Thus we have the $SU(2)\times U(1)$ invariant electroweak Lagrangian

$$\mathcal{L}_{inv} = \mathcal{L}_{YM} + \mathcal{L}_F + \mathcal{L}_\phi + \mathcal{L}_{Yuk}$$

1) $\mathcal{L}_{YM} = -\frac{1}{4}\underline{F}_{\mu\nu}\cdot\underline{F}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$

where $\underline{F}_{\mu\nu} = \partial_\mu \underline{A}_\nu - \partial_\nu \underline{A}_\mu + g_2 \underline{A}_\mu \times \underline{A}_\nu$

$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

2) $\mathcal{L}_F = \bar{l}_{mL} i\not{D} l_{mL} + \bar{q}_{mL} i\not{D} q_{mL} + \bar{e}_{mR} i\not{D} e_{mR} + \bar{u}_{mR} i\not{D} u_{mR} + \bar{d}_{mR} i\not{D} d_{mR}$

where

$$D_\mu l_L = \left(\partial_\mu - \frac{ig_2}{2}\underline{\sigma}\cdot\underline{A}_\mu + \frac{ig_1}{2}B_\mu\right)l_L$$

$$D_\mu q_L = \left(\partial_\mu - \frac{ig_2}{2}\underline{\sigma}\cdot\underline{A}_\mu - \frac{ig_1}{6}B_\mu\right)q_L$$

$$D_\mu e_R = \left(\partial_\mu + ig_1 B_\mu\right)e_R$$

$$D_\mu u_R = \left(\partial_\mu - \frac{2i}{3}g_1 B_\mu\right)u_R$$

$$D_\mu d_R = \left(\partial_\mu + \frac{i}{3}g_1 B_\mu\right)d_R$$

3) $\mathcal{L}_\phi = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi^\dagger \phi)$

where

$$V(\phi^\dagger \phi) = m^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

$$D_\mu \phi = \left(\partial_\mu - \frac{ig_2}{2} \underline{\sigma} \cdot \underline{A}_\mu - \frac{ig_1}{2} B_\mu\right) \phi$$

4) $\mathcal{L}_{yuk} = \Gamma^e_{mn} \bar{l}_{mL} \phi e_{nR} + \Gamma^d_{mn} \bar{q}_{mL} \phi d_{nR}$

$$+ \Gamma^u_{mn} \bar{q}_{mL} \tilde{\phi} u_{nR} + \text{H.C.}$$

where

$$\tilde{\phi} = i\sigma^2 \phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$$

Let's recall the gauge transformations which leave this lagrangian invariant:

1) $$\underline{T} \cdot \underline{A}'_\mu = U(\omega) \underline{T} \cdot \underline{A}_\mu U^{-1}(\omega) - \frac{i}{g_2} (\partial_\mu U(\omega)) U^{-1}(\omega)$$

1') $\underline{A}'_\mu = \underline{A}_\mu + [\partial_\mu \underline{\omega} + g_2 \underline{A}_\mu \times \underline{\omega}]$

where

$$U(\omega) = e^{+ig_2 \underline{\omega} \cdot \underline{T}}$$

2) $l'_L = U_l(\omega,\theta)\, l_L$

2') $l'_L = l_L + \frac{i}{2} g_2 \underline{\omega}\cdot\underline{\tau}\, l_L - \frac{ig_1}{2}\theta\, l_L$

where

$$U_l(\omega,\theta) = e^{+\frac{i}{2} g_2 \underline{\omega}\cdot\underline{\tau} - \frac{ig_1}{2}\theta}$$

<u>Note:</u> $\bar{l}'_L = \bar{l}_L\, U_l^{-1}(\omega,\theta)$

$$\bar{l}'_L = \bar{l}_L - \frac{i}{2} g_2 \bar{l}_L\, \underline{\omega}\cdot\underline{\tau} - \frac{ig_1}{2}\theta\, \bar{l}_L$$

3) $q'_L = U_q(\omega,\theta)\, q_L$

3') $q'_L = q_L + \frac{i}{2} g_2 \underline{\omega}\cdot\underline{\tau}\, q_L + \frac{ig_1}{6}\theta\, q_L$

where

$$U_q(\omega,\theta) = e^{+\frac{i}{2} g_2 \underline{\omega}\cdot\underline{\tau} - \frac{ig_1}{6}\theta}$$

4) $e'_R = e^{-ig_1\theta}\, e_R$

4') $e'_R = (1 - ig_1\theta)\, e_R$

5) $u'_R = e^{+\frac{2i}{3} g_1 \theta}\, u_R$

5') $u'_R = (1 + \frac{2i}{3} g_1 \theta)\, u_R$

6) $d_R' = e^{-\frac{ig_1}{3}\theta} d_R$

6') $d_R' = (1 - \frac{ig_1}{3}\theta) d_R$

7) $\phi' = U_\phi(\omega,\theta)\phi$

7') $\phi' = [1 + \frac{ig_2}{2}\underline{\omega}\cdot\underline{\tau} + \frac{ig_1}{2}\theta]\phi$

where

$$U_\phi(\omega,\theta) = e^{+\frac{ig_2}{2}\underline{\omega}\cdot\underline{\tau} + \frac{ig_1}{2}\theta}$$

We now desire to spontaneously break the $SU(2) \times U(1)$ symmetry down to $U(1)_{EM}$.

Consider the vacuum expectation of $\langle\phi\rangle \neq 0$

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} \qquad v = \text{Real}.$$

Now

$\sigma^i\begin{pmatrix}0\\ v\end{pmatrix} \neq 0$ So the vac. is not invariant under all $SU(2)$

$Y\begin{pmatrix}0\\ v\end{pmatrix} \neq 0$ since ϕ has $Y = +1/2$ So Y is broken.

But

$$Q\begin{pmatrix}0\\v\end{pmatrix} = (T_3+Y)\begin{pmatrix}0\\v\end{pmatrix} = \left(\tfrac{1}{2}\sigma^3+Y\right)\begin{pmatrix}0\\v\end{pmatrix} = 0 = \begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{pmatrix}0\\v\end{pmatrix}$$

The Q that is E-M symmetry is not broken.
i.e. ϕ_0 has zero charge!

$$\langle 0|\phi|0\rangle = \langle 0|U^{-1}U\phi U^{-1}U|0\rangle$$

$$= \langle 0'|\phi'|0'\rangle$$

if $|0\rangle$ is invariant then $U|0\rangle = |0\rangle$

So $\langle 0|\phi|0\rangle = \langle 0|\phi'|0\rangle$

$$\frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix} = U_\phi(\omega,\theta)\frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix}$$

Contradiction! So $U|0\rangle \neq |0\rangle$
$|0\rangle$ not invariant

Now we ask to look at the minimum of the potential V for $\phi = \frac{1}{\sqrt{2}}\binom{0}{v}$

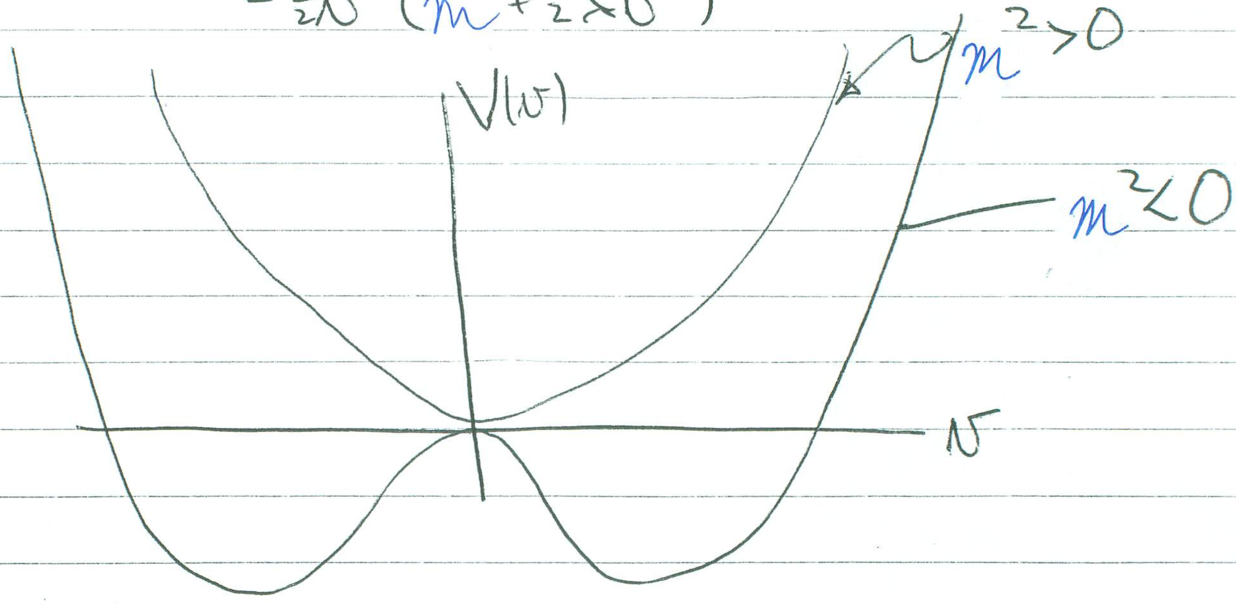
$$V(v) = m^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

$$\phi^\dagger \phi = \left(\phi^{+\dagger} \ \ \phi_0^\dagger\right) \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \phi^- \phi^+ + \phi_0^\dagger \phi_0$$

$$\langle \phi^+ \rangle = 0 \ ; \ \langle \phi^0 \rangle = v/\sqrt{2} \quad \Rightarrow \phi^\dagger \phi = \tfrac{1}{2} v^2$$

$$\Rightarrow \quad V(v) = \tfrac{1}{2} m^2 v^2 + \tfrac{1}{4} \lambda v^4$$

$$= \tfrac{1}{2} v^2 \left(m^2 + \tfrac{1}{2} \lambda v^2\right)$$



Now $V' = v(m^2 + \lambda v^2)$ if $m^2 > 0$ min. of V at $v = 0$.

Symmetric solution

if $m^2 < 0$ V has extrema a $v = 0$

$v = \sqrt{-m^2/\lambda}$

$V'' = m^2 + 3\lambda v^2$ at $v=0$

$V'' = m^2 < 0 \Rightarrow v = 0$ max

at $v = \sqrt{-m^2/\lambda} \Rightarrow V'' = m^2 - 3\lambda m^2/\lambda$

$= -2m^2 > 0$ min.

So we choose the spontaneously broken mode minimum

$$\boxed{v = \sqrt{-m^2/\lambda} \qquad m^2 < 0}$$

We can eliminate the Goldstone bosons by the Higgs-Kibble transformation: we define 4 new fields $\underline{\xi}$; η by

$$\phi = e^{\frac{-i\,\underline{\xi}\cdot\underline{\tau}}{2v}} \begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix}$$

$\underline{\xi}$ are massless G.B and η is Higgs meson.

We can now exploit the gauge invariance of the theory to transform away the $\underline{\xi}$'s, 0

Make the SU(2) gauge transformation with

$$\vec{w} = \frac{\vec{\xi}}{g_2 v}$$

then

$$\phi' = U_\phi(\theta=0, \vec{\omega} = \frac{\vec{\xi}}{g_2 v})\phi$$

$$\phi' = \begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix} \quad \text{only.}$$

Now we must re-write the Lagrangian in terms of ϕ'; A'_μ, l', q', e', u', d', $\rightarrow$ the form of the Lag. is the same so we just drop all primes and replace ϕ with $\frac{v+\eta}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}$ in the Lag.:

So, 1) $\mathcal{L}_{YM} \rightarrow \mathcal{L}_{YM}$

2) $\mathcal{L}_F \rightarrow \mathcal{L}_F$

these do not involve ϕ

3)

$$\mathcal{L}_\phi = \left[\left(\partial_\mu - \frac{ig_2}{2}\underline{\sigma}\cdot\underline{A}_\mu - \frac{ig_1}{2}B_\mu\right)\begin{pmatrix}0\\ \frac{v+\eta}{\sqrt{2}}\end{pmatrix}\right]^\dagger \times$$

$$\times \left(\partial_\mu - \frac{ig_2}{2}\underline{\sigma}\cdot\underline{A}_\mu - \frac{ig_1}{2}B_\mu\right)\begin{pmatrix}0\\ \frac{v+\eta}{\sqrt{2}}\end{pmatrix}$$

$$-\left[\frac{m^2}{2}(v+\eta)^2 + \frac{\lambda}{4}(v+\eta)^4\right]$$

$$=$$

$$\frac{1}{2}\left[(0\ \partial_\mu\eta) + \frac{ig_2}{2}(0\ \ v+\eta)\,\underline{\sigma}\cdot\underline{A}_\mu + \frac{ig_1}{2}(0\ \ v+\eta)\,B_\mu\right]\times$$

$$\times\left[\begin{pmatrix}0\\ \partial^\mu\eta\end{pmatrix} - \frac{ig_2}{2}\underline{\sigma}\cdot\underline{A}^\mu\begin{pmatrix}0\\ v+\eta\end{pmatrix} - \frac{ig_1}{2}B^\mu\begin{pmatrix}0\\ v+\eta\end{pmatrix}\right]$$

$$-V(v+\eta)$$

$$= \frac{1}{2}\Big[\partial_\mu\eta\,\partial^\mu\eta - \cancel{\frac{ig_2}{2}(0\ 1)\,\underline{\sigma}\cdot\underline{A}^\mu\begin{pmatrix}0\\1\end{pmatrix}\partial_\mu\eta\,(v+\eta)}$$

$$-\frac{ig_1}{2}B^\mu\,\partial_\mu\eta\,(v+\eta) + \cancel{\frac{ig_2}{2}(0\ 1)\,\underline{\sigma}\cdot\underline{A}^\mu\begin{pmatrix}0\\1\end{pmatrix}(v+\eta)\,\partial_\mu\eta}$$

$$+\frac{g_2^2}{4}(v+\eta)^2(0\ 1)\,\underline{\sigma}\cdot\underline{A}^\mu\,\underline{\sigma}\cdot\underline{A}_\mu\begin{pmatrix}0\\1\end{pmatrix} + \frac{g_2g_1}{4}B^\mu(v+\eta)^2(0\ 1)\,\underline{\sigma}\cdot\underline{A}_\mu\begin{pmatrix}0\\1\end{pmatrix}$$

$$+\frac{ig_1}{2}B_\mu(v+\eta)\,\partial^\mu\eta + \frac{g_2g_1}{4}(v+\eta)^2(0\ 1)\,\underline{\sigma}\cdot\underline{A}^\mu\begin{pmatrix}0\\1\end{pmatrix}B_\mu$$

$$+\frac{g_1^2}{4}B_\mu B^\mu(v+\eta)^2\Big] - V(v+\eta)$$

$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu \eta \partial^\mu \eta$$

$$+\frac{1}{8}(v+\eta)^2 \; (0\;1)\left[g_2^2\, \underline{\tau}\cdot\underline{A}^\mu\, \underline{\tau}\cdot\underline{A}_\mu + 2g_2g_1 B^\mu\, \underline{\tau}\cdot\underline{A}_\mu + g_1^2 B_\mu B^\mu\right]\begin{pmatrix}0\\1\end{pmatrix} - V(\eta+v)$$

$$= \frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \frac{1}{8}(v+\eta)^2\; (0\;1)\left[g_2\underline{\tau}\cdot\underline{A}^\mu + g_1 B^\mu\right]\left[g_2\underline{\tau}\cdot\underline{A}_\mu + g_1 B_\mu\right]\begin{pmatrix}0\\1\end{pmatrix} - V(\eta+v)$$

Now

$$g_2\underline{\tau}\cdot\underline{A}^\mu + g_1 B^\mu = \begin{bmatrix} g_2A_3^\mu + g_1 B^\mu & g_2(A_1^\mu - iA_2^\mu) \\ g_2(A_1^\mu + iA_2^\mu) & -g_2A_3^\mu + g_1B^\mu \end{bmatrix}$$

So it is convenient to define

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(A_\mu^1 \mp i A_\mu^2\right)$$

$$Z_\mu \equiv \frac{g_2 A_\mu^3 - g_1 B_\mu}{\sqrt{g_2^2+g_1^2}}$$

$$A_\mu \equiv \frac{g_2 B_\mu + g_1 A_\mu^3}{\sqrt{g_2^2+g_1^2}}$$

So

$$(0\ 1)\left[g_2 \vec{\tau}\cdot \vec{A}^\mu + g_1 B^\mu\right]^2 \begin{pmatrix}0\\1\end{pmatrix}$$

$$= (0\ 1)\begin{bmatrix} (g_2 A_3^\mu + g_1 B^\mu)^2 + g_2^2 ({A_1^\mu}^2 + {A_2^\mu}^2) & g_2^2 \text{ ——} \\ \text{——} & g_2^2 (A_1^\mu + i A_2^\mu)(A_1^\mu - i A_2^\mu) + (g_1 B^\mu - g_2 A_3^\mu)^2 \end{bmatrix}\begin{pmatrix}0\\1\end{pmatrix}$$

$$= 2 g_2^2 W_\mu^+ W^{-\mu} + (g_2^2 + g_1^2) Z_\mu Z^\mu .$$

Thus

$$\boxed{\begin{aligned} \mathcal{L}_\phi = {} & \frac{1}{2}\partial_\mu \eta\, \partial^\mu \eta - \left[\frac{m^2}{2}(v+\eta)^2 + \frac{\lambda}{4}(v+\eta)^4\right] \\ & + \frac{1}{8}(v+\eta)^2\left[2 g_2^2 W_\mu^+ W^{-\mu} + (g_2^2 + g_1^2) Z_\mu Z^\mu\right] \end{aligned}}$$

So let's ~~note~~ since $V' = 0$ the terms linear in η vanish we find

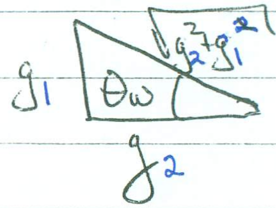
$$\mathcal{L}_\phi = \tfrac{1}{2}\partial_\mu \eta \partial^\mu \eta - \frac{m^2}{2}\eta^2 - \frac{\lambda}{4}\left[\eta^4 + 4v\eta^3 + 6v^2\eta^2\right]$$

$$+ \tfrac{1}{8}\left(v^2 + 2\eta v + \eta^2\right)\left[2g_2^2 W_\mu^+ W^{-\mu} + (g_2^2 + g_1^2) Z_\mu Z^\mu\right]$$

Now we define $M_W \equiv \frac{g_2 v}{2}$ the mass of the $W^\pm$

$$M_Z^2 = \frac{(g_2^2 + g_1^2)}{g_2^2} M_W^2$$

Now let $\frac{g_1}{g_2} = \tan\theta_W$



θ_W = weak or Weinberg $\measuredangle$

So $$\boxed{M_Z = \frac{M_W}{\cos\theta_W}}$$

Also $$e \equiv \frac{g_2 g_1}{\sqrt{g_2^2 + g_1^2}} = g_2 \sin\theta_W$$

$$Z_\mu = \cos\theta_W A^3_\mu - \sin\theta_W B_\mu$$

$$A_\mu = \sin\theta_W A^3_\mu + \cos\theta_W B_\mu$$

There is no $A_\mu A^\mu$ term so the photon is massless as it should since $U(1)_{em}$ is unbroken.

The mass^2 of η is

let $m^2 = -\mu^2 < 0$

$$-\mu^2 + 3\lambda v^2 = m_\eta^2 = -\mu^2 + 3\mu^2 = +2\mu^2$$

$$\boxed{m_\eta^2 = 2\mu^2 > 0.}$$

minimum is $+\mu^2 = \lambda v^2$

$m_\eta^2 = 2\lambda v^2$

The remaining terms are higgs meson self-interaction terms and higgs-vector interactions

So we find

$$\boxed{\begin{aligned}\mathcal{L}_\phi = {} & \tfrac{1}{2}\partial_\mu\eta\partial^\mu\eta - \tfrac{1}{2}m_\eta^2\eta^2 - \tfrac{\lambda}{4}\left(\eta^4 + 4v\eta^3\right) \\ & + g_2 M_W\, \eta W^+_\mu W^{-\mu} + \tfrac{1}{2}\frac{g_2 M_W}{\cos^2\theta_W}\eta Z_\mu Z^\mu \\ & + \tfrac{1}{4}g_2^2\eta^2 W^+_\mu W^{-\mu} + \tfrac{1}{8}\frac{g_2^2}{\cos^2\theta_W} Z_\mu Z^\mu \eta^2 \\ & + M_W^2 W^+_\mu W^{-\mu} + \tfrac{1}{2}M_Z^2 Z_\mu Z^\mu\end{aligned}}$$

We can now re-write $\mathcal{L}_{YM}$ & $\mathcal{L}_F$ in terms of $A_\mu, Z_\mu, W^\pm_\mu$

Recall

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} c & -s \\ s & c \end{pmatrix}\begin{pmatrix} A^3 \\ B \end{pmatrix}$$

$\Rightarrow$

$$\begin{pmatrix} A^3_\mu \\ B \end{pmatrix} = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}\begin{pmatrix} Z \\ A \end{pmatrix}$$

So

$$A^3_\mu = \cos\theta_W Z_\mu + \sin\theta_W A_\mu$$

$$B_\mu = -\sin\theta_W Z_\mu + \cos\theta_W A_\mu$$

So

$$-\frac{ig}{2}\underline{\sigma}\cdot\underline{A}_\mu + \frac{ig'}{2}B_\mu$$

$$= -\frac{i}{2}\begin{bmatrix} gA^3_\mu - g'B_\mu & g(A^1_\mu - iA^2_\mu) \\ g(A^1_\mu + iA^2_\mu) & -gA^3_\mu - g'B_\mu \end{bmatrix}$$

$\boxed{\text{Let } g = g_2 \quad g' = g_1}$

$$= -\frac{i}{2}\begin{bmatrix} gcZ \cancel{+gsA} + g'sZ \cancel{-g'cA} & g\sqrt{2}\,W^+ \\ g\sqrt{2}\,W^- & -(gc - g's)Z - \cancel{(g'c + gs)}A \end{bmatrix}_\mu$$

$$= -\frac{i}{2}\begin{bmatrix} Z\sqrt{g^2+g'^2} & +g\sqrt{2}\,W^+ \\ +g\sqrt{2}\,W^- & \frac{g'^2 - g^2}{\sqrt{g^2+g'^2}}Z - \frac{2gg'}{\sqrt{g^2+g'^2}}A \end{bmatrix}_\mu$$

So $\boxed{\begin{aligned} g &= g_2 \\ g' &= g_1 \end{aligned}}$

$$\bar{l}_{mL}\, i\not{D}\, l_{mL} = \left(\bar{\nu}_{mL}\ \bar{e}_{mL}\right) i\not{\partial} \begin{pmatrix} \nu_{mL} \\ e_{mL} \end{pmatrix}$$

$$+ \left(\bar{\nu}_{mL}\ \bar{e}_{mL}\right) \frac{1}{2} \begin{bmatrix} \sqrt{g^2+g'^2}\,\not{Z} & +\sqrt{2}\,g\,\not{W}^+ \\ +\sqrt{2}\,g\,\not{W}^- & \frac{g'^2-g^2}{\sqrt{g^2+g'^2}}\not{Z} - \frac{2gg'}{\sqrt{\quad}}\not{A} \end{bmatrix} \begin{pmatrix} \nu_{mL} \\ e_{mL} \end{pmatrix}$$

$$\bar{e}_R\, i\not{D}\, e_R = \bar{e}_R\, i\not{\partial}\, e_R$$

$$+ \bar{e}_R(-g')\left[-\sin\theta_W \not{Z} + \cos\theta_W \not{A}\right] e_R$$

$$= \bar{e}_R\, i\not{\partial}\, e_R + \bar{e}_R\left[\frac{g'^2}{\sqrt{\quad}}\not{Z} - \frac{gg'}{\sqrt{\quad}}\not{A}\right] e_R$$

Now we combine these two :

$$\bar{l}_{mL}\, i\not\partial\, l_{mL} = \bar{\nu}_L i\not\partial \nu_L + \bar{e}_L i\not\partial e_L + \bar{e}_R i\not\partial e_R$$

$$+ \bar{e}_R i\not\partial e_R$$

$$+ \tfrac{1}{2}\, \bar{\nu}_L \left(\sqrt{}\, \not{Z} \nu_L + \sqrt{2}\, g \not{W}^+ e_L \right)$$

$$+ \tfrac{1}{2}\, \bar{e}_L \left(\sqrt{2}\, g \not{W}^- \nu_L + \frac{g'^2 - g^2}{\sqrt{}} \not{Z} e_L - \frac{2gg'}{\sqrt{}} \not{A} e_L \right)$$

$$+ \bar{e}_R \frac{g'^2}{\sqrt{}} \not{Z} e_R - \frac{gg'}{\sqrt{}} \bar{e}_R \not{A} e_R$$

$$= \bar{\nu}_L i\not\partial \nu_L + \bar{e} i\not\partial e$$

$$+ \frac{\sqrt{g^2+g'^2}}{2} Z_\mu \left[\bar{\nu}_L \gamma^\mu \nu_L + \frac{g'^2 - g^2}{g'^2+g^2} \bar{e}_L \gamma^\mu e_L + \frac{g'^2}{g^2+g'^2} \bar{e}_R \gamma^\mu e_R \right]$$

$$- \frac{gg'}{\sqrt{g^2+g'^2}} A_\mu \left[\bar{e} \gamma^\mu e \right]$$

$$+ \frac{g}{\sqrt{2}} \left[\bar{\nu}_L \not{W}^+ e_L + \bar{e}_L \not{W}^- \nu_L \right]$$

Next we consider

$$\bar q_L i\not D q_L + \bar u_R i\not D u_R + \bar d_R i\not D d_R$$

$$= \bar u i\not\partial u + \bar d i\not\partial d$$

$$+ \frac{1}{2}\bar u_L\left(\sqrt{\ }\,\not Z u_L + \sqrt{2} g \not W^+ d_L\right)$$

$$+ \frac{1}{2}\bar d_L\left(\sqrt{2} g \not W^- u_L + \frac{g'^2-g^2}{\sqrt{\ }}\not Z d_L - \frac{2gg'}{\sqrt{\ }}\not A d_L\right)$$

$$+ \frac{2}{3}g' \overline{u_L d_L}\,\not B\begin{pmatrix} u_L \\ d_L\end{pmatrix} + \frac{2}{3}g' \bar u_R \not B u_R$$

$$- \frac{1}{3}g' \bar d_R \not B d_R$$

$$= \bar u i\not\partial u + \bar d i\not\partial d$$

$$+ \frac{\sqrt{g^2+g'^2}}{2} Z_\mu\Big[\bar u_L\gamma^\mu u_L + \frac{g'^2-g^2}{g'^2+g^2}\bar d_L\gamma^\mu d_L$$

$$- \frac{4}{3}\frac{g'^2}{g^2+g'^2}\bar u\gamma^\mu u - \frac{4}{3}\frac{g'^2}{g^2+g'^2} d_L\gamma^\mu d_L$$

$$+ \frac{2}{3}\frac{g'^2}{g^2+g'^2} d_R\gamma^\mu d_R\Big]$$

$$+ \frac{gg'}{\sqrt{g^2+g'^2}} A_\mu\Big[- \bar d_L\gamma^\mu d_L + \frac{2}{3}\bar u\gamma^\mu u$$

$$+ \frac{2}{3}\bar d_L\gamma^\mu d_L - \frac{1}{3}\bar d_R\gamma^\mu d_R\Big]$$

$$+ \frac{g}{\sqrt{2}}\left[\bar u_L \not W^+ d_L + \bar d_L \not W^- u_L\right]$$

So we can put this altogether:

$$\mathcal{L}_F = \bar{\nu}_L i\not{\partial}\nu_L + \bar{e}\, i\not{\partial} e + \bar{u}\, i\not{\partial} u + \bar{d}\, i\not{\partial} d$$

$$+ e A_\mu J^\mu_{em} + \frac{g_2}{2\sqrt{2}}\left(J^\mu_W W^-_\mu + J^{\mu\,+}_W W^+_\mu\right)$$

$$+ \frac{\sqrt{g_2^2 + g_1^2}}{2} J^\mu_Z Z_\mu$$

where the electromagnetic current J^μ_{em} is given by

$$J^\mu_{em} = \left[+\tfrac{2}{3}\bar{u}\gamma^\mu u - \tfrac{1}{3}\bar{d}\gamma^\mu d - \bar{e}\gamma^\mu e\right]$$

The charged weak current J^μ_W is

$$J^\mu_W = \left(\bar{e}_L \gamma^\mu \nu_L + \bar{d}_L \gamma^\mu u_L\right) 2$$

$$= \left(\bar{e}\gamma^\mu(1-\gamma_5)\nu + \bar{d}\gamma^\mu(1-\gamma_5) u\right)$$

The weak neutral current is given by

$$+J_Z^\mu = +\bar{\nu}_L\gamma^\mu\nu_L - \frac{g^2-g'^2}{g'^2+g^2}\,\bar{e}_L\gamma^\mu e_L + \frac{g'^2}{g^2+g'^2}\,\bar{e}_R\gamma^\mu e_R$$

$$-\frac{4}{3}\frac{g'^2}{g^2+g'^2}\,\bar{u}\gamma^\mu u + \bar{u}_L\gamma^\mu u_L$$

$$+\frac{-g^2-\frac{1}{3}g'^2}{g^2+g'^2}\,\bar{d}_L\gamma^\mu d_L + \frac{2}{3}\frac{g'^2}{g^2+g'^2}\,\bar{d}_R\gamma^\mu d_R$$

$$= +\bar{\nu}_L\gamma^\mu\nu_L + \sin^2\theta_W\,\bar{e}\gamma^\mu e - \cos^2\theta_W\,\bar{e}_L\gamma^\mu e_L$$

$$-\frac{4}{3}\sin^2\theta_W\,\bar{u}\gamma^\mu u + \bar{u}_L\gamma^\mu u_L$$

$$-\cos^2\theta_W\,\bar{d}_L\gamma^\mu d_L - \frac{1}{3}\sin^2\theta_W\,\bar{d}_L\gamma^\mu d_L$$

$$+\frac{2}{3}\sin^2\theta_W\,\bar{d}_R\gamma^\mu d_R$$

$$J_Z^\mu = \bar{\nu}_L\gamma^\mu\nu_L + 2\sin^2\theta_W\,\bar{e}\gamma^\mu e - \bar{e}_L\gamma^\mu e_L$$

$$-\frac{4}{3}\sin^2\theta_W\,\bar{u}\gamma^\mu u + \bar{u}_L\gamma^\mu u_L$$

$$+\frac{2}{3}\sin^2\theta_W\,\bar{d}\gamma^\mu d - \bar{d}_L\gamma^\mu d_L$$

So we have the 3 types of currents:

1) $J^\mu_W = \left(\bar{e}_m \gamma^\mu (1-\gamma_5) \nu_m + \bar{d}_m \gamma^\mu (1-\gamma_5) u_m\right)$

2) $J^\mu_{em} = q_m \bar{\psi}_m \gamma^\mu \psi_m$

$$= +\tfrac{2}{3} \bar{u}_m \gamma^\mu u_m - \tfrac{1}{3} \bar{d}_m \gamma^\mu d_m - \bar{e}_m \gamma^\mu e_m$$

3)

$$J^\mu_Z = \bar{\psi}_m \gamma^\mu T^3_m (1-\gamma_5) \psi_m - 2 q_m \bar{\psi}_m \gamma^\mu \psi_m \sin^2\theta_W$$

where T^3_m is value of T^3 for ψ_m (i.e. $\pm\frac{1}{2}$) and q_m the charge.

$$= \bar{\nu}_L \gamma^\mu \nu_L - \bar{e}_L \gamma^\mu e_L + \bar{u}_L \gamma^\mu u_L - \bar{d}_L \gamma^\mu d_L$$

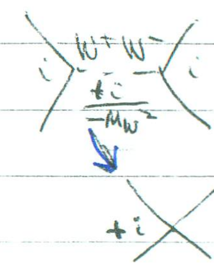
$$+ 2\sin^2\theta_W \left(\bar{e}\gamma^\mu e - \tfrac{2}{3}\bar{u}\gamma^\mu u + \tfrac{1}{3}\bar{d}\gamma^\mu d\right)$$

The masses of M_Z & M_W can be estimated by low energy effective theory. Recall $W^\pm$ has a propagator like $\sim \frac{g_{\mu\nu}}{p^2 - M_W^2}$ & Z $\frac{g_{\mu\nu}}{p^2 - M_Z^2}$

for low energy.

So for a charged current interaction we find

$$\mathcal{L}_c^{eff} = \frac{g_2^2}{8}\left(J^\mu_W J^\dagger_{W\mu} + h.c.\right)\frac{i^2}{-M_W^2}$$



$$= +\frac{g_2^2}{8 M_W^2}\left(J^\mu_W J^\dagger_{W\mu} + h.c\right)$$

$$\equiv +\frac{G_F}{\sqrt{2}}\left(J^\mu_W J^\dagger_{W\mu} + h.c.\right)$$

So

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8 M_W^2} = \frac{1}{2v^2}}$$

$$\frac{v}{\sqrt{2}} = \frac{2^{1/4}}{2\sqrt{G_F}}$$

Now recall $e = g_2 \sin\theta_W$, $e^2 = 4\pi\alpha$

So

$$M_W^2 = \frac{\sqrt{2}}{G_F}\frac{e^2}{8\sin^2\theta_W} = \frac{(37\,\text{GeV})^2}{\sin^2\theta_W}$$

$$\boxed{M_W = \frac{37\,\text{GeV}}{\sin\theta_W}}$$

$$\boxed{M_Z = \frac{M_W}{\cos\theta_W} = \frac{75\,\text{GeV}}{\sin 2\theta_W}}$$

to lowest order.

$\sin^2\theta_W(M_Z) \simeq .23122$