{\bar{\psi}}_{\dot{\alpha}}(p)\,\bar{\psi}_{\dot{\beta}}(0)|0\rangle = \frac{-2im\,\epsilon_{\dot{\alpha}\dot{\beta}}}{p^2-m^2}}$$

And diff. wrt $\eta^\alpha(y)$ $\Rightarrow$

$$-(\partial_x^2+m^2)\underbrace{\frac{\delta^2 Z^c}{i\delta\bar{\eta}^{\dot{\alpha}}(x)\,i\delta\eta^\alpha(y)}} = 2\,\partial^x_\mu\,\delta^4(x-y)\,\sigma^\mu_{\alpha\dot{\alpha}}$$

$$= -\langle 0|T\,\bar{\psi}_{\dot{\alpha}}(x)\,\psi_\alpha(y)|0\rangle$$

So we find

$$\boxed{(-\partial_x^2 - m^2)\langle 0|T\bar{\psi}_{\dot\alpha}(x)\,\psi_\alpha(y)|0\rangle = -2\partial^x_\mu \delta^4(x-y)\,\bar\sigma^\mu_{\alpha\dot\alpha}}$$

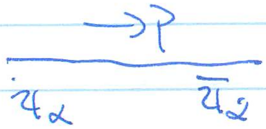
F.T. $\Rightarrow$

$$\boxed{\langle 0|T\tilde{\bar{\psi}}_{\dot\alpha}(p)\,\psi_\alpha(0)|0\rangle = \frac{+2i\not{p}_{\alpha\dot\alpha}}{p^2-m^2}}$$

So we have found all the propagators in the Feynman diagram expansion

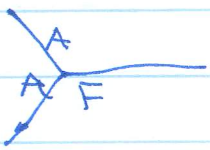
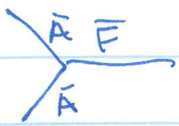
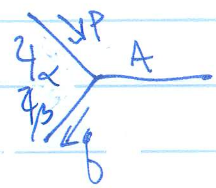
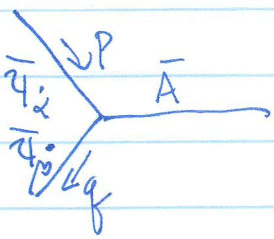
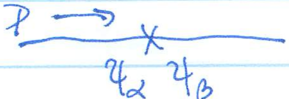
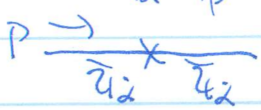
Line	Propagator Factor
$A \xleftarrow{p} \bar{A}$	$\frac{i}{p^2-m^2}$
$A \xleftarrow{p} F$	$\frac{im}{p^2-m^2}$
$\bar{A} \xleftarrow{p} \bar{F}$	$\frac{im}{p^2-m^2}$
$\psi_\beta \xrightarrow{p} \psi_\alpha$	$\frac{2im\,\epsilon_{\alpha\beta}}{p^2-m^2}$
$\bar\psi_{\dot\beta} \xrightarrow{p} \bar\psi_{\dot\alpha}$	$-\frac{2im\,\epsilon_{\dot\alpha\dot\beta}}{p^2-m^2}$
$\bar\psi_{\dot\alpha} \xrightarrow{p} \psi_\alpha$	$\frac{+2i\not{p}_{\alpha\dot\alpha}}{p^2-m^2}$

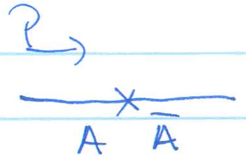
Line	Propagator Factor
$\xrightarrow{p}$ $\bar{F}$ — F	$\frac{ip^2}{p^2-m^2}$
$\xrightarrow{p}$ A — F	$\frac{im}{p^2-m^2}$
$\xrightarrow{p}$ $\bar{A}$ — $\bar{F}$	$\frac{im}{p^2-m^2}$



$$+\frac{2i\,p_{\alpha\dot{\alpha}}}{p^2-m^2}$$

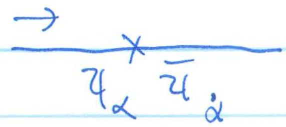
Now from Γ_{int} we read off the vertices

Vertex	Factor
	$-2iZ_g g$
	$-2iZ_g g$
	$+iZ_g g\,\epsilon_{\alpha\beta}$
	$-iZ_g g\,\epsilon_{\dot{\alpha}\dot{\beta}}$
	$-ia$
	$-ia$
	$\frac{i}{2}a\,\epsilon_{\alpha\beta}$
	$\frac{i}{2}a\,\epsilon_{\dot{\alpha}\dot{\beta}}$



ibp^2

ib



$-\frac{ib}{2}\not{p}_{\alpha\dot\alpha}$

As usual there are the other ingredients

$\int \frac{d^4k_1}{(2\pi)^4}\cdots\frac{d^4k_{m(\Gamma)}}{(2\pi)^4}$ integration over the $m(\Gamma)$-loop momenta

inclusion of the symmetry # $\alpha(\Gamma)$ and the over-all energy-momentum conserving delta functions.

$$\langle 0|T\,A(x_1)\cdots\bar{A}(\bar{x}_1)\cdots F(y_1)\cdots\bar{F}(\bar{y}_1)\cdots\psi_{\alpha_1}(z_1)\cdots\bar{\psi}_{\dot\alpha_1}(\bar{z}_1)\cdots|0\rangle_{1PI}$$

$$= \int \frac{d^4p_1}{(2\pi)^4}\cdots\frac{d^4\bar{z}_{n_{\bar\psi}}}{(2\pi)^4}\, e^{-i(p_1x_1+\cdots+q_{n_{\bar\psi}}z_{n_{\bar\psi}})}\times$$

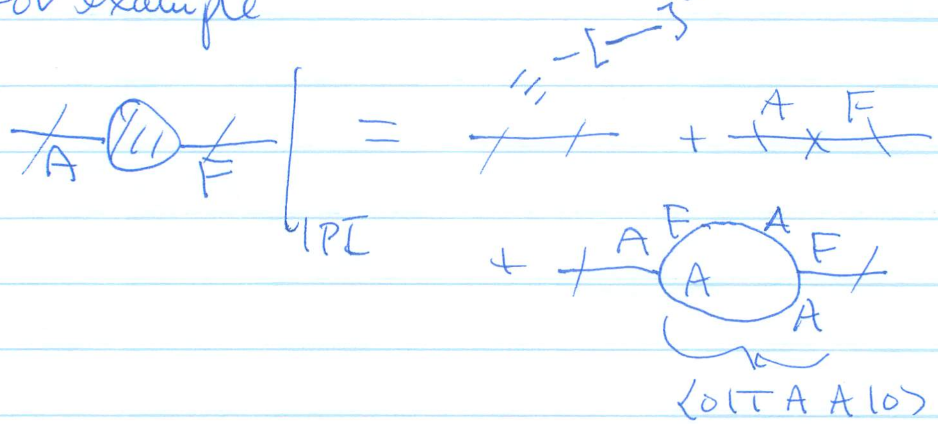
$$\times(2\pi)^4\delta^4(p_1+\cdots+q_{n_{\bar\psi}})\sum_{\Gamma\in G_{1PI}^{(n_A,n_{\bar{A}},\ldots,n_{\bar\psi})}}\alpha(\Gamma)\times$$

$$\times\int\frac{d^4k_1}{(2\pi)^4}\cdots\frac{d^4k_{m(\Gamma)}}{(2\pi)^4}\, I_\Gamma(p,\ldots,q,k)$$

where the Feynman integrand I_Γ is built from the above diagrammatic lines & vertices.

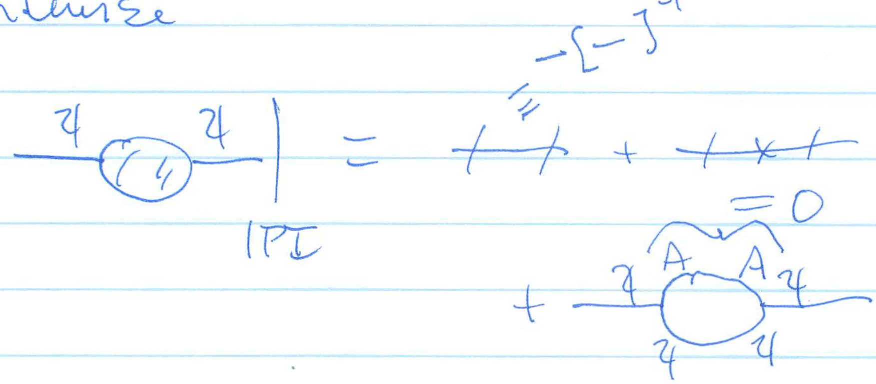
So let's begin by finding the RGE β & γ for the model in one-loop. We first note that the pure chiral 1-PI functions vanish at zero momentum (and in general in one-loop). This is a property in all orders as we will see when we consider supergraphs.

For example



$$= -i(m+a)$$

Likewise



$$= +\frac{i}{2}(m+a)$$

Hence the normalization condition $\Rightarrow$

$$\langle 0|T\tilde{A}(p)F(0)|0\rangle^{1PI}\Big|_{p=0} \equiv -im = -i(m+a)$$

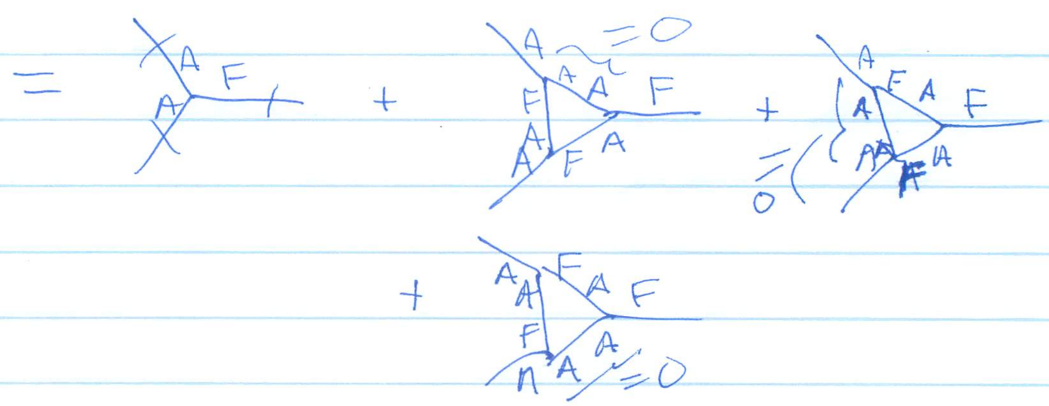
$$\Rightarrow \boxed{a=0}$$ This is true to all orders

although we have shown this in 1-loop here.

(The SUSY implies the consistent normalization condition for $\langle 0|T\tilde{\psi}(p)\psi(0)|0\rangle^{1PI} = \frac{i}{2}m$ with $a=0$)

Similarly, the 1-PI pure chiral vertex has no radiative corrections at zero momentum

$$\langle 0|T\tilde{A}(p_1)\tilde{A}(p_2)F(0)|0\rangle^{1PI}\Big|_{p_1=p_2=0} \equiv -ig$$



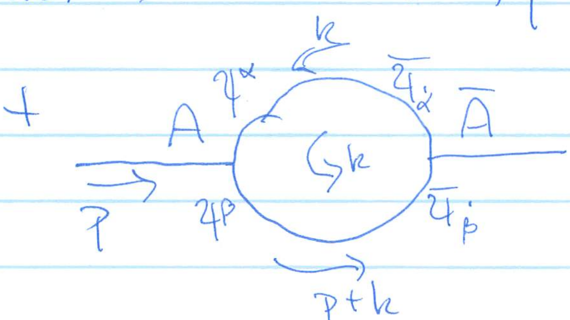
$$= -iZ_g g \Rightarrow \boxed{Z_g = 1}$$

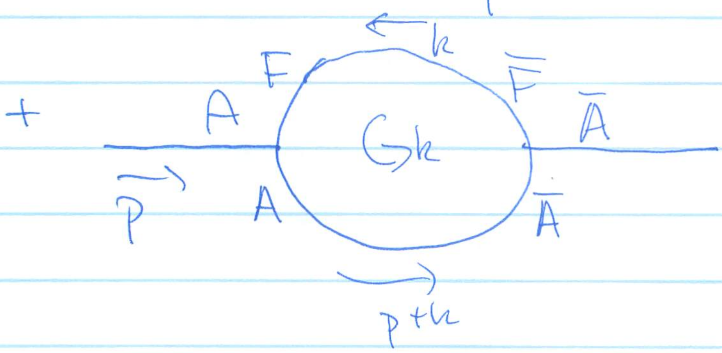
This condition persists to all orders.
Pure (anti-) chiral 1-PI functions have no radiative corrections at zero momentum.

This is known as the no renormalization theorem for the superpotential since $a=0$ & $Z_\phi=1$ there are no radiative corrections to $W(\phi)$ (at zero momentum) wysiwyg.

On the other hand mixed 1PI functions have radiative corrections in one-loop and more

$$\langle 0|T\tilde{A}(p)\bar{A}(0)|0\rangle^{PI} = +i(1+b)p^2$$





$$\langle 0| T\, \tilde{A}(p)\, \tilde{A}(0)\, |0\rangle^{1PI} = i(1+b)\, p^2$$

$$+ \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} (ig)^2 \frac{2i(\not{k})^{\dot\alpha}{}_{\alpha}}{k^2 - m^2} \frac{2i(\bar{\not{p}} + \bar{\not{k}})^{\dot\alpha}{}_{\alpha}}{(p+k)^2 - m^2}$$

Symmetry number from combinatorics

$$+ (-2ig)^2 \int \frac{d^4k}{(2\pi)^4} \frac{ik^2}{k^2 - m^2} \frac{i}{(p+k)^2 - m^2}$$

$$= i(1+b)\, p^2 + \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2} \frac{1}{(p+k)^2 - m^2} \left[4g^2\right] \times$$

$$\times \left[k^2 + \frac{1}{2} \underbrace{\not{k}^{\alpha}{}_{\dot\alpha} (\bar{\not{p}} + \bar{\not{k}})^{\dot\alpha}{}_{\alpha}}_{= -2(k\cdot p + k^2)} \right]$$

$$= i(1+b)\, p^2 + 4g^2 \int \frac{d^4k}{(2\pi)^4} \frac{(+k^2 - p\cdot k - k^2)}{(k^2 - m^2)((p+k)^2 - m^2)}$$

Now recall

$$\frac{-p\cdot k}{((p+k)^2 - m^2)(k^2 - m^2)} = \int_0^1 d\alpha \frac{-p\cdot k}{\left[\alpha((p+k)^2 - m^2) + (1-\alpha)(k^2 - m^2)\right]^2}$$

Now the denominator $= \alpha(p+k)^2 - \alpha m^2 + k^2 - m^2 - \alpha(k^2 - m^2)$

$$= \cancel{\alpha k^2} - \cancel{\alpha k^2} + 2p\cdot k\,\alpha + \alpha p^2 - \cancel{\alpha m^2} + \cancel{\alpha m^2} + k^2 - m^2$$

$$= \alpha(2p\cdot k + p^2) + k^2 - m^2$$

Let $l = k + xp$; $k^2 = l^2 + x^2p^2 - 2x\,l\cdot p$

$k = l - xp$; $2p\cdot k = 2p\cdot l - 2xp^2$

So

$$\text{Den.} = \alpha\left[2p\cdot l - 2xp^2 + p^2\right] + l^2 + x^2p^2 - 2x\,l\cdot p - m^2$$

$$= 2p\cdot l\left[\alpha - x\right] + l^2 + p^2(\alpha + x^2 - 2\alpha x) - m^2$$

Let $\alpha = x$ $\Rightarrow$

$$\text{Den.} = l^2 + \alpha(1-\alpha)p^2 - m^2$$

$\Rightarrow$ $(x = \alpha)$

$$\frac{+p\cdot k}{\left[(p+k)^2 - m^2\right]\left[k^2 - m^2\right]} = \int_0^1 d\alpha \, \frac{+\alpha p^2 - p\cdot l}{\left[l^2 + \alpha(1-\alpha)p^2 - m^2\right]^2}$$

So let

$$\langle 0|T\tilde{A}(p)\,A(0)|0\rangle^{\text{1PI}} = i(1+b)p^2 \, i\Pi'(p^2)$$

$\Rightarrow$

$$-i\Pi'(p^2) = 4g^2\int\frac{d^4k}{(2\pi)^4}\,\frac{-p\cdot k}{(k^2-m^2)\left((p+k)^2 - m^2\right)}$$

$$= 4g^2\int_0^1 d\alpha\int\frac{d^4l}{(2\pi)^4}\,\frac{\alpha p^2 \overbrace{- l\cdot p}^{\text{odd in } l \;\to 0}}{\left[l^2 + \alpha(1-\alpha)p^2 - m^2\right]^2}$$

$$= 4g^2\int_0^1 d\alpha\int\frac{d^4l}{(2\pi)^4}\,\frac{\alpha p^2}{\left[l^2 + \alpha(1-\alpha)p^2 - m^2\right]^2}$$

Now recall the d-dimensional integrals

$$\int \frac{d^d l}{(2\pi)^d} \frac{1}{(l^2-\Delta)^2} = \frac{i}{(4\pi)^{d/2}} \frac{\Gamma(2-\frac{d}{2})}{\Gamma(2)} \left(\frac{1}{\Delta}\right)^{2-d/2}$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{l^2}{(l^2-\Delta^2)^2} = \frac{-i}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(2-\frac{d}{2}-1)}{\Gamma(2)} \left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}-1}$$

$$\Delta = m^2 - \alpha(1-\alpha)p^2$$

let $d = 4-\epsilon \Rightarrow 2-\frac{d}{2} = \epsilon/2 \; ; \; 1-\frac{d}{2} = \frac{\epsilon}{2}-1$

$$\boxed{-i\Pi'(p^2) = p^2 4g^2 \int_0^1 d\alpha\, \alpha \frac{i}{(4\pi)^{d/2}} \frac{\Gamma(\epsilon/2)}{\Gamma(2)} \left(\frac{1}{\Delta}\right)^{\epsilon/2}}$$

$\Rightarrow$

$$\boxed{\begin{aligned}\langle 0|T\tilde{\bar{A}}(p)\bar{A}(0)|0\rangle^{1PI} &= i(1+b)p^2 \\ &+ ip^2 4g^2 \int_0^1 d\alpha\, \alpha \frac{\Gamma(\epsilon/2)}{(4\pi)^{d/2}} e^{-\frac{\epsilon}{2}\ln\Delta}\end{aligned}}$$

$$\Delta = m^2 - \alpha(1-\alpha)p^2$$

Now this normalization condition $\Rightarrow$

$$\frac{\partial}{\partial p^2} \langle 0|T\tilde{\bar{A}}(p)\bar{A}(0)|0\rangle^{1PI}\Big|_{p^2=-\mu^2} \equiv i$$

$$\doteq i(1+b) + i\frac{4g^2}{(4\pi)^2}\int_0^1 d\alpha\, \alpha\, e^{-\frac{\epsilon}{2}\ln(m^2+\alpha(1-\alpha)\mu^2)}\, \Gamma(\epsilon/2) \times$$

$$\times\left[1 - \frac{\epsilon}{2}\frac{\alpha(1-\alpha)\mu^2}{m^2+\alpha(1-\alpha)\mu^2}\right]$$

$$\Rightarrow \boxed{b = -\frac{g^2}{(2\pi)^2}\int_0^1 d\alpha\, \alpha\, \Gamma\left(\frac{\epsilon}{2}\right) e^{-\frac{\epsilon}{2}\ln[m^2+\alpha(1-\alpha)\mu^2]} \times \left[1 - \frac{\epsilon}{2}\frac{\alpha(1-\alpha)\mu^2}{m^2+\alpha(1-\alpha)\mu^2}\right]}$$

Let's isolate the divergent part & finite terms

$$b = -\frac{g^2}{(2\pi)^2}\int_0^1 d\alpha\,\alpha\left(\frac{2}{\epsilon} - \gamma + O(\epsilon)\right)\times\left[1-\frac{\epsilon}{2}\frac{\alpha(1-\alpha)\mu^2}{m^2+\alpha(1-\alpha)\mu^2}\right]\times\left[1 - \frac{\epsilon}{2}\ln[m^2+\alpha(1-\alpha)\mu^2] + \dots\right]$$

$$\boxed{b = -\frac{g^2}{(2\pi)^2}\int_0^1 d\alpha\,\alpha\left[\frac{2}{\epsilon} - \ln[m^2+\alpha(1-\alpha)\mu^2] - \gamma - \frac{\alpha(1-\alpha)\mu^2}{m^2+\alpha(1-\alpha)\mu^2} + O(\epsilon)\right]}$$

Thus the field wavefunction renormalization is the only divergence we encounter — in one-loop we find

$$\boxed{Z = 1 + b = 1 - \frac{g^2}{(2\pi)^2}\int_0^1 d\alpha\,\alpha\left[\frac{2}{\epsilon} - \gamma - \ln[m^2+\alpha(1-\alpha)\mu^2] - \frac{\alpha(1-\alpha)\mu^2}{m^2+\alpha(1-\alpha)\mu^2}\right]}$$

Now recall the renormalization group eq. is

$$\left(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} - \gamma(N_A + N_{\bar{A}} + N_\psi + N_{\bar{\psi}} + N_F + N_{\bar{F}}) + \gamma_m m\frac{\partial}{\partial m}\right) \times \Gamma^{(N_A+\cdots+N_{\bar{F}})}(p_1,\ldots,p_{\bar{F}}) = 0$$

applying this to the normalization condition we find for the A-F 2pt. function

$$\left(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} - 2\gamma + \gamma_m m\frac{\partial}{\partial m}\right)\Gamma_{AF}(p)\Big|_{p=0} = 0$$

Now $\langle 0|T\tilde{A}(p)F(0)|0\rangle^{1PI}\Big|_{p=0} = -im$

$\Rightarrow$

$$\left(-2\gamma + \gamma_m m\frac{\partial}{\partial m}\right)(-im) = 0$$

$$\Rightarrow 2\gamma(-im) = -i\gamma_m m$$

$$\Rightarrow \boxed{\gamma_m = 2\gamma}$$

Applied to the vertex function $\Rightarrow$

$$0 = \left(\beta\frac{\partial}{\partial g} - 3\gamma\right)\langle 0|T\tilde{A}(p_1)\tilde{A}(p_2)F(0)|0\rangle^{1PI}\Big|_{p_1=0=p_2}$$

$$= \left(\beta\frac{\partial}{\partial g} - 3\gamma\right)(-ig)$$

So $\Rightarrow$

$$\boxed{\beta = 3g\gamma}$$

Finally applying the RGE to the wavefunction renormalization

$$\left(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} + \gamma_m m\frac{\partial}{\partial m} - 2\gamma\right)\langle 0|T\tilde{A}(p)\bar{A}(0)|0\rangle^{1PI} = 0$$

So $\frac{\partial}{\partial p^2}$ and $\Big|_{p^2=-\mu^2} \Rightarrow$

$$\left(\mu\frac{\partial}{\partial\mu}\frac{\partial}{\partial p^2}\langle 0|T\tilde{A}(p)\bar{A}(0)|0\rangle^{1PI}\right)\Bigg|_{p^2=-\mu^2}$$

$$+\beta\frac{\partial}{\partial g}(i) + \gamma_m m\frac{\partial}{\partial m}(i) - 2\gamma i = 0$$

$$\Rightarrow \boxed{2i\gamma = \left(\mu\frac{\partial}{\partial\mu}\frac{\partial}{\partial p^2}\langle 0|T\tilde{A}(p)\bar{A}(0)|0\rangle^{1PI}\right)\Bigg|_{p^2=-\mu^2}}$$

Now p.-347-

all μ-dependence here in 1-loop

$$\frac{\partial}{\partial p^2}\langle 0|T\tilde{A}(p)\bar{A}(0)|0\rangle^{1PI} = iZ$$

$$+\underbrace{\frac{\partial}{\partial p^2}\left[ip^2 4g^2\int_0^1 d\alpha\,\frac{\Gamma(\epsilon/2)}{(4\pi)^2}\,e^{-\frac{\epsilon}{2}\ln\left[m^2-\alpha(1-\alpha)p^2\right]}\right]}_{\text{indep. of }\mu\text{ in 1-loop!}}$$

So $2i\gamma = i\mu \frac{\partial}{\partial\mu} Z \Rightarrow \boxed{\gamma = \frac{1}{2}\mu\frac{\partial Z}{\partial\mu}}$

$$\gamma = \frac{1}{2}\frac{g^2}{(2\pi)^2}\int_0^1 d\alpha\,\alpha\,\mu\frac{\partial}{\partial\mu}\left[\ln[m^2+\alpha(1-\alpha)\mu^2] + \frac{\alpha(1-\alpha)\mu^2}{m^2+\alpha(1-\alpha)\mu^2}\right]$$

$$= \frac{1}{2}\frac{g^2}{(2\pi)^2}\int_0^1 d\alpha\,\alpha\left[\frac{2\alpha(1-\alpha)\mu^2}{m^2+\alpha(1-\alpha)\mu^2} + \frac{2\alpha(1-\alpha)\mu^2}{m^2+\alpha(1-\alpha)\mu^2} - \frac{2[\alpha(1-\alpha)\mu^2]^2}{[m^2+\alpha(1-\alpha)\mu^2]^2}\right]$$

$$\boxed{\gamma = \frac{g^2}{(2\pi)^2}\int_0^1 d\alpha\,\alpha\left(\frac{[\alpha(1-\alpha)\mu^2]^2+2\alpha(1-\alpha)\mu^2 m^2}{[m^2+\alpha(1-\alpha)\mu^2]^2}\right)}$$

Suppose $\mu^2 >> m^2 \Rightarrow$

$$\boxed{\gamma \approx \frac{g^2}{(2\pi)^2}\int_0^1 d\alpha\,\alpha\cdot 1 = \frac{g^2}{8\pi^2}}$$

$\Rightarrow \boxed{\beta = 3\frac{g^3}{8\pi^2}} > 0$ (not asymptotically free as expected of a linear sigma model with fermions)

All throughout we have assumed a supersymmetric regulation and renormalization procedure — not a trivial assumption!