The Wess-Zumino Model (Chiral Model)

The W-Z model consists of one chiral superfield $\phi(x,\theta,\bar\theta)$ and its adjoint anti-chiral superfield $\bar\phi(x,\theta,\bar\theta) = \phi^\dagger$

$$\bar D_{\dot\alpha}\phi = 0 = D_\alpha \bar\phi$$

$$\Rightarrow \quad \phi_1(x,\theta) = A(x) + \theta\psi(x) + \theta^2 F(x)$$

$$\bar\phi_2(x,\bar\theta) = \bar A(x) + \bar\theta\bar\psi(x) + \bar\theta^2 \bar F(x) .$$

In the real representation we have

$$\phi(x,\theta,\bar\theta) = e^{-i\theta\gamma\bar\theta}(A + \theta\psi + \theta^2 F)$$

$$\bar\phi(x,\theta,\bar\theta) = e^{+i\theta\gamma\bar\theta}(\bar A + \bar\theta\bar\psi + \bar\theta^2\bar F) ,$$

Since the mass dimension of $\theta, \bar\theta$ is $-\frac{1}{2}$, x^μ is -1 and ∂_μ is $+1$, $D, \bar D$ are $+\frac{1}{2}$, then if the scalar field A is to be dimension $+1$ (the same as ϕ) ψ has dimension $+\frac{3}{2}$ & F has dimension $+2$. Hence we list all SUSY invariants we can construct from ϕ & $\bar\phi$ (i.e. products integrated over the appropriate measure) that are also renormalizable, that is dimension ≤ 4: They are

	Monomial	Character	dim	SUSY Invariant
1)	ϕ	chiral	1	$\int dS\, \phi$
2)	$\bar\phi$	anti-chiral	1	$\int d\bar S\, \bar\phi$
3)	$\phi\bar\phi$	vector	2	$\int dV\, \phi\bar\phi$
4)	ϕ^2	chiral	2	$\int dS\, \phi^2$
5)	$\bar\phi^2$	anti-chiral	2	$\int d\bar S\, \bar\phi^2$
6)	ϕ^3	chiral	3	$\int dS\, \phi^3$
7)	$\bar\phi^3$	anti-chiral	3	$\int dS\, \bar\phi^3$.

We stop at cubic terms since we have demanded renormalizability. for example

$$\int dV\, \phi\bar\phi = \int d^4x\, \underbrace{\underbrace{DD\bar D\bar D}_{\text{dim}=2}\ \ \underbrace{\phi\bar\phi}_{\text{dim}=2}}_{\text{dim}=4}$$

so renormalizable

$$\int dS\, \phi^3 = \int d^4x\, \underbrace{\underbrace{DD}_{\text{dim}=1}\ \ \underbrace{\phi^3}_{\text{dim}=3}}_{\text{dim}=4}$$

so renormalizable.

Thus our SUSY invariant action is

$$-i\Gamma = \int dV\, K(\phi,\bar\phi) + \int dS\, W(\phi) + \int d\bar S\, \overline{W}(\bar\phi)$$

where $K(\phi,\bar\phi) = Z\phi\bar\phi$ = Kähler Potential

$W(\phi) = 4m\phi^2 + g\phi^3 + f\phi$ = (chiral) SuperPotential

$\overline{W}(\bar\phi) = 4m\bar\phi^2 + g\bar\phi^3 + f\bar\phi$ = (anti-chiral) Superpotential.

where m, g, f are the (real) parameters of the model and Z gives the field (propagator) normalization.

We shall expand the W-Z action in terms of the component fields. Then find the absolute minimum of the potential and quantize about it.

<u>Kinetic Terms</u>

$$\phi\bar\phi = \left[e^{-i\theta\not\partial\bar\theta}(A+\theta\psi+\theta^2F)\right]\left[e^{+i\theta\not\partial\bar\theta}(\bar A+\bar\theta\bar\psi+\bar\theta^2\bar F)\right]$$

where both ϕ & $\bar\phi$ are in the same (real) representation.

Using $e^{\pm i\theta\not\partial\bar\theta} = 1 \pm i\theta\not\partial\bar\theta - \frac{1}{4}\theta^2\bar\theta^2\partial^2$

we have, recalling $\theta^\alpha\theta^\beta\theta^\gamma = 0 = \bar\theta_\alpha\bar\theta_\beta\bar\theta_\gamma$

$$\phi = \left[1 - i\theta\not\partial\bar\theta - \tfrac{1}{4}\theta^2\bar\theta^2\partial^2\right]\left[A+\theta\psi+\theta^2F\right]$$

$$= A+\theta\psi+\theta^2F - i\theta\not\partial\bar\theta A - i\theta\not\partial\bar\theta\theta\psi - \tfrac{1}{4}\theta^2\bar\theta^2\partial^2A$$

$$\bar{\phi} = [1 + i\theta\gamma\bar{\theta} - \tfrac{1}{4}\theta^2\bar{\theta}^2\gamma^2][\bar{A} + \bar{\theta}\bar{\psi} + \bar{\theta}^2\bar{F}]$$

$$= \bar{A} + \bar{\theta}\bar{\psi} + \bar{\theta}^2\bar{F} + i\theta\gamma\bar{\theta}\bar{A} + i\theta\gamma\bar{\theta}\bar{\theta}\bar{\psi} - \tfrac{1}{4}\theta^2\bar{\theta}^2\gamma^2\bar{A}$$

Now multiplying these together we have a large number of terms however we are only interested in the $\theta^2\bar{\theta}^2$ terms since the lower powers will vanish in integrated over the vector measure.

$$\phi\bar{\phi} = A\bar{A} + A\bar{\theta}\bar{\psi} + A\bar{\theta}^2\bar{F} + iA\theta\gamma\bar{\theta}\bar{A}$$

$$+ iA\theta\gamma\bar{\theta}\bar{\theta}\bar{\psi} - \tfrac{1}{4}\theta^2\bar{\theta}^2 A\gamma^2\bar{A} + \theta\psi\bar{A} + \theta\psi\bar{\theta}\bar{\psi}$$

$$+ \theta\psi\bar{\theta}^2\bar{F} + i\theta\psi\theta\gamma\bar{\theta}\bar{A} + i\theta\psi\theta\gamma\bar{\theta}\bar{\theta}\bar{\psi} + \theta^2 F\bar{A}$$

$$+ \theta^2 F\bar{\theta}\bar{\psi} + \theta^2\bar{\theta}^2 F\bar{F} - i\theta\gamma\bar{\theta}A\bar{A} - i\theta\gamma\bar{\theta}A\bar{\theta}\bar{\psi}$$

$$+ (\theta\gamma\bar{\theta})A(\theta\gamma\bar{\theta}\bar{A}) - i\theta\gamma\bar{\theta}\theta\psi\bar{A} - i\theta\gamma\bar{\theta}\theta\psi\bar{\theta}\bar{\psi}$$

$$- \tfrac{1}{4}\theta^2\bar{\theta}^2\gamma^2 A\bar{A}$$

$$= A\bar{A} + A\bar{\theta}\bar{\psi} + \theta\psi\bar{A} + \ldots$$

$$+ \Big[-\tfrac{1}{4}\theta^2\bar{\theta}^2 A\gamma^2\bar{A} + i\theta\psi\theta\gamma\bar{\theta}\bar{\theta}\bar{\psi} + \theta^2\bar{\theta}^2 F\bar{F}$$

$$+ (\theta\gamma\bar{\theta})A(\theta\gamma\bar{\theta})\bar{A} - i\theta\gamma\bar{\theta}\theta\psi\bar{\theta}\bar{\psi} - \tfrac{1}{4}\theta^2\bar{\theta}^2\gamma^2 A\bar{A}\Big]$$

So using $\theta^\alpha \theta^\beta = -\frac{1}{2}\epsilon^{\alpha\beta}\theta^2$
$\bar\theta^{\dot\alpha}\bar\theta^{\dot\beta} = +\frac{1}{2}\epsilon^{\dot\alpha\dot\beta}\bar\theta^2$ we obtain

$$\theta\psi\,\theta\not\partial\bar\theta\,\bar\theta\bar\psi = \theta^\alpha\psi_\alpha\,\theta^\beta\not\partial_{\beta\dot\beta}\bar\theta^{\dot\beta}\,\bar\psi_{\dot\alpha}\bar\theta^{\dot\alpha}$$

$$= -\theta^\alpha\theta^\beta\,\psi_\alpha\not\partial_{\beta\dot\beta}\bar\psi_{\dot\alpha}\,\bar\theta^{\dot\alpha}\bar\theta^{\dot\beta}$$

$$= \frac{1}{4}\theta^2\bar\theta^2\,\epsilon^{\alpha\beta}\psi_\alpha\not\partial_{\beta\dot\beta}\epsilon^{\dot\alpha\dot\beta}\bar\psi_{\dot\alpha}$$

$$= \frac{1}{4}\theta^2\bar\theta^2\,\psi^\beta\not\partial_{\beta\dot\beta}\bar\psi^{\dot\beta} = \frac{1}{4}\theta^2\bar\theta^2\,\psi\not\partial\bar\psi$$

$$\theta\not\partial\bar\theta\,\theta\psi\,\bar\theta\bar\psi = \theta^\alpha\not\partial_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\,\theta^\beta\psi_\beta\,\bar\psi_{\dot\beta}\bar\theta^{\dot\beta}$$

$$= -\theta^\alpha\theta^\beta\not\partial_{\alpha\dot\alpha}\psi_\beta\,\bar\psi_{\dot\beta}\,\bar\theta^{\dot\alpha}\bar\theta^{\dot\beta} = \frac{1}{4}\theta^2\bar\theta^2\not\partial_{\alpha\dot\alpha}\psi^\alpha\bar\psi^{\dot\alpha}$$

$$= \frac{1}{4}\theta^2\bar\theta^2\,\partial_\mu\psi\sigma^\mu\bar\psi$$

$$\theta\not\partial\bar\theta A\,\theta\not\partial\bar\theta\bar A = \theta^\alpha\not\partial_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}A\,\theta^\beta\not\partial_{\beta\dot\beta}\bar\theta^{\dot\beta}\bar A$$

$$= -\theta^\alpha\theta^\beta\not\partial_{\alpha\dot\alpha}A\not\partial_{\beta\dot\beta}\bar A\,\bar\theta^{\dot\alpha}\bar\theta^{\dot\beta} = \frac{1}{4}\theta^2\bar\theta^2\not\partial_{\alpha\dot\alpha}A\not\partial^{\dot\alpha\alpha}\bar A$$

$$= \frac{1}{4}\theta^2\bar\theta^2\,\partial_\mu A\partial_\nu\bar A\,\underbrace{\sigma^\mu_{\alpha\dot\alpha}\bar\sigma^{\nu\dot\alpha\alpha}}_{=\mathrm{Tr}[\sigma^\mu\bar\sigma^\nu] = 2g^{\mu\nu}}$$

$$= \frac{1}{2}\theta^2\bar\theta^2\,\partial_\mu A\partial^\mu\bar A$$

So

$$\phi\bar{\phi} = A\bar{A} + \cdots + \theta^2\bar{\theta}^2\Big[F\bar{F} + \frac{i}{4}\psi\overleftrightarrow{\not{\partial}}\bar{\psi} - \frac{1}{4}A\partial^2\bar{A} - \frac{1}{4}\partial^2 A\bar{A} + \frac{1}{2}\partial_\mu A\partial^\mu\bar{A}\Big]$$

$$\boxed{\phi\bar{\phi} = A\bar{A} + \cdots + \theta^2\bar{\theta}^2\Big[F\bar{F} + \frac{i}{4}\psi\overleftrightarrow{\not{\partial}}\bar{\psi} - \frac{1}{4}\partial^2(A\bar{A}) + \partial_\mu A\partial^\mu\bar{A}\Big]}$$

Integrating over the vector measure $\int dV = \int d^4x\, d^2\theta\, d^2\bar{\theta}$
$= \int d^4x \frac{\partial^2}{\partial\theta^2}\frac{\partial^2}{\partial\bar{\theta}^2}$

we find

$$\boxed{Z\int dV\,\phi\bar{\phi} = 16Z\int d^4x\Big[\partial_\mu A\partial^\mu\bar{A} + \frac{i}{4}\psi\overleftrightarrow{\not{\partial}}\bar{\psi} + F\bar{F}\Big]}$$

where the total space-time divergence $\partial^2(A\bar{A})$ integrates to zero.

Next consider the chiral superpotential terms $\int dS\, W(\phi)$. Since all fields are chiral here we can evaluate this term i

The chiral representation

$$\int dS\, W(\phi) = \int dS\, e^{-i\theta\gamma\bar{\theta}}\, W(\phi_1)$$

(Total derivative integrates to zero)

$$= \int dS\, W(\phi_1) = \int dS\, W(A + \theta\psi + \theta^2 F)$$

$$= \int d^4x \frac{\partial^2}{\partial\theta^2}\Big[W(A) + W'(A)(\theta\psi + \theta^2 F) + \tfrac{1}{2} W''(A)(\theta\psi + \theta^2 F)^2 \Big]$$

$$= \int d^4x \frac{\partial^2}{\partial\theta^2}\Big[W'(A)\,\theta^2 F + \tfrac{1}{2} W''(A)\,\theta\psi\theta\psi \Big]$$

Now

$$\theta\psi\theta\psi = -\theta^\alpha\theta^\beta\psi_\alpha\psi_\beta = \tfrac{1}{2}\theta^2\epsilon^{\alpha\beta}\psi_\alpha\psi_\beta$$

$$= -\tfrac{1}{2}\theta^2(\psi\psi)$$

$\Rightarrow$

$$\int dS\, W(\phi) = \int d^4x \frac{\partial^2}{\partial\theta^2}\theta^2\Big[W'(A) F - \tfrac{1}{4} W''(A)\psi^2 \Big]$$

$$\boxed{\int dS\, W(\phi) = -4\int d^4x \Big[W'(A) F - \tfrac{1}{4} W''(A)\psi^2 \Big]}$$

For the W-Z model $W(A) = 4m\,A^2 + gA^3 + fA$

So

$$W'(A) = 8mA + 3gA^2 + f$$
$$W''(A) = 8m + 6gA$$

Hence

$$\int dS\, W(\phi) = -4\int d^4x \Big[8mAF + 3gA^2F + fF$$
$$-\tfrac{1}{2}4m\psi^2 - \tfrac{3}{2}gA\psi^2 \Big]$$

$$\boxed{\int dS\, W(\phi) = -16\int d^4x \Big[2mAF - \tfrac{1}{2}m\psi\psi + \tfrac{1}{4}fF + \tfrac{3}{4}gAAF - \tfrac{3}{8}gA\psi\psi \Big]}$$

Likewise the anti-chiral superpotential yields

$$\int d\bar{S}\, \bar{W}(\bar{\phi}) = \int d\bar{S}\, e^{+i\theta\sigma\bar{\theta}}\, \bar{W}(\bar{\Phi}_2)$$
$$= \int d\bar{S}\, \bar{W}(\bar{A} + \bar{\theta}\bar{\psi} + \bar{\theta}^2\bar{F})$$
$$= \int d^4x \frac{\partial^2}{\partial\bar{\theta}^2} \Big[\bar{W}(\bar{A}) + \bar{W}'(\bar{A})(\bar{\theta}\bar{\psi} + \bar{\theta}^2\bar{F})$$
$$+ \tfrac{1}{2}\bar{W}''(\bar{A})(\bar{\theta}\bar{\psi} + \bar{\theta}^2\bar{F})^2 \Big]$$
$$= \int d^4x \frac{\partial^2}{\partial\bar{\theta}^2} \Big[\bar{\theta}^2\, \bar{W}'(\bar{A})\bar{F} + \tfrac{1}{2}\bar{W}''(\bar{A})\, \bar{\theta}\bar{\psi}\,\bar{\theta}\bar{\psi} \Big]$$

but

$$\bar{\theta}\bar{\psi}\,\bar{\theta}\bar{\psi} = -\bar{\psi}_{\dot\alpha}\bar{\theta}^{\dot\alpha}\bar{\theta}^{\dot\beta}\bar{\psi}_{\dot\beta} = -\tfrac{1}{2}\bar{\theta}^2\,\bar{\psi}\bar{\psi}$$

$$\boxed{\int d\bar{S}\,\bar{W}(\bar{\Phi}) = -4\int d^4x\left[\bar{W}'(\bar{A})\bar{F} - \tfrac{1}{4}\bar{W}''(\bar{A})\bar{\psi}\bar{\psi}\right]}$$

For the W-Z model $\bar{W}(\bar{A}) = 4m\bar{A}^2 + g\bar{A}^3 + f\bar{A}$

$$\bar{W}'(\bar{A}) = 8m\bar{A} + 3g\bar{A}^2 + f$$
$$\bar{W}''(\bar{A}) = 8m + 6g\bar{A}$$

So

$$\boxed{\begin{aligned}\int d\bar{S}\,\bar{W}(\bar{\Phi}) = -4\int d^4x\Big[&8m\bar{A}\bar{F} + 3g\bar{A}^2\bar{F} + f\bar{F}\\ &-2m\bar{\psi}\bar{\psi} - \tfrac{3}{2}g\bar{A}\bar{\psi}\bar{\psi}\Big]\end{aligned}}$$

$$\boxed{\begin{aligned}= \int d^4x\Big[&-16\left(2m\bar{A}\bar{F} - \tfrac{1}{2}m\bar{\psi}\bar{\psi}\right)\\ &-12g\left(\bar{A}\bar{A}\bar{F} - \tfrac{1}{2}\bar{A}\bar{\psi}\bar{\psi}\right) - 4f\bar{F}\Big]\end{aligned}}$$

So we obtain the W-Z model in components:

$$-i\Gamma = \int dV\, K + \int dS\, W + \int d\bar{S}\, \bar{W}$$

$$= Z\int dV\, \phi\bar{\phi} + \int dS\,[4m\phi^2 + g\phi^3 + f\phi] + \int d\bar{S}\,\{4m\bar{\phi}^2 + g\bar{\phi}^3 + f\bar{\phi}\}$$

$$= \int d^4x\,\Big\{16Z\big[\partial_\mu A\partial^\mu\bar{A} + \tfrac{i}{4}\psi\overleftrightarrow{\not{\partial}}\bar{\psi} + F\bar{F}\big] - 16m\big[2AF + 2\bar{A}\bar{F} - \tfrac{1}{2}\psi\psi - \tfrac{1}{2}\bar{\psi}\bar{\psi}\big] - 12g\big[AAF + \bar{A}\bar{A}\bar{F} - \tfrac{1}{2}A\psi\psi - \tfrac{1}{2}\bar{A}\bar{\psi}\bar{\psi}\big] - 4f(F+\bar{F})\Big\}$$

Recall that the component action can be checked to be invariant under the SUSY transformations for the component fields

$$\delta^Q(\xi,\bar{\xi})A = \xi\psi \qquad \delta^Q(\xi,\bar{\xi})\bar{A} = \bar{\xi}\bar{\psi}$$

$$\delta^Q(\xi,\bar{\xi})\psi_\alpha = 2\xi_\alpha F - 2i(\not{\partial}\bar{\xi})_\alpha A \qquad \delta^Q(\xi,\bar{\xi})\bar{\psi}^{\dot{\alpha}} = 2\bar{\xi}^{\dot{\alpha}}\bar{F} + 2i(\xi\not{\partial})^{\dot{\alpha}}\bar{A}$$

$$\delta^Q(\xi,\bar{\xi})F = \partial_\mu\psi\sigma^\mu\bar{\xi} \qquad \delta^Q(\xi,\bar{\xi})\bar{F} = -\xi\not{\partial}\bar{\psi}$$

(Isn't superspace simpler!!)

The F & F^{+} fields have no derivatives acting on them in the action. Hence they are "auxiliary fields" — their equations of motion are completely algebraic and will allow us to eliminate F & F^{+} from the action

$$1)\ \frac{-i\,\delta\Gamma}{\delta F(x)} = 0 = 16Z\,F(x) - 4f - 32m\bar{A} - 12g\bar{A}^2$$

$$2)\ \frac{-i\,\delta\Gamma}{\delta \bar{F}(x)} = 0 = 16Z\,\bar{F}(x) - 4f - 32mA - 12gA^2$$

$$\Rightarrow \boxed{\begin{array}{l} 1)\ 16ZF = 4f + 32m\bar{A} + 12g\bar{A}^2 \\ 2)\ 16Z\bar{F} = 4f + 32mA + 12gA^2 \end{array}}$$

Hence the action takes the form

$$-i\Gamma = \int d^4x \Big\{ 16Z\left[\partial_\lambda A\,\partial^\lambda \bar{A} + \tfrac{i}{4}\psi\overleftrightarrow{\partial}\bar{\psi}\right] + 16m\left[\tfrac{1}{2}\psi\psi + \tfrac{1}{2}\bar{\psi}\bar{\psi}\right] + 12g\left[\tfrac{1}{2}\psi\psi A + \tfrac{1}{2}\bar{\psi}\bar{\psi}\bar{A}\right] - V(A,\bar{A})\Big\}$$

where $V(A,\bar{A})$ is the potential for the W-Z model:

$$V = -16Z F\bar{F} + 16m[2AF + 2\bar{A}\bar{F}]$$

$$+ 12g[A^2F + \bar{A}^2\bar{F}] + 4f[F + \bar{F}]$$

$$= -16ZF\bar{F} + [4f + 32mA + 12gA^2]F$$

$$+ [4f + 32m\bar{A} + 12g\bar{A}^2]\bar{F}$$

using the $F, \bar{F}$ field equations $\Rightarrow$

$$V = -16ZF\bar{F} + 16ZF\bar{F} + 16ZF\bar{F}$$

$$\boxed{V = +16ZF\bar{F}}$$

Recall the "general" chiral model

$$-i\Gamma = \int d^4x \{ 16Z[\partial_\mu A \partial^\mu \bar{A} + \tfrac{i}{4}\psi \overleftrightarrow{\not\partial} \bar{\psi} + F\bar{F}]$$

$$-4[W'(A)F - \tfrac{1}{4}W''(A)\psi^2]$$

$$-4[\bar{W}'(\bar{A})\bar{F} - \tfrac{1}{4}\bar{W}''(\bar{A})\bar{\psi}^2] \}$$

$$\Rightarrow \quad \boxed{V = -16ZF\bar{F} + 4W'(A)F + 4\bar{W}'(\bar{A})\bar{F}}$$

and $\frac{-i\delta\Gamma}{\delta \bar{F}} = -\frac{\delta V}{\delta \bar{F}} = +16ZF - 4W'(A) = 0$

$\frac{-i\delta\Gamma}{\delta F} = -\frac{\delta V}{\delta F} = +16Z\bar{F} - 4\bar{W}'(\bar{A}) = 0$

And we find a general property of chiral models in SUSY

$$\boxed{V = +16 Z F\bar{F}}$$

$$= \frac{1}{16Z} 16 W'(A)\bar{W}'(\bar{A}) = \frac{1}{Z} W'(A)\bar{W}'(\bar{A})$$

So we see that $V \geq 0$ with absolute minimum at $\langle F\rangle = 0 = \langle \bar{F}\rangle$. From the $F, \bar{F}$ equations of motion we have the VEV:

$$16Z\langle F\rangle = 4f + 32m\langle\bar{A}\rangle + 12g\langle\bar{A}\rangle^2 = 0$$
$$16Z\langle\bar{F}\rangle = 4f + 32m\langle A\rangle + 12g\langle A\rangle^2 = 0$$

for the absolute minimum. So if $\langle A\rangle = 0$ $\Rightarrow f = 0$; the linear term in ϕ must be excluded from the action in order to quantize about the absolute minimum of the potential.

However a constant field $\phi = a$ is also a superfield since $[Q, a] = 0$ and so does the differential operator representation of Q "Q"$a = 0$ or $\delta^{Q}(\xi, \bar{\xi})a = 0$. So we can always shift the superfields $(A \to A + a, \bar{A} \to \bar{A} + \bar{a})$ by a θ-independent constant and obtain another SUSY invariant action.

So let $\phi \to \phi + a$, $\bar{\phi} \to \bar{\phi} + a$ we find $\Gamma \to \Gamma'$

$$-i\Gamma' = Z\int dV \phi\bar{\phi} + [4m + 3ga]\left[\int dS \phi^2 + \int d\bar{S} \bar{\phi}^2\right]$$
$$+ g\left[\int dS \phi^3 + \int d\bar{S} \bar{\phi}^3\right]$$
$$+ [f + 8ma + 3ga^2]\left[\int dS \phi + \int d\bar{S} \bar{\phi}\right].$$

So if Γ has a minimum at $\langle A \rangle = \alpha$ then

Γ' has a minimum at $\langle A \rangle = \alpha + a$. Hence even if $\langle A \rangle \neq 0$, we can shift to $\langle A \rangle = 0$ by $\alpha = -a$. Then the minimum require $f = 0$. Thus we can always choose $f = 0$ in the W-Z model. (One could start with $m = 0$, $g \neq 0$, $f \neq 0$ and generate the mass term by shifting, also.)

In general the $F, \bar{F}$ eq. of motion allow them to be eliminated

$$16ZF = 4W'(A) \; ; \; 16Z\bar{F} = 4\bar{W}'(\bar{A})$$

So

$$V = \frac{1}{2} W'(A) \bar{W}'(\bar{A})$$ and the action

becomes

$$-i\Gamma = \int d^4x \Big\{ 16Z\Big[\partial_\mu A \partial^\mu \bar{A} + \frac{i}{4}\psi \overleftrightarrow{\not\partial}\bar{\psi}\Big] + 16m\Big[\frac{1}{2}\psi\psi + \frac{1}{2}\bar{\psi}\bar{\psi}\Big] + 12g\Big[\frac{1}{2}\psi\psi A + \frac{1}{2}\bar{\psi}\bar{\psi}\bar{A}\Big] - \frac{1}{Z}W'(A)\bar{W}'(\bar{A})\Big\}$$

$$= \int d^4x \Big\{ 16Z\Big[\partial_\mu A^\dagger \partial^\mu A - \frac{4m^2}{Z^2}A^\dagger A\Big] + 16Z\Big[\frac{i}{4}\psi\overleftrightarrow{\not\partial}\bar{\psi} + \frac{1}{2}\frac{m}{Z}\psi\psi + \frac{1}{2}\frac{m}{Z}\bar{\psi}\bar{\psi}\Big] + 6g\Big[\psi\psi A + \bar{\psi}\bar{\psi}A^\dagger\Big] - 24g\frac{m}{Z}(A+A^\dagger)(A^\dagger A) - \frac{9g^2}{Z}(A^\dagger A)^2\Big\}$$

Thus both A & ψ have the same mass, $\left(\frac{2m}{Z}\right)$, even though different spin. (let $\psi \to \sqrt{2}\psi$, $\bar{\psi} \to \sqrt{2}\bar{\psi}$ so that the KE is canonical $\frac{i}{4}\psi\overleftrightarrow{\not\partial}\bar{\psi} \to \frac{i}{2}\psi\overleftrightarrow{\not\partial}\bar{\psi}$ and the mass terms $\frac{1}{2}\frac{m}{Z}\psi\psi + \frac{1}{2}\frac{m}{Z}\bar{\psi}\bar{\psi} \to \frac{1}{2}\left(\frac{2m}{Z}\right)[\psi^2 + \bar{\psi}^2]$)

This model is just a special case of a linear σ-model with fermions — there being a definite relation between masses and a definite relation between Yukawa & scalar self-interaction terms.

Before introducing gauge invariance in SUSY theories, let's consider perturbation theory and Feynman rules in superspace for chiral models.

We begin by considering the Feynman rules for the component model with the auxiliary fields F & $\bar{F}$ $(f=0)$

$$-i\Gamma = \int d^4x \Big\{ 16Z\big[\partial_\mu A \partial^\mu \bar{A} + \tfrac{i}{4}\psi \overleftrightarrow{\not\partial} \bar{\psi} + F\bar{F}\big]$$
$$-16m\big[2AF + 2\bar{A}\bar{F} - \tfrac{1}{2}\psi\psi - \tfrac{1}{2}\bar{\psi}\bar{\psi}\big]$$
$$-12g\big[AAF + \bar{A}\bar{A}\bar{F} - \tfrac{1}{2}A\psi\psi - \tfrac{1}{2}\bar{A}\bar{\psi}\bar{\psi}\big]\Big\}$$

The time ordered functions will be given by the Feynman path integral

$$Z[J,\bar{J},\eta,\bar{\eta},K,\bar{K}] = \int [dA][d\bar{A}][d\psi][d\bar{\psi}][dF][d\bar{F}] \times$$
$$\times e^{i\int d^4x\left[\mathcal{L} + JA + \bar{J}\bar{A} + \eta^\alpha\psi_\alpha + \bar{\eta}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} + KF + \bar{K}\bar{F}\right]}$$

with $Z[0] \equiv 1$