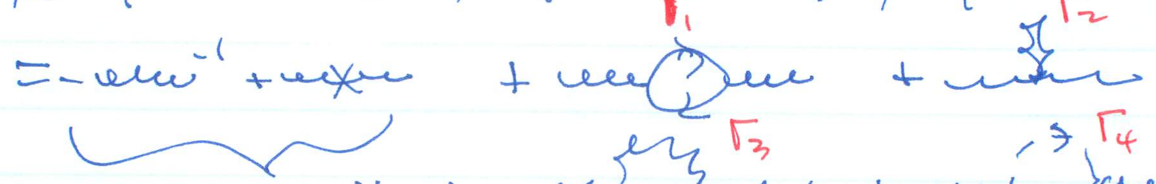
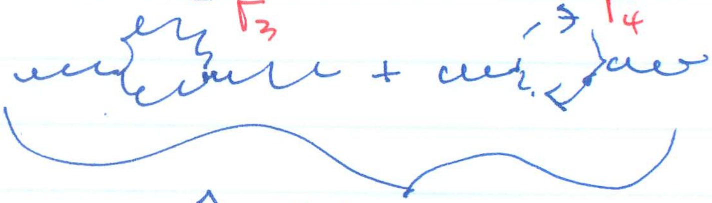
I.B. | Finally we must consider Z_3 p. -43-

$$\Gamma^{(0,0,2,0,0)}_{\substack{ij\\ \mu\nu}}(p) = \Gamma_T(p^2)\, P^T_{\mu\nu}(p)\, \delta^{ij} + \Gamma_L(p^2)\, P^L_{\mu\nu}(p)\, \delta^{ij}$$



$$= -i Z_3 p^2 P^T_{\mu\nu} \delta^{ij} - i \frac{Z_\alpha}{\alpha} p^2 P^L_{\mu\nu} \delta^{ij} +$$



$$\hat{\Pi}_{\mu\nu}(p)\,\delta^{ij} \equiv \left[\hat{\Pi}_T P^T_{\mu\nu} + \hat{\Pi}_L P^L_{\mu\nu}\right]\delta^{ij}$$

So

$$\Gamma_T(p^2) = -i Z_3 p^2 + \hat{\Pi}_T(p^2)$$

Normalization condition for "residue"

$$\left.\frac{\partial}{\partial p^2}\Gamma_T\right|_{p^2=-\mu^2} \equiv -i$$

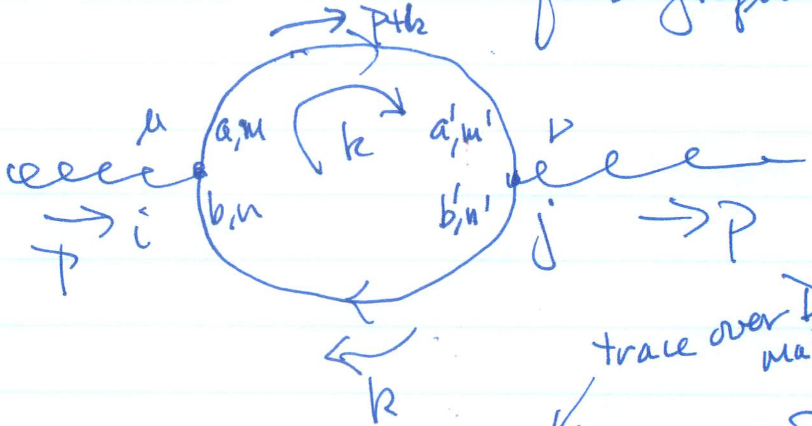
$$\Rightarrow \quad \boxed{Z_3 = 1 - i \left.\frac{\partial}{\partial p^2}\hat{\Pi}_T(p^2)\right|_{p^2=-\mu^2}}$$

So we must calculate

$$\Gamma^{(0,0,2,0,0)}_{\substack{ij\\ \mu\nu}}(p)$$

I.B.) Consider the contribution from the first graph Γ_1

$$\Gamma_{1\,ij}^{\mu\nu} \equiv$$



closed loop of Fermions

trace over Dirac matrices

$$= (-1)\int \frac{d^4k}{(2\pi)^4}\; i Z_1^F g_3 T^i_{ab}\,\delta_{mn}\,\mathrm{Tr}\Big[\gamma^\mu\Big(\frac{i\delta_{bb'}\delta^{nn'}}{\not k}\Big)\times$$
$$\times\, i Z_1^F g_3 T^j_{b'a'}\,\delta_{m'n'}\,\gamma^\nu\Big(\frac{i\delta_{aa'}\delta^{mm'}}{(\not p+\not k)}\Big)\Big]$$

$$= -g_3^2\,\mathrm{Tr}[T^iT^j]\,\delta_{mm}\int\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\Big[\gamma^\mu\frac{\not k}{k^2}\gamma^\nu\frac{(\not p+\not k)}{(p+k)^2}\Big]$$

Note:
$$\mathrm{Tr}\{T^iT^j\} = \tfrac{1}{2}\delta^{ij}$$
&
$$\delta_{mm} = \sum_{m=1}^{6}\delta_{mm} = N_F = 6 = \#\text{ of Flavours of quarks}$$

So

$$\Gamma_{1\,ij}^{\mu\nu} = -g_3^2\,\tfrac{1}{2}\delta^{ij}N_F\int\frac{d^4k}{(2\pi)^4}\,\frac{\mathrm{Tr}[\gamma^\mu\not k\gamma^\nu(\not p+\not k)]}{k^2(p+k)^2}$$

Now recall
$$\mathrm{Tr}\{\gamma^\mu\gamma^\rho\gamma^\nu\gamma^\sigma\} = 4\{g^{\mu\rho}g^{\nu\sigma} - g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\rho\nu}\}$$
(Should do in d-dimensions but get same results)

I.B.) So

$$\Gamma_{1\,ij}^{\mu\nu} = -g_3^2 \frac{4}{2} \delta^{ij} N_F \int \frac{d^4k}{(2\pi)^4} \frac{\left[k^\mu (p+k)^\nu - g^{\mu\nu}(k\cdot p + k^2) + (p+k)^\mu k^\nu \right]}{k^2 (p+k)^2}$$

$$= -2 g_3^2 \delta^{ij} N_F \int \frac{d^4k}{(2\pi)^4} \frac{\left[2k^\mu k^\nu - k^2 g^{\mu\nu} + p^\mu k^\nu + k^\mu p^\nu - g^{\mu\nu} k\cdot p \right]}{k^2 (p+k)^2}$$

Now combine denominators

$$\frac{1}{k^2 (p+k)^2} = \int_0^1 dx \frac{1}{\left[x(p+k)^2 + (1-x) k^2 \right]^2}$$

let $l^\mu = k^\mu + x p^\mu$: $k^\mu = l^\mu - x p^\mu$; $p^\mu + k^\mu = l^\mu + (1-x) p^\mu$

$\Rightarrow$

$$\Gamma_{1\,ij}^{\mu\nu} = -2 g_3^2 \delta^{ij} N_F \int \frac{d^4 l}{(2\pi)^4} \int_0^1 dx \frac{1}{\left[l^2 + p^2 (1-x) x \right]^2} \times$$

$$\times \Big[(l - xp)^\mu (l + (1-x)p)^\nu - g^{\mu\nu} [l - xp] \cdot [l + (1-x)p] + [l - xp]^\nu [l + (1-x)p]^\mu \Big]$$

I.B)

$$\text{Numerator} = l^\mu l^\nu - x(1-x)p^\mu p^\nu - x p^\mu l^\nu + (1-x) l^\mu p^\nu$$
$$- g^{\mu\nu}\left[l^2 - x(1-x)p^2 - x p\cdot l + (1-x) p\cdot l \right]$$
$$+ l^\mu l^\nu + (1-x) p^\mu l^\nu - x l^\mu p^\nu - x(1-x) p^\mu p^\nu$$
$$= 2 l^\mu l^\nu - g^{\mu\nu} l^2 - 2x(1-x) p^\mu p^\nu + g^{\mu\nu} x(1-x) p^2$$
$$+ \underbrace{l^\mu(\ldots) + l^\nu(\ldots)}_{\text{linear in } l}$$

So the terms linear in l^μ integrate to zero and we have

$$\Gamma^{\mu\nu}_{1\,ij} = -2 g_3^2 \delta^{ij} N_F \int_0^1 dx \int \frac{d^4 l}{(2\pi)^4} \frac{\left[2 l^\mu l^\nu - g^{\mu\nu} l^2 + x(1-x)\left[p^2 g^{\mu\nu} - 2 p^\mu p^\nu \right]\right]}{\left[l^2 + p^2 (1-x) x \right]^2}$$

Dimensionally regulate!

$$\Gamma^{\mu\nu}_{1\,ij} = -2 g_3^2 \delta^{ij} N_F \int_0^1 dx \int \frac{d^d l}{(2\pi)^d} \frac{\left[2 l^\mu l^\nu - g^{\mu\nu} l^2 + x(1-x)\left[p^2 g^{\mu\nu} - 2 p^\mu p^\nu \right]\right]}{\left[l^2 + p^2 (1-x) x \right]^2}$$

I.B.| Now $\int d^d\ell\, \ell^\mu \ell^\nu f(\ell^2) = \frac{1}{d}\int d^d\ell\, \ell^2 f(\ell^2) g^{\mu\nu}$

$$\Gamma^{\mu\nu}_{1\,ij} = -2g_3^2\delta^{ij}N_F\int_0^1 dx \int \frac{d^d\ell}{(2\pi)^d} \frac{\left[\left(\frac{2}{d}-1\right)\ell^2 g^{\mu\nu} + x(1-x)\left[p^2 g^{\mu\nu} - 2p^\mu p^\nu\right]\right]}{\left[\ell^2 + p^2(1-x)x\right]^2}$$

Wick rotate $\Rightarrow$

$$\Gamma^{\mu\nu}_{1\,ij} = -2ig_3^2\delta^{ij}N_F\int_0^1 dx \int \frac{d^d\ell_E}{(2\pi)^d} \times$$

$$\times \frac{\left[\left(1-\frac{2}{d}\right)\ell_E^2 g^{\mu\nu} + x(1-x)\left[p^2 g^{\mu\nu} - 2p^\mu p^\nu\right]\right]}{\left[\ell_E^2 - p^2 x(1-x)\right]^2}$$

$$= -2ig_3^2\delta^{ij}N_F\int_0^1 dx\, \Omega_d \int_0^\infty d\ell\, \ell^{d-1} \times$$

$$\times \left[\frac{\left(1-\frac{2}{d}\right)\ell^2 g^{\mu\nu} + x(1-x)\left\{p^2 g^{\mu\nu} - 2p^\mu p^\nu\right\}}{\left[\ell^2 - p^2 x(1-x)\right]^2}\right]$$

Now we have done these integrals we get

$$\int \frac{d^d\ell_E}{(2\pi)^d} \frac{1}{\left[\ell_E^2 + M\right]^n} = \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(n-\frac{d}{2})}{\Gamma(n)} \left(\frac{1}{M}\right)^{n-\frac{d}{2}}$$

$$\int \frac{d^d\ell_E}{(2\pi)^d} \frac{\ell_E^2}{\left[\ell_E + M\right]^n} = \frac{1}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(n-\frac{d}{2}-1)}{\Gamma(n)} \left(\frac{1}{M}\right)^{n-\frac{d}{2}-1}$$

I.B.) $\Gamma^{\mu\nu}_{1\,ij} = -2ig_3^2\,\delta^{ij} N_F \int_0^1 dx \times \frac{1}{(4\pi)^{d/2}} \frac{1}{\Gamma(2)}$

$$\times \Big[\frac{d}{2}\Gamma(1-\tfrac{d}{2}) \left(\frac{1}{M}\right)^{2-\frac{d}{2}-1} (1-\tfrac{2}{d}) g^{\mu\nu}$$

$$+ x(1-x)\left[p^2 g^{\mu\nu} - 2p^\mu p^\nu\right]\Gamma(2-\tfrac{d}{2})\left(\frac{1}{M}\right)^{2-d/2}\Big]$$

$$\boxed{M \equiv -p^2 x(1-x)}$$

$$\boxed{\begin{aligned}\Gamma^{\mu\nu}_{1\,ij} &= -2ig_3^2\,\delta^{ij} N_F \frac{1}{(4\pi)^2}\int_0^1 dx \times \\ &\times\Big[-(1-\tfrac{d}{2})\Gamma(1-\tfrac{d}{2})\left(\frac{1}{M}\right)^{1-\frac{d}{2}} g^{\mu\nu} \\ &+ x(1-x)\left[p^2 g^{\mu\nu} - 2p^\mu p^\nu\right]\Gamma(2-\tfrac{d}{2})\left(\frac{1}{M}\right)^{2-\frac{d}{2}}\Big]\end{aligned}}$$

Now $(1-\frac{d}{2})\Gamma(1-\frac{d}{2}) = \Gamma(2-\frac{d}{2})$ $\left(z\Gamma(z) = \Gamma(z+1)\right)$

$$\Gamma^{\mu\nu}_{1\,ij} = -2ig_3^2\,\delta^{ij} N_F \frac{1}{(4\pi)^2}\int_0^1 dx \times$$

$$\times\left[+\Gamma(2-\tfrac{d}{2})\left(\frac{1}{M}\right)^{2-\frac{d}{2}}\right]\left[-Mg^{\mu\nu} + x(1-x)\left[p^2 g^{\mu\nu} - 2p^\mu p^\nu\right]\right]$$

So we want the divergent part

$\epsilon = 4-d$
$\frac{\epsilon}{2} = 2 - d$

I.B.)
$$\Gamma^{\mu\nu}_{1\,div\,ij} \xrightarrow[\epsilon\to 0]{d\to 4} -2ig_3^2\delta^{ij}N_F\frac{1}{(4\pi)^2}\int_0^1 dx\left[\Gamma\left(\frac{\epsilon}{2}\right)e^{-\frac{\epsilon}{2}\ln M}\right]\times$$
$$\left[-Mg^{\mu\nu} + x(1-x)\left[p^2 g^{\mu\nu} - 2p^\mu p^\nu\right]\right]$$

$$= -2ig_3^2\delta^{ij}N_F\frac{1}{(4\pi)^2}\int_0^1 dx\left[\frac{2}{\epsilon}\left[1-\frac{\epsilon}{2}\ln M\right]\right]\times$$
$$\times\left[-Mg^{\mu\nu} + x(1-x)\left[p^2 g^{\mu\nu} - 2p^\mu p^\nu\right]\right]$$

Recall $M = -p^2 x(1-x)$.
$\Rightarrow$

$$\Gamma^{\mu\nu}_{1\,div\,ij} = -2ig_3^2\delta^{ij}N_F\frac{1}{(4\pi)^2}\int_0^1 dx\left[2 - \ln M\right]\times$$
$$\times\left[p^2 x(1-x)g^{\mu\nu} + x(1-x)\left[p^2 g^{\mu\nu} - 2p^\mu p^\nu\right]\right]$$

$$\boxed{\Gamma^{\mu\nu}_{1\,div\,ij} = -2ig_3^2\delta^{ij}N_F\frac{1}{(4\pi)^2}\int_0^1 dx\left[\frac{2}{\epsilon} - \ln M\right]\cdot 2\cdot x(1-x)\times \underbrace{\left[p^2 g^{\mu\nu} - p^\mu p^\nu\right]}_{= p^2 P_T^{\mu\nu}(p)}}$$

purely transverse

So we find

I.B.) $\hat{\Pi}_{\mu\nu}(p^2) = -4ip^2 g_3 N_F \frac{1}{(4\pi)^2}\int_0^1 dx \left(\frac{2}{\epsilon} - \ln M\right) x(1-x)$

diagram 1

We eventually want the μ dependent term

$$\hat{\Pi}_{\mu\nu}(p^2) = -\frac{4ip^2 g_3 N_F}{(4\pi)^2}\int_0^1 dx \left[\frac{2}{\epsilon} - \ln(-p^2) - \ln(x(1-x))\right] x(x)(1-x)$$

So

$$Z_3^{(1)} = 1 - i\frac{\partial}{\partial p^2}\hat{\Pi}_{\mu\nu}(p^2)\Big|_{p^2=-\mu^2}$$

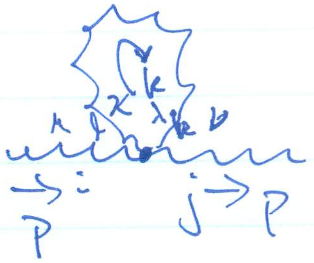
diagram 1

$$= 1 + \frac{4 g_3 N_F}{(4\pi)^2}\ln\mu^2 \underbrace{\int_0^1 dx\, x(1-x)}_{=1/6} + \overbrace{\cdots}^{\mu \text{ indep.}}$$

$$\boxed{Z_3^{(1)} = 1 + \frac{4}{3}\frac{g_3 N_F}{(4\pi)^2}\ln\mu + \cdots}$$

Next let's note

$\Gamma_{2\,ij}^{\mu\nu} =$



Combinatoric factor

$$= \frac{1}{2}\, -i Z_1^{4G} g_3^2 \times \int \frac{d^4k}{(2\pi)^4} \left(\frac{-i\delta^{kl} g^{\kappa\lambda}}{k^2} \right) \times$$

$$\times \Big[f_{hij} f_{hkl} (g_{\mu\lambda} g_{\nu\kappa} - g_{\mu\kappa} g_{\nu\lambda}) + f_{hik} f_{hlj} (g_{\mu\kappa} g_{\lambda\nu} - g_{\mu\nu} g_{\lambda\kappa}) + f_{hil} f_{hjk} (g_{\mu\nu} g_{\lambda\kappa} - g_{\mu\lambda} g_{\nu\kappa}) \Big]$$

$$= -\frac{1}{2} Z_1^{4G} g_3^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \times \Big[\delta^{kl} \cancelto{0}{f_{hij} f_{hkl} (g_{\mu\lambda} g_\nu{}^\lambda - g_\mu{}^\lambda g_{\nu\lambda})}$$

$$+ f_{hik} f_{hkj} (g_{\mu\nu} - g_{\mu\nu} \; g_\lambda{}^\lambda) + f_{hik} f_{hjk} (g_{\mu\nu} g^\lambda{}_\lambda - g_{\mu\nu}) \Big]$$

d-dimensions

$$= -\frac{1}{2} g_3^2 \int \frac{d^dk}{(2\pi)^d} \frac{(1-d) g_{\mu\nu}}{k^2} \Big[f_{hik} f_{hjk} (-1) - f_{hik} f_{hjk} \Big]$$

$$= +g_3^2 (1-d) g_{\mu\nu} \big[f_{lik} f_{ljk} \big] \int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2}$$

Recall $f_{lik} f_{ljk} = + \mathrm{Tr}[T_{(8)}^i T_{(8)}^j]$

$$= C_2(8)\,\delta_{ij} = 3\delta_{ij}$$

So

$$\boxed{\Gamma_{2\,ij}^{\mu\nu} = g_3^2 (1-d)\, C_2(8)\, g^{\mu\nu} \delta_{ij} \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2}}$$

So we see No p^2 dependence — it will not contribute to β — but let's keep it anyway to show the massless corrections to gluon propagator.

Multiply by $\frac{(p+k)^2}{(p+k)^2} \Rightarrow$

$$\Gamma_{2\,ij}^{\mu\nu} = g_3^2 (1-d)\, C_2(8)\, g^{\mu\nu} \delta_{ij} \int \frac{d^d k}{(2\pi)^d} \frac{(p+k)^2}{k^2 (p+k)^2}$$

$$= g_3^2 (1-d)\, C_2(8)\, g^{\mu\nu} \delta_{ij} \int_0^1 dx \int \frac{d^d l}{(2\pi)^d} \times$$

$$\times \frac{[l^2 + (1-x)^2 p^2 + 2(1-x)\, p\cdot l]}{[l^2 + p^2 (1-x) x]^2}$$ (see p. -64-)

(linear term integrates to zero)

So Wick rotate $\Rightarrow$

$$\Gamma_{2\,ij}^{\mu\nu} = i g_3^2 (1-d)\, C_2(8)\, g^{\mu\nu} \delta_{ij} \int_0^1 dx \int \frac{d^d l_E}{(2\pi)^d} \times \frac{\times \{-l_E^2 + (1-x)^2 p^2\}}{[l_E^2 - p^2(1-x)x]^2}$$

(see p. -66-)

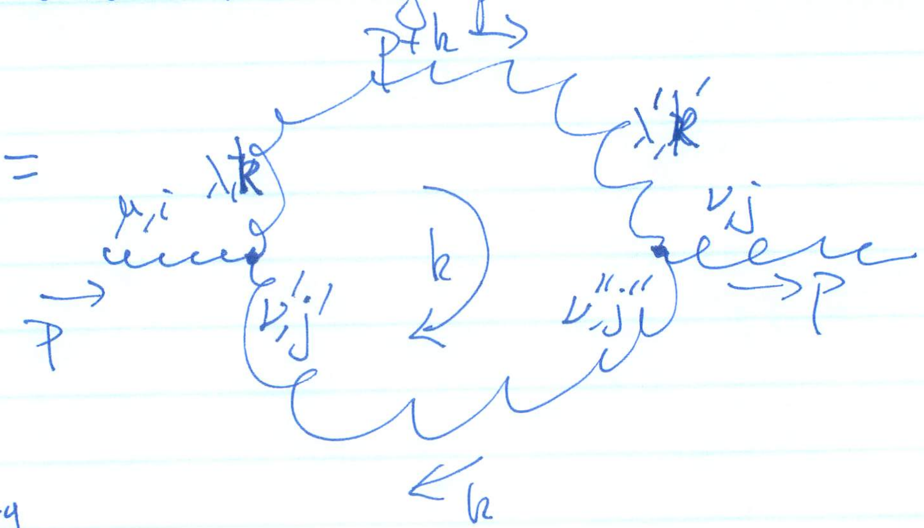
$$= i g_3^2 (1-d)\, C_2(8)\, g^{\mu\nu} \delta_{ij} \int_0^1 dx \left(\frac{1}{(4\pi)^{d/2}}\right) \cdot \left(-\frac{d}{2} \frac{\Gamma(1-\frac{d}{2})}{\Gamma(2)} \left(\frac{1}{M}\right)^{1-\frac{d}{2}} + (1-x)^2 p^2 \frac{\Gamma(2-\frac{d}{2})}{\Gamma(2)} \left(\frac{1}{M}\right)^{2-\frac{d}{2}}\right)$$

$$\boxed{M = -p^2(1-x)x}$$

$$\boxed{\Gamma_{2\,ij}^{\mu\nu} = \frac{i g_3^2}{(4\pi)^{d/2}} (1-d)\, C_2(8)\, g^{\mu\nu} \delta_{ij} \int_0^1 dx \left(\frac{1}{M}\right)^{2-\frac{d}{2}} \cdot \left[-\frac{d}{2}\Gamma(1-\tfrac{d}{2})\left(-p^2(1-x)x\right) + \Gamma(2-\tfrac{d}{2})(1-x)^2 p^2\right]}$$

Next consider the Γ_3 graph:

$\Gamma_{3\,ij}^{\mu\nu} =$



Symmetry Number

$$= \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\, Z_1^{3G} g_3 f_{ij'k}\left[(p-k)^\lambda g^{\mu\nu'} + (k+p+k)^\mu g^{\nu'\lambda} + (-p-k-p)^{\nu'} g^{\mu\lambda}\right]\cdot$$

$$\cdot\left(\frac{-i\delta^{kk'}g_{\lambda\lambda'}}{(p+k)^2}\right)\left(\frac{-i\delta^{j'j''}g_{\nu'\nu''}}{k^2}\right)\cdot$$

$$Z_1^{3G} g_3 f_{jj''k'}\left[(-p+k)^{\lambda'} g^{\nu\nu''} + (-k-p-k)^{\nu} g^{\nu''\lambda'} + (p+k+p)^{\nu''} g^{\nu\lambda'}\right]$$

$$= -\frac{1}{2} g_3^2 f_{ij'k} f_{jj'k} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(p+k)^2} \times$$

$$\times\left[(p-k)^\lambda g^{\mu\nu'} + (p+2k)^\mu g^{\nu'\lambda} - (2p+k)^{\nu'} g^{\mu\lambda}\right]\cdot$$

$$\cdot\left[(k-p)_\lambda g^{\nu}{}_{\nu'} - (p+2k)^\nu g_{\lambda\nu'} + (2p+k)_{\nu'} g^{\nu}{}_{\lambda}\right]$$

Recall $f_{j'ik} f_{j'jk} = C_2(8)\,\delta_{ij}$

& $\dfrac{1}{k^2(p+k)^2} = \int_0^1 dx \dfrac{1}{[\ell^2 + p^2(1-x)x]^2}$ with $\ell = k + xp$ (p.-64-)

So

$$\Gamma_{3\,ij}^{\mu\nu} = -\tfrac{1}{2} g_3^2 C_2(8)\,\delta_{ij} \int \frac{d^4\ell}{(2\pi)^4} \int_0^1 \frac{dx}{[\ell^2 + p^2(1-x)x]^2} \cdot$$

$$N^{\mu\nu} \equiv \left\{ \begin{aligned} &\cdot\big[(k-p)^2(-g^{\mu\nu}) - (p-k)^\mu (p+2k)^\nu + (p-k)^\nu (2p+k)^\mu \\ &- (p-k)^\nu (p+2k)^\mu - (p+2k)^\mu (p+2k)^\nu g^\lambda{}_\lambda \\ &+ (p+2k)^\mu (2p+k)^\nu + (p-k)^\mu (2p+k)^\nu \\ &+ (2p+k)^\mu (p+2k)^\nu - (2p+k)^2 g^{\mu\nu}\big] \end{aligned} \right.$$

Simplify numerator $N^{\mu\nu}$ $\quad k-p = \ell - (1+x)p \,;\; p+2k = 2\ell + (1-2x)p$

$2p+k = \ell + (2-x)p$

$$\begin{aligned} N^{\mu\nu} = \Big[&\tfrac{1}{2}(\ell - (1+x)p)^2(-g^{\mu\nu}) - \frac{g^{\mu\nu}}{2}(\ell + (2-x)p)^2 \\ &+ (\ell - (1+x)p)^\mu (2\ell + (1-2x)p)^\nu - (\ell - (1+x)p)^\mu (\ell + (2-x)p)^\nu \\ &+ (2\ell + (1-2x)p)^\mu (\ell + (2-x)p)^\nu - \tfrac{1}{2} g^\lambda{}_\lambda (2\ell + (1-2x)p)^\mu (2\ell + (1-2x)p)^\nu \\ &+ (\mu \leftrightarrow \nu) \Big] \end{aligned}$$

Now when we integrate over l odd in l terms vanish & we have effectively

$$N^{\mu\nu} = \Big[-\tfrac{1}{2} g^{\mu\nu} \left(l^2 + (1+x)^2 p^2 \right) - \tfrac{1}{2} g^{\mu\nu} \left(l^2 + (2-x)^2 p^2 \right)$$
$$+ 2 l^\mu l^\nu - (1+x)(1-2x) p^\mu p^\nu - l^\mu l^\nu + (1+x)(2-x) p^\mu p^\nu$$
$$+ 2 l^\mu l^\nu + (1-2x)(2-x) p^\mu p^\nu - \tfrac{1}{2} g^\lambda_\lambda \big(4 l^\mu l^\nu + (1-2x)^2 p^\mu p^\nu \big)$$
$$+ (\mu \leftrightarrow \nu) \Big]$$

Also in d-dimensions $l^\mu l^\nu \to \frac{1}{d} g^{\mu\nu} l^2$, So

$$N^{\mu\nu} = -2 g^{\mu\nu} l^2 - \left[(1+x)^2 + (2-x)^2 \right] g^{\mu\nu} p^2$$
$$+ \frac{6}{d} g^{\mu\nu} l^2 + 2 p^\mu p^\nu \left[(1+x)(2-x) - (1+x)(1-2x) + (1-2x)(2-x) \right.$$
$$- g^\lambda_\lambda \frac{4}{d} g^{\mu\nu} l^2 - g^\lambda_\lambda (1-2x)^2 p^\mu p^\nu$$

$$\boxed{\begin{aligned} N^{\mu\nu} = & -2 g^{\mu\nu} \left[1 - \frac{3}{d} + 2 \right] l^2 \\ & - \left[(1+x)^2 + (2-x)^2 \right] g^{\mu\nu} p^2 \\ & + p^\mu p^\nu \left[(1+x)^2 + (1-2x)(2-x) - \tfrac{1}{2} d (1-2x)^2 \right] 2 \end{aligned}}$$

So finally

$$\boxed{N^{\mu\nu} = -6g^{\mu\nu}\left(1-\frac{1}{d}\right)l^2 - \left[(1+x)^2+(2-x)^2\right]g^{\mu\nu}p^2 + p^\mu p^\nu\left[(2-d)(1-2x)^2+2(1+x)(2-x)\right]}$$

So we have

$$\Gamma_{3\,ij}^{\mu\nu} = -\frac{1}{2}g_3^2 C_2(8)\delta_{ij}\int\frac{d^dl}{(2\pi)^d}\int_0^1 dx\frac{1}{\left[l^2+p^2(1-x)x\right]^2}\cdot$$

$$\cdot\left[-6g^{\mu\nu}\left(1-\frac{1}{d}\right)l^2 - \left[(1+x)^2+(2-x)^2\right]g^{\mu\nu}p^2 + p^\mu p^\nu\left[(2-d)(1-2x)^2+2(1+x)(2-x)\right]\right]$$

Wick Rotate

$$\Gamma_{3\,ij}^{\mu\nu} = -\frac{i}{2}g_3^2 C_2(8)\delta_{ij}\int_0^1 dx\int\frac{d^dl_E}{(2\pi)^d}\frac{1}{\left[l_E^2+M\right]^2}\cdot$$

$$\cdot\left[6g^{\mu\nu}\left(1-\frac{1}{d}\right)l_E^2 - \left[(1+x)^2+(2-x)^2\right]g^{\mu\nu}p^2 + p^\mu p^\nu\left[(2-d)(1-2x)^2+2(1+x)(2-x)\right]\right]$$

---

as usual $M = -p^2(1-x)x$

This yields:

$$\Gamma_{3ij}^{\mu\nu} = -\frac{i}{2}\frac{g_3^2}{(4\pi)^{d/2}} C_2(8)\delta_{ij}\int_0^1 dx \Bigg[\frac{d}{2}\frac{\Gamma(2-\frac{d}{2}-1)}{\Gamma(2)}\left(\frac{1}{M}\right)^{1-\frac{d}{2}} \cdot 6 g^{\mu\nu}\left(1-\frac{1}{d}\right) + \left[\frac{\Gamma(2-\frac{d}{2})}{\Gamma(2)}\left(\frac{1}{M}\right)^{2-\frac{d}{2}}\right]\Big[-g^{\mu\nu}p^2\left[(1+x)^2+(2-x)^2\right] + p^\mu p^\nu\left\{(2-d)(1-2x)^2 + 2(1+x)(2-x)\right\}\Big]\Bigg]$$

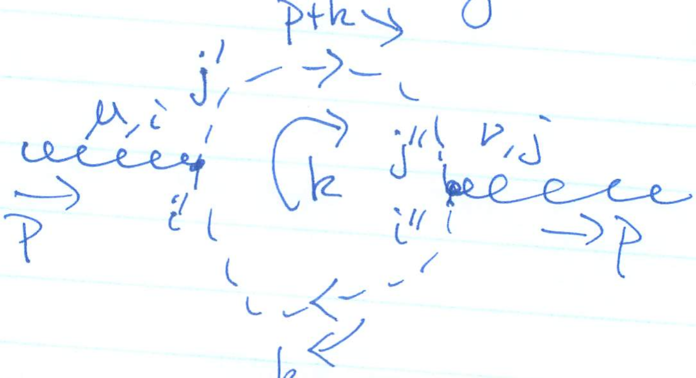
$$\boxed{\Gamma_{3ij}^{\mu\nu} = -\frac{i}{2}\frac{g_3^2}{(4\pi)^{d/2}} C_2(8)\delta_{ij}\int_0^1 dx \Big\{ 3 g^{\mu\nu}(d-1)\Gamma(1-\tfrac{d}{2})\left(\tfrac{1}{M}\right)^{1-d/2} + \Gamma(2-\tfrac{d}{2})\left(\tfrac{1}{M}\right)^{2-d/2}\Big[-g^{\mu\nu}p^2\left[(1+x)^2+(2-x)^2\right] + p^\mu p^\nu\left\{(2-d)(1-2x)^2 + 2(1+x)(2-x)\right\}\Big]\Big\}}$$

factor out $\left(\frac{1}{M}\right)^{2-d/2} \Rightarrow$

$$\boxed{\begin{aligned}\Gamma_{3\,ij}^{\mu\nu} = +i\frac{g_3^2}{(4\pi)^{d/2}} C_2(8)\,\delta_{ij}\int_0^1 dx\left(\frac{1}{M}\right)^{2-d/2}\cdot \\ \cdot\Big\{\Gamma\left(1-\tfrac{d}{2}\right) g^{\mu\nu} p^2\left[\tfrac{3}{2}(d-1)\,x(1-x)\right] \\ +\Gamma\left(2-\tfrac{d}{2}\right) g^{\mu\nu} p^2\left[\tfrac{1}{2}(2-x)^2+\tfrac{1}{2}(1+x)^2\right] \\ -\Gamma\left(2-\tfrac{d}{2}\right) p^\mu p^\nu\left[\left(1-\tfrac{d}{2}\right)(1-2x)^2+(1+x)(2-x)\right]\Big\}\end{aligned}}$$

Finally consider the ϕ-π ghost loop:

$\Gamma_{4\,ij}^{\mu\nu} =$



closed loop of anti-commuting ϕ-π ghosts

$$= (-1)\int\frac{d^4k}{(2\pi)^4}\,\frac{Z_1^c g_3}{\alpha}(p+k)^\mu f_{j'ii'}\left(\frac{-i\delta^{j'j''}\alpha}{(p+k)^2}\right)\cdot$$

$$\cdot\left(\frac{-i\delta^{i'i''}\alpha}{k^2}\right)\frac{Z_1^c g_3}{\alpha}k^\nu f_{i''jj''}$$

$$\Gamma^{\mu\nu}_{4ij} = g_3^2 (f_{j'ii'} f_{i'jj'}) \int \frac{d^4k}{(2\pi)^4} \frac{(p+k)^\mu k^\nu}{k^2 (p+k)^2}$$

$$= g_3^2 (-C_2(8) \delta_{ij}) \int_0^1 dx \int \frac{(l+(1-x)p)^\mu (l-xp)^\nu}{[l^2 - M]^2} \frac{d^4 l}{(2\pi)^4}$$

odd in l^μ vanish

$$= -g_3^2 C_2(8) \delta_{ij} \int_0^1 dx \int \frac{d^4 l}{(2\pi)^4} \frac{l^\mu l^\nu - x(1-x) p^\mu p^\nu}{[l^2 - M]^2}$$

Wick Rotate

$$= -i g_3^2 C_2(8) \delta_{ij} \int_0^1 dx \int \frac{d^d l_E}{(2\pi)^d} \frac{\left(-\frac{g^{\mu\nu}}{d} l_E^2\right) - x(1-x) p^\mu p^\nu}{[l_E^2 + M]^2}$$

$$= i \frac{g_3^2}{(4\pi)^{d/2}} C_2(8) \delta_{ij} \int_0^1 dx \left(\frac{1}{M}\right)^{2-d/2} \cdot \left[\Gamma(1-\tfrac{d}{2}) \frac{g^{\mu\nu}}{2} M + \Gamma(2-\tfrac{d}{2}) p^\mu p^\nu x(1-x)\right]$$

$$\boxed{\Gamma^{\mu\nu}_{4ij} = i \frac{g_3^2}{(4\pi)^{d/2}} C_2(8) \delta_{ij} \int_0^1 dx \left(\frac{1}{M}\right)^{2-d/2} \cdot \left[-\Gamma(1-\tfrac{d}{2}) g^{\mu\nu} p^2 \left[\tfrac{1}{2} x(1-x)\right] + \Gamma(2-\tfrac{d}{2}) p^\mu p^\nu \left[x(1-x)\right]\right]}$$

Now we can add the 3 diagrams:

$$\Gamma_{2\,ij}^{\mu\nu} + \Gamma_{3\,ij}^{\mu\nu} + \Gamma_{4\,ij}^{\mu\nu}$$

$$= i\,\frac{g_3^2}{(4\pi)^{d/2}}\, C_2(8)\,\delta_{ij} \int_0^1 dx \left(\frac{1}{M}\right)^{2-d/2}$$

$$\cdot \Big\{ \Gamma\left(1-\tfrac{d}{2}\right)\left[g^{\mu\nu} p^2\right] \underbrace{\left[+ \tfrac{d}{2}(1-d)\, x(1-x) + \tfrac{3}{2}(d-1)\,x(1-x) - \tfrac{1}{2}\,x(1-x) \right]}_{=\left[(d-2)\left(1-\frac{d}{2}\right)\right]x(1-x)}$$

$$+ \Gamma\left(2-\tfrac{d}{2}\right) g^{\mu\nu} p^2 \left[(1-d)(1-x)^2 + \tfrac{1}{2}(2-x)^2 + \tfrac{1}{2}(1+x)^2 \right]$$

$$- \Gamma\left(2-\tfrac{d}{2}\right) p^\mu p^\nu \underbrace{\left[\left(1-\tfrac{d}{2}\right)(1-2x)^2 + (1+x)(2-x) - x(1-x) \right]}_{=\left(1-\frac{d}{2}\right)(1-2x)^2 + 2} \Big\}$$

Now this becomes using $z\Gamma(z) = \Gamma(z+1)$

$$\Rightarrow \left(1-\tfrac{d}{2}\right)\Gamma\left(1-\tfrac{d}{2}\right) = \Gamma\left(2-\tfrac{d}{2}\right)$$

$$\Gamma_{2\,ij}^{\mu\nu} + \Gamma_{3\,ij}^{\mu\nu} + \Gamma_{4\,ij}^{\mu\nu}$$

$$= i \frac{g_3^2}{(4\pi)^{d/2}} C_2(8)\,\delta_{ij} \int_0^1 dx \left(\frac{1}{M}\right)^{2-d/2} \cdot \Gamma\left(2-\frac{d}{2}\right)$$

$$\Big(\cdot \left\{ g^{\mu\nu} p^2 - p^\mu p^\nu \right\} \left[\left(1-\frac{d}{2}\right)(1-2x)^2 + 2 \right]$$

$$+ g^{\mu\nu} p^2 \underbrace{\Big[-\left(1-\frac{d}{2}\right)(1-2x)^2 - 2 + (1-d)(1-x)^2 + \frac{1}{2}(2-x)^2 + \frac{1}{2}(1+x)^2 + (d-2)x(1-x) \Big]}_{= \cancel{d \cdot \left(x-\frac{1}{2}\right) + \left(\frac{1}{2}-x\right)}} \Big)$$

Now the last set of terms must vanish $= (d-1)\left(x-\frac{1}{2}\right)$

Now the x-integrand M is symmetric wrt $x \leftrightarrow (1-x)$

So we have $\int_0^1 dx\, f(x(1-x)) \underbrace{\left[\frac{1}{2}-x\right]}_{= \frac{1}{2}(1-x) - \frac{1}{2}x}$

So in first term let $y = 1-x$ (so $x = 1-y$)

$$= \int_0^1 dx\, f(x(1-x))\left(-\frac{1}{2}x\right) + \int_0^1 dy\, f(y(1-y))\,\frac{1}{2}y$$

$$= 0$$

So we have the transversality of the gluon self-energy corrections. ϕ-Π terms essential

So we can combine all the results

p.-67-

$$\Gamma_{1\,ij}^{\mu\nu} = -2i\frac{g_3^2}{(4\pi)^{d/2}}\delta_{ij}N_F\int_0^1 dx\left(\frac{1}{M}\right)^{2-d/2}\Gamma\left(2-\frac{d}{2}\right)\cdot$$

$$\cdot\left[2\left(g^{\mu\nu}p^2-p^\mu p^\nu\right)\left[x(1-x)\right]\right]$$

$$\Gamma_{2\,ij}^{\mu\nu}+\Gamma_{3\,ij}^{\mu\nu}+\Gamma_{4\,ij}^{\mu\nu} = i\frac{g_3^2}{(4\pi)^{d/2}}C_2(8)\delta_{ij}\int_0^1 dx\left(\frac{1}{M}\right)^{2-d/2}\Gamma\left(2-\frac{d}{2}\right)\cdot$$

$$\cdot\left(g^{\mu\nu}p^2-p^\mu p^\nu\right)\left[\left(1-\frac{d}{2}\right)(1-2x)^2+2\right]$$

So $\Rightarrow$

p.-62-

$$\boxed{\begin{aligned}\hat{\Pi}_T(p^2) &= i\frac{g_3^2}{(4\pi)^{d/2}}\int_0^1 dx\left(\frac{1}{M}\right)^{2-d/2}\Gamma\left(2-\frac{d}{2}\right)\cdot\\ &\cdot\left[C_2(8)\left[\left(1-\frac{d}{2}\right)(1-2x)^2+2\right]-4N_F\left[x(1-x)\right]\right]p^2\end{aligned}}$$

The normalization condition p-62- is

$$Z_3 = 1 - i\frac{\partial}{\partial p^2}\hat{\Pi}_T(p^2)\Big|_{p^2=-\mu^2}$$

$$= 1 + \frac{\partial}{\partial p^2}\Big\{ p^2 \frac{g_3^2}{(4\pi)^{d/2}}\int_0^1 dx\, \Gamma(2-\tfrac{d}{2}) e^{-(2-\frac{d}{2})\ln M} \cdot$$

$$\cdot \Big[C_2(8)\big[(1-\tfrac{d}{2})(1-2x)^2+2\big] - 4N_F\big[x(1-x)\big]\Big]\Big\}\Big|_{p^2=-\mu^2}$$

We are only interested in the divergent part of this
$\epsilon = 4-d$; $M = -p^2 x(1-x)$

$$Z_3 \underset{\epsilon\to 0}{\overset{d\to 4}{=}} \frac{g_3^2}{(4\pi)^2}\int_0^1 dx\, \frac{2}{\epsilon}\Big[1-\frac{\epsilon}{2}\ln M\Big]\cdot\Big|_{p^2=-\mu^2}\cdot$$

$$\cdot\Big[C_2(8)\big[(-1)(1-2x)^2+2\big] - 4N_F\big[x(1-x)\big]\Big]$$

$$= \frac{g_3^2}{(4\pi)^2}\int_0^1 dx\,\Big[\frac{2}{\epsilon} - \ln\mu^2 - \ln x(1-x)\Big]\cdot + \cdots$$

$$\cdot\Big\{ (2-(1-2x)^2)C_2(8) - 4N_F\, x(1-x)\Big\}$$

So

$$Z_3 = 1 + \frac{g_3^2}{(4\pi)^2}(-\ln\mu^2)\int_0^1 dx\left\{(2-(1-2x)^2)C_2(8) - 4N_F\, x(1-x)\right\} + \cdots$$

$$\boxed{Z_3 = 1 + \frac{g_3^2}{(4\pi)^2}(-\ln\mu^2)\left[\frac{5}{3}C_2(8) - \frac{2}{3}N_F\right] + \cdots}$$

Recall the expression for β : p. -61-

$$\beta = g_3\left[\mu\frac{\partial}{\partial\mu}\,\frac{1}{2}Z_3 - \frac{g_3^2}{(4\pi)^2}\left[2(C_2(3)+C_2(8)) - 2C_2(3)\right]\right]$$

$$= g_3\left[-\frac{g_3^2}{(4\pi)^2}\right]\left[\frac{5}{3}C_2(8) - \frac{2}{3}N_F + 2(C_2(3)+C_2(8)) - 2C_2(3)\right]$$

$$\boxed{\beta = -\frac{g_3^3}{(4\pi)^2}\left[\frac{11}{3}C_2(8) - \frac{2}{3}N_F\right]}$$

$$= -\frac{g_3^3}{(4\pi)^2}\left[11 - \frac{2}{3}N_F\right]$$

$$= -\frac{g_3^3}{(4\pi)^2}\left[11 - \frac{2}{3}\cdot 6\right] < 0$$