Standard Model Summary:

The Standard Model of electroweak and strong interactions based on the gauge symmetry group $SU(3)\times SU(2)\times U(1)$ has dynamics described by the gauge invariant Lagrangian

$$\mathcal{L}_{SM} = \mathcal{L}_{YM} + \mathcal{L}_F + \mathcal{L}_\phi + \mathcal{L}_{Yuk}$$

1) $$\mathcal{L}_{YM} = -\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$

where the anti-symmetric covariant field strength tensors are

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g_2 \epsilon_{ijk} A^j_\mu A^k_\nu \quad ; \; i=1,2,3$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$G^m_{\mu\nu} = \partial_\mu G^m_\nu - \partial_\nu G^m_\mu + g_3 f_{mnp} G^n_\mu G^p_\nu \quad , \; m=1,2,\ldots,8$$

with f_{mnp} the $SU(3)$ structure constants.

2) $\mathcal{L}_F = \bar{l}_L^w\, i\not{D}\, l_L^w + \bar{q}_L^w\, i\not{D}\, q_L^w + \bar{e}_R^w\, i\not{D}\, e_R^w + \bar{u}_R^w\, i\not{D}\, u_R^w$
$+ \bar{d}_R^w\, i\not{D}\, d_R^w$

where the covariant derivatives are

$$D_\mu l_L^w = \left[\partial_\mu - \frac{ig_2}{2}\vec{\tau}\cdot\vec{A}_\mu + \frac{ig_1}{2}B_\mu\right] l_L^w$$

$$D_\mu^{ab} q_L^{wb} = \Big[\left(\partial_\mu - \frac{ig_2}{2}\vec{\tau}\cdot\vec{A}_\mu - \frac{ig_1}{6}B_\mu\right)\delta^{ab} - \frac{ig_3}{2}(\vec{\lambda}\cdot\vec{G}_\mu)_{ab}\Big] q_L^{wb}$$

$$D_\mu e_R^w = (\partial_\mu + i g_1 B_\mu) e_R^w$$

$$D_\mu^{ab} u_R^{wb} = \left[\left(\partial_\mu - \frac{2i}{3}g_1 B_\mu\right)\delta^{ab} - \frac{ig_3}{2}(\vec{\lambda}\cdot\vec{G}_\mu)_{ab}\right] u_R^{wb}$$

$$D_\mu^{ab} d_R^{wb} = \left[\left(\partial_\mu + \frac{i}{3}g_1 B_\mu\right)\delta^{ab} - \frac{ig_3}{2}(\vec{\lambda}\cdot\vec{G}_\mu)_{ab}\right] d_R^{wb}$$

where $a, b = 1, 2, 3$ = color indices of the quarks.

---

3) $\mathcal{L}_\phi = (D_\mu\phi)^\dagger (D^\mu\phi) - V(\phi^\dagger\phi)$

$$D_\mu\phi = \left(\partial_\mu - \frac{ig_2}{2}\vec{\tau}\cdot\vec{A}_\mu - \frac{ig_1}{2}B_\mu\right)\phi$$

& $V(\phi^\dagger\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$

$\phi = \begin{bmatrix}\phi^+\\ \phi^0\end{bmatrix}$ complex scalar doublet. $(\phi^+)^\dagger = \phi^-$
$(\phi^0)^\dagger = \phi^{0\dagger}$

4) $$\mathcal{L}_{Yuk} = \Gamma^{e}_{mn} \bar{l}^{w}_{mL} \phi e^{w}_{nR} + \Gamma^{d}_{mn} \bar{q}^{w}_{mL} \phi d^{w}_{nR} + \Gamma^{u}_{mn} \bar{q}^{w}_{mL} \tilde{\phi} u^{w}_{nR} + h.c.$$

with
$$\tilde{\phi} = i\sigma^2 \phi^* = \begin{bmatrix} \phi^{0*} \\ -\phi^- \end{bmatrix}$$ a $(2, -\frac{1}{2})$ representation

---

Since $-\mu^2 = m^2 < 0$ the $SU(2)\times U(1)$ symmetry is spontaneously broken to $U(1)_{em}$ with the unbroken electric charge given by

$$Q = T^3 + y \quad \text{for each field.}$$

Hence the minimum of V is at $\langle 0|\phi|0\rangle = \begin{bmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{bmatrix}$

And we can express the SM in the <u>unitary gauge</u> where $\phi = \begin{bmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{bmatrix}$

$\eta(x) = H(x)$ the Higgs field.

& $$W^{\pm}_{\mu} \equiv \frac{1}{\sqrt{2}}\left(A^1_{\mu} \mp i A^2_{\mu}\right) \; ; \quad A^1_{\mu} = \frac{1}{\sqrt{2}}(W^+_{\mu} + W^-_{\mu}), \quad A^2_{\mu} = \frac{i}{\sqrt{2}}(W^+_{\mu} - W^-_{\mu})$$

and $$\begin{pmatrix} Z_{\mu} \\ A_{\mu} \end{pmatrix} \equiv \begin{bmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{bmatrix} \begin{pmatrix} A^3_{\mu} \\ B_{\mu} \end{pmatrix}, \quad \tan\theta_w = \frac{g_1}{g_2}$$

with $\begin{pmatrix} A^3_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$.

So $\mathcal{L}_{SM}$ in the unitary gauge becomes

1) $\mathcal{L}_{YM}$:

$$F^i_{\mu\nu} = \begin{cases} \frac{1}{\sqrt{2}}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu) + \frac{1}{\sqrt{2}}(\partial_\mu W^-_\nu - \partial_\nu W^-_\mu) \\ \quad + \frac{i g_2}{\sqrt{2}}\left[(W^+_\mu - W^-_\mu)(\cos\theta_W Z_\nu + s\theta_W A_\nu) - (\mu \leftrightarrow \nu)\right] & i=1 \\ \frac{i}{\sqrt{2}}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu) - \frac{i}{\sqrt{2}}(\partial_\mu W^-_\nu - \partial_\nu W^-_\mu) \\ \quad + \frac{g_2}{\sqrt{2}}\left[(c\theta_W Z_\mu + s\theta_W A_\mu)(W^+_\nu + W^-_\nu) - (\mu \leftrightarrow \nu)\right] & i=2 \\ \cos\theta_W(\partial_\mu Z_\nu - \partial_\nu Z_\mu) + \sin\theta_W(\partial_\mu A_\nu - \partial_\nu A_\mu) \\ \quad + i g_2\left[W^-_\mu W^+_\nu - W^+_\mu W^-_\nu\right] & i=3 \end{cases}$$

$$B_{\mu\nu} = -\sin\theta_W(\partial_\mu Z_\nu - \partial_\nu Z_\mu) + \cos\theta_W(\partial_\mu A_\nu - \partial_\nu A_\mu)$$

---

2) $$\mathcal{L}_F = \bar{\nu}_L i\not{\partial}\nu_L + \bar{e} i\not{\partial} e + \bar{u}\, i\left[\not{\partial} - \frac{i g_3}{2}(\vec{\lambda}\cdot\vec{\not{G}})\right]u + \bar{d}\, i\left[\not{\partial} - \frac{i g_3}{2}(\vec{\lambda}\cdot\vec{\not{G}})\right]d$$

$$+ e A_\mu J^\mu_{em} + \frac{e}{2\sqrt{2}\sin\theta_W}\left[J^\mu_W W^-_\mu + J^{\mu\,\dagger}_W W^+_\mu\right] + \frac{e}{\sin 2\theta_W} J^\mu_Z Z_\mu$$

2) where $e = g_2 \sin\theta_W = g_1 \cos\theta_W$

and 1) The electromagnetic current

$$J^\mu_{em} = q_M \bar{\psi}_M \gamma^\mu \psi_M$$

M sums over all fields with q_M their electric charge

$$= +\frac{2}{3}\bar{u}_m \gamma^\mu u_m - \frac{1}{3}\bar{d}_m \gamma^\mu d_m - \bar{e}_m \gamma^\mu e_m$$

2) Charged Weak Current

$$J^\mu_W = \bar{e}\gamma^\mu(1-\gamma_5)\nu + \bar{d}\gamma^\mu(1-\gamma_5) A_{CKM} u$$

3) Neutral Weak Current

$$J^\mu_Z = \bar{\psi}_M \gamma^\mu T^3_M (1-\gamma_5)\psi_M - 2 q_M \bar{\psi}_M \gamma^\mu \psi_M$$

$$= \bar{\nu}_L \gamma^\mu \nu_L - \bar{e}_L \gamma^\mu e_L + \bar{u}_L \gamma^\mu u_L - \bar{d}_L \gamma^\mu d_L + 2\sin^2\theta_W\left(\bar{e}\gamma^\mu e - \frac{2}{3}\bar{u}\gamma^\mu u + \frac{1}{3}\bar{d}\gamma^\mu d\right)$$

where A_{CKM} is the Cabbibo-Kobayashi-Maskawa matrix

2) Let

$$V = A^{+}_{CKM} = (A^{d+}_L A^{u}_L)^{+}$$

$$= (A^{u+}_L A^{d}_L)$$

$$= \begin{array}{c|ccc} & d & s & b \\ u & V_{ud} & V_{us} & V_{ub} \\ c & V_{cd} & V_{cs} & V_{cb} \\ t & V_{td} & V_{ts} & V_{tb} \end{array}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & C_2 & -S_2 \\ 0 & S_2 & C_2 \end{pmatrix} \begin{pmatrix} C_1 & -S_1 & 0 \\ S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & C_3 & S_3 \\ 0 & -S_3 & C_3 \end{pmatrix}$$

$$= \begin{pmatrix} C_1 & -S_1 C_3 & -S_1 S_3 \\ S_1 C_2 & C_1 C_2 C_3 - S_2 S_3 e^{i\delta} & C_1 C_2 S_3 + S_2 C_3 e^{i\delta} \\ S_1 S_2 & C_1 S_2 C_3 + C_2 S_3 e^{i\delta} & C_1 S_2 S_3 - C_2 C_3 e^{i\delta} \end{pmatrix}$$

$$\doteq \begin{bmatrix} 0.9738-0.9750 & 0.218-0.224 & 0.001-0.007 \\ 0.218-0.224 & 0.9734-0.9752 & 0.030-0.058 \\ 0.003-0.019 & 0.029-0.058 & 0.9983-0.9996 \end{bmatrix}$$

3) $$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu \eta \partial^\mu \eta - \frac{1}{2} m_H^2 \eta^2 - \frac{\lambda}{4}(\eta^4 + 4v\eta^3)$$

$$+ M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu$$

$$+ g_2 \left[M_W \eta + \frac{1}{4} g_2 \eta^2 \right] W_\mu^+ W^{-\mu}$$

$$+ \frac{1}{2}\left[\frac{g_2 M_W}{\cos^2\theta_W} \eta + \frac{1}{4} \frac{g_2^2}{\cos^2\theta_W} \eta^2 \right] Z_\mu Z^\mu$$

where

$$M_W = \frac{g_2 v}{2} \quad \left(\approx \frac{37}{\sin\theta_W} \text{ GeV} \right)$$

$$M_Z = \frac{M_W}{\cos\theta_W} \quad \left(\approx \frac{75}{\sin 2\theta_W} \text{ GeV} \right)$$

$$m_H^2 = 2\mu^2 = 2\lambda v^2$$

---

4) $$\mathcal{L}_{yuk} = -\left[1 + \frac{g_2}{2M_W}\eta \right] \left[m_u \bar{u}u + m_c \bar{c}c \right.$$

$$+ m_t \bar{t}t + m_d \bar{d}d + m_s \bar{s}s + m_b \bar{b}b$$

$$\left. + m_e \bar{e}e + m_\mu \bar{\mu}\mu + m_\tau \bar{\tau}\tau \right]$$
