IB.) So we need only to calculate the counter-terms.

(This is the case of $\phi = Z^{1/2}\phi_0$ Bare theory)

Recall $g_3 = Z_1^{F\,-1} \cdot Z_2 Z_3^{1/2} g_3^0$

$$\boxed{\beta = \mu \frac{\partial}{\partial\mu} g_3 = g_3\left[-\mu \frac{\partial \ln Z_1^F}{\partial \mu} + \mu \frac{\partial \ln Z_2}{\partial\mu} + \frac{1}{2}\mu \frac{\partial \ln Z_3}{\partial \mu}\right] = g_3\left[2\gamma_q + \gamma_G - \mu \frac{\partial}{\partial\mu} \ln Z_1^F\right]}$$ ✓

where we relate to bare theory

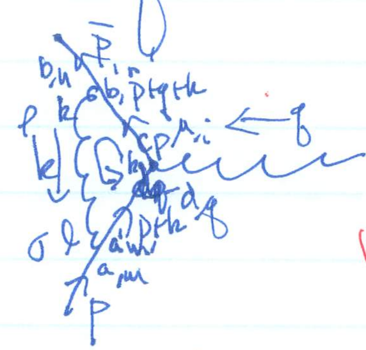
above is the $\mu \to \mu'$ theory.

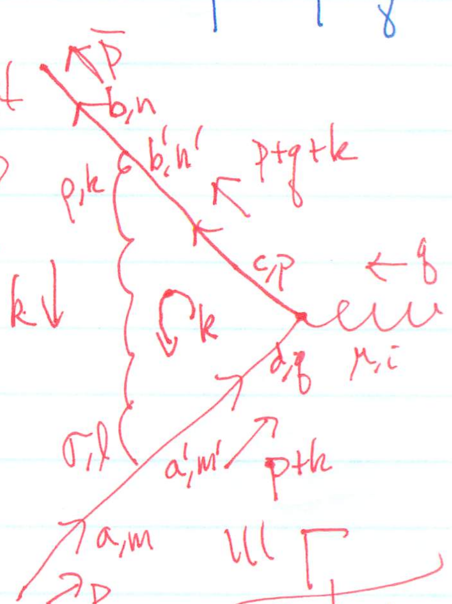
So now to calculate the counter-terms:

Now the Z's are divergent – so we only need these parts — we might as well put $m_{(m)} = 0$ $(\mu >> m)$ in the quark masses and choose a convenient gauge — say Feynman gauge $\alpha = 1$.

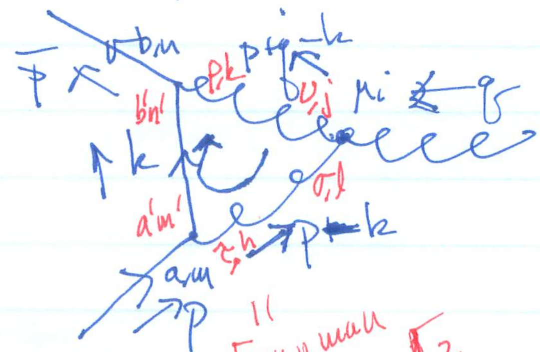
(all can be shown rigorously)

Consider the vertex first: Σ-Minkow. $\bar{p}^\mu = p^\mu + q^\mu$

$\hat{\Gamma}^{ab\,\mu}_{mn\,i}(p, \bar{p}, q) =$  blow it up bigger

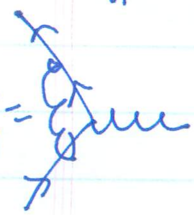


$\parallel\ \Gamma$ Feynman diagram

$+$ 

$\parallel$ Feynman diagram Γ_2

I.B.) $$\hat{\Gamma}^{ab\,\mu}_{1\;mn\;i} = \int \frac{d^4k}{(2\pi)^4}\, iZ_1^F g_3 T^k_{bb'}\, \delta_{nn'}\, \gamma^\rho \frac{i}{\not p+\not q+\not k}\, \delta^{b'c}\delta_{n'p} \cdot iZ_1^F g_3 T^i_{cd}\, \delta_{pq}\, \gamma^\mu \frac{i}{\not p+\not k}\, \delta^{a'd}\delta_{qm'} \cdot iZ_1^F g_3 T^l_{a'a}\, \delta^{m'm}\, \gamma^\sigma \left(\frac{-i\delta^{kl}}{k^2} g_{\rho\sigma}\right)$$

$\Gamma_1 =$ 

$$= Z_1^{F\,3} g_3^3 \left(T^k T^i T^k\right)_{ba} \delta_{mn} \times \int \frac{d^4k}{(2\pi)^4}\, \gamma^\rho \frac{(\not p+\not q+\not k)}{(p+q+k)^2}\, \gamma^\mu \frac{(\not p+\not k)}{(p+k)^2}\, \gamma_\rho \frac{1}{k^2}$$

Now we are interested in the divergent part which comes from the $\mu^2 = p^2$ $q^2 = 0$ etc. term with $k^\mu k^\nu$ in the numerator

$$\boxed{\hat{\Gamma}_{1\,div}\Big|_{NP} = Z_1^{F\,3} g_3^3 \left(T^k T^i T^k\right)_{ba} \delta_{mn} \times \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\rho \not k \gamma^\mu \not k \gamma_\rho}{k^2\,(k^2-\mu^2)^2}}$$

(Rough argument can be made more rigorous)

i.e. $$\frac{1}{(p+k)^2} = \frac{1}{(p^2+k^2+2p\cdot k)} = \frac{1}{p^2+k^2}\,\frac{1}{\left[1+\frac{2p\cdot k}{p^2+k^2}\right]}$$

$$\underset{\substack{k\to\infty \\ p^2=-\mu^2}}{=} \frac{1}{k^2-\mu^2}\left[1-\frac{2p\cdot k}{k^2}+\dots\right] \simeq \frac{1}{k^2-\mu^2}$$

↑ Euclidean

$$\gamma^\rho\gamma^\mu\gamma_\rho = -\gamma^\mu\gamma^\rho\gamma_\rho + 2\gamma^\mu = (2-d)\gamma^\mu$$
$$\gamma^\rho\gamma_\rho = g^\rho{}_\rho = d$$

I.B.) Now we evaluate by dimensional regulation:

$$I = \int \frac{d^4k}{(2\pi)^4}\frac{\gamma^\rho \not{k}\gamma^\mu\not{k}\gamma_\rho}{k^2(k^2-\mu^2)^2} = \lim_{d\to4}\int\frac{d^dk}{(2\pi)^d}\frac{\gamma^\rho\not{k}\gamma^\mu\not{k}\gamma_\rho}{k^2(k^2-\mu^2)^2}$$

$$\gamma^\rho\not{k}\gamma^\mu\not{k}\gamma_\rho = -\gamma^\rho\gamma^\mu\not{k}\not{k}\gamma_\rho + 2k^\mu\gamma^\rho\not{k}\gamma_\rho$$
$$= -k^2\gamma^\rho\gamma^\mu\gamma_\rho + 2k^\mu\gamma^\rho\not{k}\gamma_\rho$$
$$= (d-2)k^2\gamma^\mu + 2(2-d)\not{k}k^\mu$$

Now $\int d^dk\, k^\mu k^\nu f(k^2) = A g^{\mu\nu} \Rightarrow A = \frac{1}{d}\int d^dk\, k^2 f(k^2)$

$\Rightarrow$

$$I = \lim_{d\to4}\int\frac{d^dk}{(2\pi)^d}\frac{(d-2)k^2\gamma^\mu - \frac{(d-2)2}{d}k^2\gamma^\mu}{k^2(k^2-\mu^2)^2}$$

$$= \int\frac{d^dk}{(2\pi)^d}(d-2)\gamma^\mu\left[1-\frac{2}{d}\right]\frac{1}{(k^2-\mu^2)^2} = \int\frac{d^dk_E}{(2\pi)^d}\frac{(d-2)\gamma^\mu\left[1-\frac{2}{d}\right](i)}{(k_E^2+\mu^2)^2}$$

$$I = i\int\frac{d^dk_E}{(2\pi)^d}\frac{(d-2)^2}{d}\frac{\gamma^\mu}{(k_E^2+\mu^2)^2} = \frac{\Omega_d\, i}{(2\pi)^d}\frac{(d-2)^2}{d}\gamma^\mu\int_0^\infty\frac{k^{d-1}dk}{(k^2+\mu^2)^2}$$

$\boxed{\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}}$ and $\int_0^\infty \frac{k^{d-1}dk}{(k^2+\mu^2)^2} = \frac{1}{2}\int_0^\infty\frac{x^{\frac{d-2}{2}}dx}{(x+\mu^2)^2}$

let $y = \frac{+\mu^2}{(x+\mu^2)}$ $\quad x=\infty, y=0$ $\quad x=0, y=1$ $\quad = \frac{+1}{2}\int_0^1 dy\frac{1}{\mu^2}x^{\frac{d}{2}-1}$

$dy = \frac{-\mu^2}{(x+\mu^2)^2}dx$; $(x+\mu^2) = \frac{\mu^2}{y}$; $x = \frac{\mu^2}{y}-\mu^2$

$x = \mu^2\left(\frac{1-y}{y}\right)$

$$I = \frac{\Omega_d\, i}{(2\pi)^d}\frac{(d-2)^2}{2d\mu^2}\gamma^\mu\int_0^1 dy\,\mu^{d-2}\left(\frac{1-y}{y}\right)^{\frac{d-2}{2}}$$

$$= \frac{\Omega_d}{(2\pi)^d}i\frac{(d-2)^2}{2d\mu^2}\gamma^\mu\mu^{d-2}\underbrace{\int_0^1 dy\, y^{1-d/2}(1-y)^{\frac{d}{2}-1}}_{=\frac{\Gamma(2-\frac{d}{2})\Gamma(\frac{d}{2})}{\Gamma(2)}}$$

I.B.)

$$\underline{I} = \frac{2\pi^{d/2}}{\cancel{\Gamma(d/2)}} \frac{i}{(2\pi)^d} \frac{(d-2)^2}{2d\mu^2} \gamma^\mu \mu^{d-2} \frac{\Gamma(2-\frac{d}{2})\cancel{\Gamma(\frac{d}{2})}}{\Gamma(2)}$$

$$\underline{I} = \frac{i\,\gamma^\mu}{2^d\,\pi^{d/2}} \frac{(d-2)^2}{d} \mu^{d-4} \frac{\Gamma(2-d/2)}{\Gamma(2)}$$

Now let $\boxed{\epsilon = 4-d}$

So $\Gamma(2-\frac{d}{2}) = \Gamma(\frac{\epsilon}{2}) = \frac{2}{\epsilon} - \gamma + O(\epsilon) \approx \frac{2}{\epsilon}$

$\gamma \approx .5772$
Euler-Mascheroni constant.

Now we have

$$\boxed{\underline{I} = \frac{i\,\gamma^\mu}{2^d\,\pi^{d/2}} \frac{(d-2)^2}{d} \mu^{-\epsilon} \frac{\Gamma(\frac{\epsilon}{2})}{\Gamma(2)}}$$, $\Gamma(2) = 1$, $\Gamma(n+1) = n!$

Now

$$\mu^{-\epsilon} = e^{-\epsilon \ln\mu} \approx 1 - \epsilon\ln\mu + O(\epsilon^2)$$

So

$$\underline{I} \overset{d\to 4}{=} \frac{i\gamma^\mu}{2^4\pi^2} \frac{2^2}{4} \left(\frac{2}{\epsilon}\right)(1-\epsilon\ln\mu) + \cdots$$

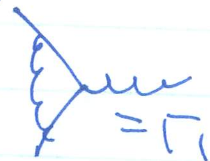
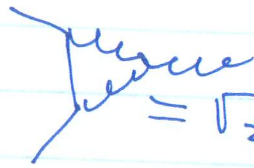
$$\underline{I} \overset{d\to 4}{=} \frac{i\gamma^\mu}{16\pi^2} 2 \cdot \left[\frac{1}{\epsilon} - \ln\mu + \cdots\right]$$

So

$$\hat{\Gamma}_{1\,div}\Big|_{NP} = g_3^3 (T^k T^i T^k)_{ba} \delta_{mn} \gamma^\mu \frac{i}{8\pi^2}\left[\frac{1}{\epsilon} - \ln\mu\right]$$

I.B.) Recall $\Gamma^{ab\,(1,1,1,0,0)\,\mu}_{mn\,(p,\bar{p},q)\,i}\Big|_{NP} \equiv i g_3 T^i_{ba}\delta_{mn}\gamma^\mu$

$$= i Z_1^F g_3 T^i_{ba}\delta_{mn}\gamma^\mu$$

$+$  $= \Gamma_1$ $+$  $= \Gamma_2$

$\Rightarrow$

$$\boxed{Z_1^F = 1 - \frac{1}{g_3}\hat{\Gamma}_{1div}\Big|_{NP} - \frac{1}{g_3}\hat{\Gamma}_{2div}\Big|_{NP}}$$

Some need to finish $\hat{\Gamma}_{1div}\Big|_{NP}$

$$\hat{\Gamma}_{1div}\Big|_{NP} = i g_3^3 \left(T^k T^i T^k\right)_{ba}\delta_{mn}\gamma^\mu \frac{1}{8\pi^2}\left[\frac{1}{\epsilon} - \ln\mu\right]$$

Now

$$T^k T^i T^k = T^k T^k T^i + T^k [T^i, T^k]$$

p.-22'- $= \left[C_2(3) - \frac{1}{2}C_2(8)\right]T^i$

$$\left(= \left[C_2(r) - \frac{1}{2}C_2(\text{adjoint})\right]T^i_{(r)}\right)$$

So

$$\boxed{\hat{\Gamma}_{1div}\Big|_{NP} = i g_3^3\left[C_2(3) - \frac{1}{2}C_2(8)\right]T^i_{ba}\delta_{mn}\gamma^\mu\left[\frac{1}{8\pi^2}\left(\frac{1}{\epsilon} - \ln\mu\right)\right]}$$

$\Rightarrow$

$$\boxed{= i g_3 T^i_{ba}\delta_{mn}\gamma^\mu\, \hat{\hat{\Gamma}}_{1div}\Big|_{NP}}$$

I.B.1 So
$$\boxed{\mu\frac{\partial}{\partial\mu}\hat{\Gamma}_{1\,div}\Big|_{NP} = -\frac{g_3^2}{8\pi^2}\left[C_2(3)-\frac{1}{2}C_2(8)\right]}$$

Next we need $\hat{\Gamma}_{2\,div}\Big|_{NP}$

$$\hat{\Gamma}^{ab\mu}_{2\;mni} = \int\frac{d^4k}{(2\pi)^4}\, iZ_1^F g_3 T^k_{bb'}\delta_{nn'}\gamma^\rho \frac{i\delta_{b'a'}\delta_{m'n'}}{\not k}$$
$$\times iZ_1^F g_3 T^h_{a'a}\delta_{mm'}\gamma^\tau\left(\frac{-i\delta^{kj}g^{\rho\nu}}{(p+q-k)^2}\right)\times$$
$$\times\left(\frac{-i\delta^{hl}g^{\tau\sigma}}{(p-k)^2}\right)\left[Z_1^{3G}g_3\right]f_{ijl}\times$$
$$\times\Big[(q+p+q-k)_\sigma g_{\mu\nu}$$
$$+(-p-q+k-p+k)_\mu g_{\sigma\nu}+(p-k-q)_\nu g_{\mu\sigma}\Big]$$

$$= +iZ_1^{F^2}Z_1^{3G}g_3^3\, T^j_{bc}T^k_{ca}\delta_{nm}f_{ijk}\times$$
$$\times\int\frac{d^4k}{(2\pi)^4}\gamma^\nu\frac{\not k}{k^2}\gamma^\rho\frac{1}{(p+q-k)^2}\frac{1}{(p-k)^2}\times$$
$$\times\Big[(p+2q-k)_\rho g_{\mu\nu}+(2k-2p-q)_\mu g_{\nu\rho}$$
$$+(p-q-k)_\nu g_{\mu\rho}\Big].$$

I.B.) So

$$\boxed{\begin{aligned}\hat{\Gamma}^{ab\mu}_{2\,mni} &= +ig_3^3\,\delta_{mn}\,f_{ijk}\left(T^jT^k\right)_{ba}\times\\ &\times\int\frac{d^4k}{(2\pi)^4}\,\frac{\gamma^\nu \not{k}\gamma^\rho}{k^2(p-k)^2(p+q-k)^2}\times\\ &\times\Big[(p+2q-k)_\rho g_{\mu\nu}+(2k-2p-q)_\mu g_{\nu\rho}\\ &\qquad+(p-q-k)_\nu g_{\mu\rho}\Big].\end{aligned}}$$

Again we are interested in the divergent part. So we need the $k^\mu k^\nu$ in numerator

$$\boxed{\begin{aligned}\hat{\Gamma}_{2\,div}\Big|_{NP} &= +ig_3^3\,\delta_{mn}\,f_{ijk}\,\tfrac{1}{2}\left([T^j,T^k]\right)_{ba}\times\\ &\times\int\frac{d^4k}{(2\pi)^4}\,\frac{\gamma^\nu\not{k}\gamma^\rho\left[-k_\rho g_{\mu\nu}+2k_\mu g_{\nu\rho}-k_\nu g_{\mu\rho}\right]}{k^2(k^2-\mu^2)(k^2-\mu^2)}\end{aligned}}$$

Now $\frac{1}{2}f_{ijk}[T^j,T^k]=\frac{i}{2}f_{ijk}f_{jkl}T^l$

$$=\frac{i}{2}\,\underset{8=(1,1)}{C_2(\underset{\shortparallel}{\text{adjoint}})}\,T^i$$

Recall $C_2(8)=3$

$C_2(3)=4/3$

I.B.)

$$\boxed{\hat{\Gamma}_{2\,div}\Big|_{NP} = +\frac{1}{2} g_3^3 C_2(8) T^i_{ba} \delta_{mn} \times \int \frac{d^4k}{(2\pi)^4} \frac{\left[+\gamma^\mu \not{k}\not{k} - 2k^\mu \gamma^\rho \not{k} \gamma_\rho + \not{k}\not{k}\gamma^\mu\right]}{k^2(k^2-\mu^2)^2}}$$

$$\gamma^\rho \not{k} \gamma_\rho = -\not{k}\gamma^\rho\gamma_\rho + 2\not{k} = (2-d)\not{k}$$

$$\boxed{\hat{\Gamma}_{2\,div}\Big|_{NP} = +\frac{1}{2} g_3^3 C_2(8) T^i_{ba} \delta_{mn} \times \int \frac{d^dk}{(2\pi)^d} \frac{\left[2k^2\gamma^\mu - 2(2-d)k^\mu \not{k}\right]}{k^2(k^2-\mu^2)^2}}$$

$$= g_3^3 C_2(8) T^i_{ba} \delta_{mn} \gamma^\mu \times \int \frac{d^dk}{(2\pi)^d} \frac{\left[k^2 - (2-d)\frac{k^2}{d}\right]}{k^2(k^2-\mu^2)^2}$$

$$= g_3^3 C_2(8) T^i_{ba} \delta_{mn} \gamma^\mu \int \frac{d^dk}{(2\pi)^d} \left(\frac{2d-2}{d}\right) \frac{1}{(k^2-\mu^2)^2}$$

$$= \frac{g_3^3 C_2(8) T^i_{ba} \delta_{mn} \gamma^\mu}{(2\pi)^d} \frac{(2d-2)}{d} \Omega_d \, i \int_0^\infty \frac{k^{d-1}dk}{(k^2+\mu^2)^2}$$

$$= +i \frac{g_3^3 C_2(8) T^i_{ba} \delta_{mn} \gamma^\mu \Omega_d (2d-2)}{(2\pi)^d \, d \cdot 2} \int_0^\infty dx \frac{x^{\frac{d-2}{2}}}{(x+\mu^2)^2}$$

I.B.) As before $\int_0^\infty dx \frac{x^{\frac{d-2}{2}}}{(x+\mu^2)^2} = \int_0^1 \frac{1}{\mu^2} dy\, \mu^{d-2} \left(\frac{1-y}{y}\right)^{\frac{d-2}{2}}$

$$= \mu^{+(d-4)} \int_0^1 dy\, y^{+1-\frac{d}{2}} (1-y)^{\frac{d}{2}-1}$$

$$= \mu^{(d-4)} \frac{\Gamma(2-\frac{d}{2})\Gamma(\frac{d}{2})}{\Gamma(2)}$$

$\epsilon = 4-d \qquad = \mu^{-\epsilon} \frac{\Gamma(\epsilon/2)\Gamma(d/2)}{1}$

So

$$\boxed{\hat{\Gamma}_{2div}\Big|_{NP} = i g_3^3 C_2(8) T^i_{ba} \delta_{mn} \gamma^\mu \frac{2\pi^{d/2}}{\cancel{\Gamma(d/2)}} \frac{(2d-2)}{2^d \pi^d 2.d} \mu^{-\epsilon} \Gamma(\tfrac{\epsilon}{2}) \cancel{\Gamma(\tfrac{d}{2})} \overset{d\to 4}{=} i g_3^3 C_2(8) T^i_{ba} \delta_{mn} \gamma^\mu \frac{1}{(8\pi)^2} \mu^{-\epsilon} \Gamma(\epsilon/2) \underbrace{(2d-2)}_{=6}}$$

$$\Rightarrow \hat{\Gamma}_{2div}\Big|_{NP} = i g_3 T^i_{ba} \delta_{mn} \gamma^\mu \hat{\Gamma}_{2div}\Big|_{NP}$$

$$\boxed{\hat{\Gamma}_{2div}\Big|_{NP} = \frac{g_3^2 C_2(8) 6}{(8\pi)^2} \Gamma\left(\frac{\epsilon}{2}\right) \mu^{-\epsilon}}$$

$$= \frac{6 g_3^2 C_2(8)}{(8\pi)^2} \left[\frac{2}{\epsilon}\right]\left[1-\epsilon \ln\mu\right]$$

$$\boxed{\hat{\Gamma}_{2div}\Big|_{NP} = \frac{6 g_3^2 C_2(8)}{2(4\pi)^2} \left[\frac{1}{\epsilon} - \ln\mu\right]}$$

$$\boxed{\mu \frac{\partial}{\partial\mu} \hat{\Gamma}_{2div}\Big|_{NP} = -\frac{g_3^2\, 3}{(4\pi)^2} C_2(8)}$$

I.B.1 So we obtain

$$\boxed{\begin{aligned} Z_1^F &= 1 - \frac{1}{g_3}\hat{\Gamma}_{1\,div}\Big|_{np} - \frac{1}{g_3}\hat{\Gamma}_{2\,div}\Big|_{np} \\ &= 1 - \frac{g_3^2}{(4\pi)^2}\left[2\left(C_2(3) - \tfrac{1}{2}C_2(8)\right) + 3C_2(8)\right]\left(\tfrac{1}{\epsilon} - \ln\mu\right) \end{aligned}}$$

$\Rightarrow$

$$\mu\partial_\mu Z_1^F = \frac{g_3^2}{(4\pi)^2}\left[2\left(C_2(3) - \tfrac{1}{2}C_2(8)\right) + 3C_2(8)\right]$$

$$\boxed{\mu\partial_\mu Z_1^F = \frac{g_3^2}{(4\pi)^2}\left[2C_2(3) + 2C_2(8)\right]}$$

$$\boxed{= \frac{g_3^2}{(4\pi)^2}2\left[C_2(3) + C_2(8)\right]}$$

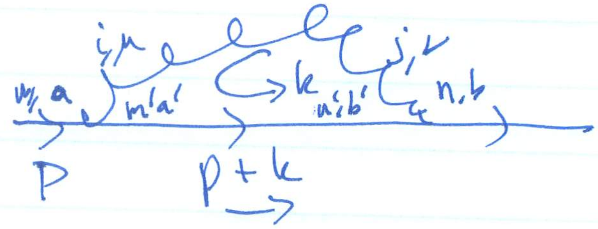
So find so far

$$\boxed{\begin{aligned} \beta &= g_3\mu\frac{\partial}{\partial\mu}\left[Z_2 + \tfrac{1}{2}Z_3 - Z_1^F\right] \\ &= g_3\left[\mu\frac{\partial}{\partial\mu}\left(Z_2 + \tfrac{1}{2}Z_3\right) - \frac{g_3^2}{(4\pi)^2}2\left(C_2(3) + C_2(8)\right)\right] \end{aligned}}$$

I.B.) Onward to Z_2 :  :

$$\Gamma^{ab\,(1,1,0,0,0)}_{mn}(p) = i Z_2 \not p\, \delta^{ab}\delta_{mn} - i\left(m_{(m)}+\delta m_{(m)}\right)\delta_{mn}\delta^{ab}$$
$$- i\hat{\Sigma}_c(p)\,\delta_{mn}\delta^{ab}$$

where

$$-i\hat{\Sigma}_c(p)\,\delta_{mn}\delta^{ab} =$$ 

$$= \int \frac{d^4k}{(2\pi)^4}\, i Z_1^F g_3 T^i_{bb'}\delta_{nn'}\gamma^\nu \left(\frac{i}{\not p+\not k}\delta^{a'b'}\delta_{m'n'}\right)\times$$
$$\times \gamma^\mu\, i Z_1^F g_3 T^j_{a'a}\delta_{mm'}\times$$
$$\times\left(\frac{-i g_{\mu\nu}\delta^{ij}}{k^2}\right)$$

$$= -g_3^2 \underbrace{\left(T^i T^i\right)_{ba}}_{=C_2(3)\delta_{ab}}\delta_{mn}\int\frac{d^4k}{(2\pi)^4}\gamma^\rho\frac{1}{\not p+\not k}\gamma_\rho\frac{1}{k^2}$$

$$= -g_3^2 C_2(3)\delta^{ab}\delta_{mn}\int\frac{d^4k}{(2\pi)^4}\frac{\gamma^\rho(\not k+\not p)\gamma_\rho}{(p+k)^2k^2}$$

$$= -g_3^2 C_2(3)\delta^{ab}\delta_{mn}\int\frac{d^4k}{(2\pi)^4}\frac{\gamma^\rho\not p\gamma_\rho+\gamma^\rho\not k\gamma_\rho}{(p+k)^2k^2}$$

$$= -i\hat{\Sigma}_c(p)\,\delta_{mn}\delta^{ab}$$

IF$_a$)

So

$$\boxed{\hat{\Sigma}_1(p) = -ig_3^2 C_2(3)\int\frac{d^4k}{(2\pi)^4}\frac{\gamma^\rho \not{p}\gamma_\rho + \gamma^\rho \not{k}\gamma_\rho}{(k+p)^2 k^2}}$$

Now we have

$$\frac{1}{(p+k)^2}\frac{1}{k^2} = \int_0^1 dx \frac{1}{[x(p+k)^2+(1-x)k^2]^2}$$

$$\Big(\text{i.e. } \frac{1}{AB} = \int_0^1 d\alpha \frac{1}{[\alpha A+(1-\alpha)B]^2} = \int_0^1 d\alpha \frac{1}{[\alpha(A-B)+B]^2}$$

$$= -\left(\frac{1}{A-B}\right)\left[\frac{1}{\alpha(A-B)+B}\right]\Big|_0^1 = -\frac{1}{A-B}\left[\frac{1}{A-\cancel{B}+\cancel{B}} - \frac{1}{B}\right]$$

$$= -\frac{1}{A-B}\frac{B-A}{AB} = \frac{1}{AB}\checkmark\Big)$$

$$\frac{1}{(p+k)^2k^2} = \int_0^1 dx\frac{1}{[x\{(p+k)^2-k^2\}+k^2]^2} = \int_0^1\frac{dx}{[x[p^2+2k\cdot p]+k^2]^2}$$

So

$$\hat{\Sigma}_1(p) = -ig_3^2 C_2(3)\int_0^1 dx\int\frac{d^4k}{(2\pi)^4}\frac{\gamma^\rho\not{p}\gamma_\rho+\gamma^\rho\not{k}\gamma_\rho}{[x(p+k)^2+(1-x)k^2]^2}$$

Let

$$l^\mu = k^\mu + xp^\mu \;;\; l^2 = k^2+x^2p^2+2xk\cdot p$$

$$\hat{\Sigma}_1(p) = -ig_3^2 C_2(3)\int_0^1 dx\int\frac{d^4l}{(2\pi)^4}\frac{\gamma^\rho\not{p}\gamma_\rho+\gamma^\rho(\not{l}-x\not{p})\gamma_\rho}{[l^2+p^2x(1-x)]^2}$$

$$\boxed{\hat{\Sigma}_1(p) = -ig_3^2 C_2(3)\int_0^1 dx\int\frac{d^4l}{(2\pi)^4}\frac{(1-x)\gamma^\rho\not{p}\gamma_\rho}{[l^2+p^2(1-x)x]^2}}$$

($\not{l}$ is odd)

I 3.) Wick rotate

$$\hat{\Sigma}_1(p) = +g_3^2 C_2(3) \int_0^1 dx \int \frac{d^d l}{(2\pi)^d} \frac{(1-x) \overbrace{\gamma^\rho \not{p} \gamma_\rho}^{(2-d)\not{p}}}{[l^2 - p^2(1-x)x]^2}$$

So

$$\frac{\partial}{\partial \not{p}} \hat{\Sigma}_1(p)\Big|_{p^2=-\mu^2} = g_3^2 C_2(3) \int_0^1 dx \int \frac{d^d l_E}{(2\pi)^d} \frac{(1-x) \quad (2-d)}{[l_E^2 + \mu^2(1-x)x]^2}$$

$$\frac{\partial}{\partial \not{p}} \hat{\Sigma}_1(p)\Big|_{p^2=-\mu^2} = \frac{g_3^2 C_2(3)}{2^d \pi^d} \frac{2\pi^{d/2}}{\Gamma(d/2)} \int_0^1 dx\,(1-x)(2-d) \int_0^\infty \frac{l^{d-1}\,dl}{[l^2+\mu^2(1-x)x]^2}$$

again $$\int_0^\infty \frac{l^{d-1}\,dl}{[l^2+\mu^2(1-x)x]^2} = \frac{1}{2}\int_0^\infty \frac{x^{\frac{d-2}{2}}\,dx}{[x+\mu^2(1-x)x]^2}$$

$$= \frac{1}{2}\left(\mu\sqrt{(1-x)x}\right)^{-\epsilon} \frac{\Gamma(\epsilon/2)\,\Gamma(d/2)}{\Gamma(2)}$$

$$\frac{\partial}{\partial \not{p}} \hat{\Sigma}_1(p)\Big|_{p^2=-\mu^2} = \frac{g_3^2 C_2(3)}{2^d \pi^d} \frac{2\pi^{d/2}(2-d)}{2} \int_0^1 dx\,(1-x) *$$

$$* \left[\frac{2}{\epsilon}\right]\left[1 - \epsilon \ln \mu\sqrt{x(1-x)}\right]$$

$$= \frac{g_3^2 C_2(3)\,2}{2^d \pi^{d/2}} (2-d) \int_0^1 dx\,(1-x)\left[\frac{1}{\epsilon} - \ln\mu - \ln\sqrt{x(1-x)}\right]$$

I.B.) So $\boxed{\left.\frac{\partial}{\partial \not{p}}\hat{\Sigma}_1(p)\right|_{p^2=-\mu^2} \overset{d\to 4}{=} -\frac{4g_3^2 C_2(3)}{(4\pi)^2}\left[\frac{1}{2\epsilon} - \frac{1}{2}\ln\mu - \cdots\right]}$

ignore $\ln\sqrt{x(1-x)}$ indp. of μ term

$\Rightarrow$

Go p. -42-

$$Z_2 = 1 + \left.\frac{\partial \hat{\Sigma}_1}{\partial \not{p}}\right|_{\not{p}=\mu}$$

&

$$2\gamma_q = \mu\frac{\partial}{\partial\mu} Z_2$$

$\Rightarrow$

$$\boxed{2\gamma_q = \mu\frac{\partial}{\partial\mu}\left(\left.\frac{\partial\hat{\Sigma}_1}{\partial\not{p}}\right|_{\not{p}=\mu}\right) = +\frac{2g_3^2 C_2(3)}{(4\pi)^2}}$$

So we have another term for β

$$\boxed{\beta = g_3\left[\mu\frac{\partial}{\partial\mu}\frac{1}{2}Z_3 - \frac{g_3^2}{(4\pi)^2}\left[2\left(C_2(3)+C_2(8)\right) - 2C_2(3)\right]\right]}$$