7A) 4) Finally we must evaluate the Yukawa lagrangian to determine the fermion masses.

$$\mathcal{L}_{yuk} = \Gamma^e_{mn} \bar{l}_{mL} \begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix} e_{nR} + \Gamma^d_{mn} \bar{q}_{mL} \begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix} d_{nR}$$

$$+ \Gamma^u_{mn} \bar{q}_{mL} \begin{pmatrix} \frac{v+\eta}{\sqrt{2}} \\ 0 \end{pmatrix} u_{nR} \quad + H.C.$$

$$= \Gamma^e_{mn} \left(\frac{v+\eta}{\sqrt{2}}\right) \bar{e}_{mL} e_{nR} + \Gamma^d_{mn} \left(\frac{v+\eta}{\sqrt{2}}\right) \bar{d}_{mL} d_{nR}$$

$$+ \Gamma^u_{mn} \left(\frac{v+\eta}{\sqrt{2}}\right) \bar{u}_{mL} u_{nR} \quad + h.c.$$

$$= -\bar{e}_L M^e e_R + \bar{e}_L h^e e_R \eta$$

$$- \bar{u}_L M^u u_R + \bar{u}_L h^u u_R \eta$$

$$- \bar{d}_L M^d d_R + \bar{d}_L h^d d_R \eta \quad + h.c.$$

where

$$M^e_{mn} = \frac{-v}{\sqrt{2}} \Gamma^e_{mn} \text{ ; similarly for } M^u \,\&\, M^d$$

and

$$h^e_{mn} = \frac{1}{\sqrt{2}} \Gamma^e_{mn} = \frac{-1}{v} M^e_{mn}$$

$$= \frac{-g}{2M_W} M^e_{mn}. \text{ Similarly for } h^u \,\&\, h^d.$$

So we see that the mass matrix is in general not diagonal; i.e. the weak interaction fields we've been using are not the "physical" i.e. mass matrix eigenfields fields. In general the Γ_{mn} are complex and non-hermitian. So we can still diagonalize the mass matrix by a left-right transformation

for $M^{e,u,d}$ we have 3x3 matrices with real, positive diagonal eigenvalues m_1, m_2, m_3 call it M_{diag}

$$M_{diag.} = \begin{pmatrix} m_1 & & 0 \\ & m_2 & \\ 0 & & m_3 \end{pmatrix} = A_L^\dagger M A_R$$

(i.e. $(m_1, m_2, m_3) = (m_e, m_\mu, m_\tau)$ or (m_u, m_c, m_t) or (m_d, m_s, m_b)).

where A_L & A_R are 3x3 unitary matrices ($A_L = A_R$ if M is hermitean)

further we can almost determine A_L and A_R uniquely by noting that $MM^\dagger$ and $M^\dagger M$ are hermitean.

$$(MM^\dagger)^\dagger = MM^\dagger \; ; \; (M^\dagger M)^\dagger = M^\dagger M \, .$$

and that

$$A_L^\dagger M M^\dagger A_L = A_L^\dagger M A_R A_R^\dagger M^\dagger A_L$$

$$= M_{diag} M_{diag}^\dagger = \begin{pmatrix} m_1^2 & & 0 \\ & m_2^2 & \\ 0 & & m_3^2 \end{pmatrix}$$

$$A_R^\dagger M^\dagger M A_R = A_R^\dagger M^\dagger A_L A_L^\dagger M A_R = M_{diag}^\dagger M_{diag}$$

$$= \begin{pmatrix} m_1^2 & & 0 \\ & m_2^2 & \\ 0 & & m_3^2 \end{pmatrix}$$

So we can determine A_L and A_R by diagonalizing $MM^\dagger$, $M^\dagger M$ by simple linear alg. This fixes A_L & A_R only up to 3 arb. phases for R & 3 for L.

i.e. let $A_L' = A_L K_L$; $A_R' = A_R K_R$

$$K_{\genfrac{}{}{0pt}{}{L}{R}} = \begin{bmatrix} e^{i\varphi_{1\genfrac{}{}{0pt}{}{L}{R}}} & & 0 \\ & e^{i\varphi_{2\genfrac{}{}{0pt}{}{L}{R}}} & \\ 0 & & e^{i\varphi_{3\genfrac{}{}{0pt}{}{L}{R}}} \end{bmatrix} ;$$

Then $A_L'^{+} M m^{+} A_L'$

$$= K_L^{+} A_L^{+} m m^{+} A_L K_L = K_L^{+} M_{diag}^{2} K_L$$

$$= M_{diag}^{2}.$$

& $A_R'^{+} M^{+} M A_R' = M_{diag}^{2}$ also.

However we still can use

$A_L^{+} M A_R = M_{diag}$ to determine the

phase differences $\varphi_{iL} - \varphi_{iR}$ since

$$A_L'^{+} M A_R' = K_L^{+} M_{diag} K_R$$

$$= \begin{pmatrix} e^{-i\varphi_{1L}} & & 0 \\ & e^{-i\varphi_{2L}} & \\ 0 & & e^{-i\varphi_{3L}} \end{pmatrix} \begin{pmatrix} m_1 & & 0 \\ & m_2 & \\ 0 & & m_3 \end{pmatrix} \begin{pmatrix} e^{i\varphi_{1R}} & & 0 \\ & e^{i\varphi_{2R}} & \\ 0 & & e^{i\varphi_{3R}} \end{pmatrix}$$

$$= \begin{bmatrix} m_1 e^{-i(\varphi_{1L}-\varphi_{1R})} & & 0 \\ & m_2 e^{-i(\varphi_{2L}-\varphi_{2R})} & \\ 0 & & m_3 e^{-i(\varphi_{3L}-\varphi_{3R})} \end{bmatrix}$$

* (Alternatively we can redefine phases of fields
$u_L' = K_L^{+} u_L$
$u_R' = K_R^{+} u_R$; Lagrangian form is the same if $K_L = K_R$.)

So each term must be real and positive

$$\Rightarrow \boxed{\varphi_{iL} = \varphi_{iR} \equiv \varphi_i} \; !$$

So $K_L = K_R = K$ are arb. *

So back to mass eigenstates.

Now we can define new fields (concentrating of u_L, u_R for a moment)

$$u_L^W = A_L^u u_L$$
$$u_R^W = A_R^u u_R$$

where I've now put a superscript W (each) on all the fields in the $SU(2)\times U(1)$ interaction basis — i.e. call the fields we've been working with in the Lagrangian superscript W

The mass-eigenfields now have no superscript.

Similarly for

$$d^W_L = A^d_L d_L \qquad e^W_L = A^e_L e_L$$

$$d^W_R = A^d_R d_R \qquad e^W_R = A^e_R e_R .$$

The mass & Yukawa interaction terms now become diagonalized.

$$\mathcal{L}_{Yuk} = -\bar{u}^W_L M^u u^W_R - \bar{u}^W_L h^u u^W_R \eta + \cdots$$

$$= -\bar{u}_L A^{u\dagger}_L M^u A^u_R u_R - \frac{g_2}{2M_W} \bar{u}_L A_L M^u A_R u_R \eta$$

$$+ \cdots$$

$$= -\bar{u}_L \begin{pmatrix} m_u & & 0 \\ & m_c & \\ 0 & & m_t \end{pmatrix} u_R - \frac{g_2}{2M_W}\eta \bar{u}_L \begin{pmatrix} m_u & & 0 \\ & m_c & \\ 0 & & m_t \end{pmatrix} u_R$$

$$+ \cdots$$

$$= -\left[1 + \frac{g_2}{2M_W}\eta\right] \left\{ (\bar{u}_L \; \bar{c}_L \; \bar{t}_L) \begin{pmatrix} m_u & & 0 \\ & m_c & \\ 0 & & m_t \end{pmatrix} \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} \right.$$

$$\left. + (\bar{u}_R \; \bar{c}_R \; \bar{t}_R) \begin{pmatrix} m_u & & 0 \\ & m_c & \\ 0 & & m_t \end{pmatrix} \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} \right\} + \cdots$$

h.c.

So $\mathcal{L}_{Yuk}$ only measures the eigenvalues — i.e. the masses. we now would like to determine the rest of $\Gamma_{mn}^{e,d,u}$ — i.e. A_L, A_R

So

$$\boxed{\begin{aligned}\mathcal{L}_{Yuk} = -\left[1 + \frac{g_2}{2M_W}\eta\right]\Big[& m_u \bar{u}u + m_c \bar{c}c + m_t \bar{t}t \\ & + m_d \bar{d}d + m_s \bar{s}s + m_b \bar{b}b \\ & + m_e \bar{e}e + m_\mu \bar{\mu}\mu + m_\tau \bar{\tau}\tau \Big]\end{aligned}}$$

We can now also express the gauge couplings in $\mathcal{L}_F$ in terms of the mass eigenstate fields:

Since T^3 & Y (hence Q) are the same for each generation we find

J^μ_{em}, J^μ_Z have the same form (no change)

Thus the GWS with left fermi fields in doublets has no flavor changing neutral currents (no strangeness changing neutral currents GIM).

However the J^μ_W changes:

$$J^\mu_W = \bar{e}^W \gamma^\mu (1-\gamma_5) \nu^W + \bar{d}^W \gamma^\mu (1-\gamma_5) u^W$$

$$= \bar{e}\, \gamma^\mu (1-\gamma_5) A_L^{e\dagger} \nu^W + \bar{d}\, \gamma^\mu (1-\gamma_5) A_L^{d\dagger} A_L^u u$$

$$\equiv \bar{e}\, \gamma^\mu (1-\gamma_5) \nu + \bar{d}\, \gamma^\mu (1-\gamma_5) A_C u$$

where

$$A_{C\,mn} \equiv \left(A_L^{d\dagger} A_L^u\right)_{mn}$$

&

$$\nu_L \equiv A_L^{e\dagger} \nu_L^W$$ since neutrinos are massless and we just associate ν_{mL} with e_{mL}.

A_C is the generalized Cabibbo matrix — it fixes the flavor structure of J^μ_W. So thru gauge interactions we can determine some of the Γ^{edu}_{mn}.

Note that

$$l_{mL} = \begin{pmatrix} \nu_m \\ e_m \end{pmatrix}_L$$

&

$$q_{mL} = \begin{pmatrix} A_{C\,mn} u_n \\ d_m \end{pmatrix}_L$$ transform as $SU(2)$ doublets

while u_{mR}, d_{mR}, e_{mR} are SU(2) singlets.

A_R cannot be measured in the GWS model since it never appears in the currents we can only determine A_L that is some of the A_C which appears in J^μ_W.

A_C is all the higgs couplings the GWS model can measure!

Now A_C is a 3×3 unitary matrix since the A_L are

$$A_C = A_L^{d\dagger} A_L^u$$

So it has 3^2 real parameters, but recall there are 3 arb. phases for each A_L. So we can choose 6 of the A_C parameters at will — they are not observable and can be chosen for convenience. Actually 5 of the phases can be chosen to put A_C in a convenient form the other phase does not occur in the $\mathcal{L}_{ag}$ and so is irrelevant. (Perhaps due to U(1)em even a global U(1)em).

So A_C depends only on 9−5=4 real parameters.

3 of the parameters represent rotations of the families into each other. The 4th parameter — an observable phase ∡ measures the amount of CP violation in the original Yukawa couplings.

Kobayashi and Maskawa first expressed A_c this way and so A_c is called the KM matrix

A_{KM} in this 3 family (6 quark) GWS model:

$$A_{KM} = \begin{bmatrix} c_1 & -s_1 c_2 & -s_1 s_2 \\ +s_1 c_3 & c_1 c_2 c_3 + s_2 s_3 e^{-i\delta} & c_1 s_2 c_3 - c_2 s_3 e^{-i\delta} \\ s_1 s_3 & c_1 c_2 s_3 - s_2 c_3 e^{-i\delta} & +c_1 s_2 s_3 + c_2 c_3 e^{-i\delta} \end{bmatrix}$$

$\theta_1, \theta_2, \theta_3$ are the 3 rotation ∡'s and δ is the CP violating phase.

III. Phenomenology of the KM version of the GWS model.

1) A_{KM} not we determined

a) $|C_1| = .9737 \pm .0025$

$|S_3| = .28^{+.21}_{-.28}$ from β decay & semi-leptonic hyperon decay.

b) $K_L - K_S$ mass diff ⇒ $.1 < |S_2| < .7$

c) ε parameter for CP viol. in Kaons

⇒

$$S_2 S_3 \sin\delta = O(10^{-3})$$

2) Neutral current interactions ν-hadron

ν-e

e-hadron

$$\Rightarrow \sin^2\theta_W = .229 \pm .009 \ (\pm .005)$$

exp. theory

3) Higgs mass $m_\eta = \sqrt{+2\mu^2} = \sqrt{2\lambda}\, v$

$$v = 246\,\text{GeV from } G_F$$

$$m_\eta = \sqrt{2\lambda}\, 250\,\text{GeV}$$

various arguments $m_\eta < 200\,\text{GeV}.$

4) Charmed hadron decay, non-leptonic hyperon & kaon decays, $\Delta I = \frac{1}{2}$, CP-violation, $K_L - K_S$ mass difference — all seem to be compatible with SM.

Let's discuss some <u>not</u> so satisfactory issues. The first 2 concern the Higgs sector the last the gauge coupling running.

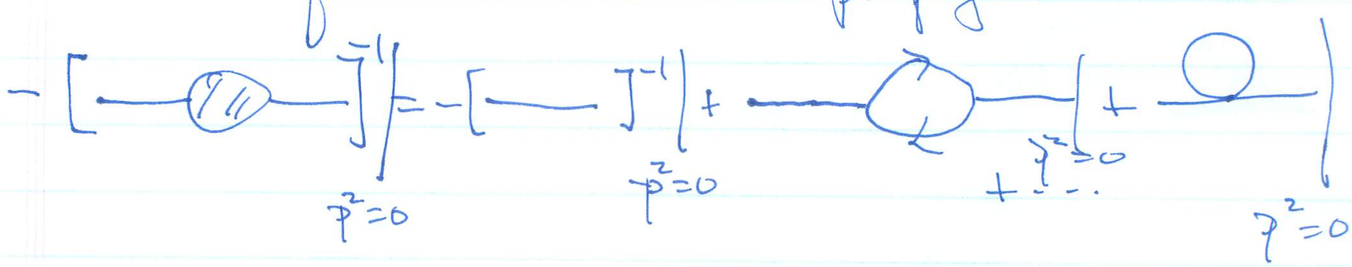
<u>Fine-Tuning Problem</u>

1) Highly sensitive Higgs Potential to heavy, high scale physics. — consider scalar field coupled to fermions & self.

$$\mathcal{L} = \bar{\psi} i \not\partial \psi - M\bar{\psi}\psi + \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + g\,\phi\bar{\psi}\psi$$

$$-\frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

The radiative corrections to the scalar mass is found from inverse propagator



$$-i(m^2+\Delta m^2) = -im^2 + (ig)^2\int\frac{d^4k}{(2\pi)^4}(-1)\,\mathrm{Tr}\left[\frac{i}{\not k - M}\,\frac{i}{\not k - M}\right]$$

$$+\frac{(-i\lambda)}{2}\int\frac{d^4k}{(2\pi)^4}\,\frac{i}{k^2-m^2}$$