I.B.) Now, as usual, these are bare fields & parameters and we re-scale all to deal with renormalized perturbation theory

bare fields

$$
\left.\begin{aligned}
G^i_{0\mu} &= Z_3^{1/2} G^i_\mu \\
q^a_{0m} &= Z_2^{1/2} q^a_m \\
c^i_0 &= Z_c^{1/2} c^i \\
\bar{c}^i_0 &= Z_c^{1/2} \bar{c}^i
\end{aligned}\right\}
$$

renormalized fields

So in detail

$$
\mathcal{L} = \sum_m Z_2 \bar{q}^a_m \left[i\left(\partial\!\!\!/ \delta^{ab} - i g_3^0 T^i_{ab} Z_3^{1/2} G\!\!\!/^i \right) - m^0_{(m)} \delta^{ab} \right] q^b_m
$$

$$
- \frac{Z_3}{4} \left[\partial_\mu G^i_\nu - \partial_\nu G^i_\mu + g_3^0 Z_3^{1/2} f_{ijk} G^j_\mu G^k_\nu \right]^2
$$

$$
+ \frac{g_3^0}{32\pi^2} \theta^0 Z_3 \left[\partial_\mu G^i_\nu - \partial_\nu G^i_\mu + g_3^0 Z_3^{1/2} f_{ijk} G^j_\mu G^k_\nu \right] \epsilon^{\mu\nu\rho\sigma} \times
$$
$$
\times \frac{1}{2} \left[\partial_\rho G^i_\sigma - \partial_\sigma G^i_\rho + g_3^0 Z_3^{1/2} f_{ijk} G^j_\rho G^k_\sigma \right]
$$

$$
- \frac{Z_3}{2\alpha_0} \left(\partial_\mu G^{i\mu} \right)^2 - \frac{Z_c}{\alpha_0} \partial^\mu \bar{c}_i \left[\partial_\mu \delta^{ij} + g_3^0 Z_3^{1/2} f_{ikj} G^k_\mu \right] c_j
$$

Now ~~we~~ introduce renormalized mass and coupling constant parameters and their counterterms

$$
Z_2 m^0_{(m)} \equiv \overset{\text{finite}}{m_{(m)}} + \overset{\text{divergent}}{\delta m_{(m)}}
$$

I.B) $Z_1^F g_3 \equiv Z_2 Z_3^{1/2} g_3^0$ with $Z_1^F = 1+\delta_1^F$ (quark covariant D_μ)

$Z_1^C g_3 \equiv Z_C Z_3^{1/2} g_3^0 Z_\alpha^G$ with $Z_1^C = 1+\delta_1^C$ ($\phi\pi$ covariant derivative)

$\frac{Z_\alpha^G}{\alpha} = \frac{Z_3}{\alpha_0}$ with $Z_\alpha = 1+\delta_\alpha$ (gaugefixing)

$Z_1^{3G} g_3 \equiv Z_3^{3/2} g_3^0$ with $Z_1^{3G} = 1+\delta_1^{3G}$ (3 Gluon vertex)

$Z_1^{4G} g_3^2 = Z_3^2 (g_3^0)^2$ with $Z_1^{4G} = 1+\delta_1^{4G}$ (4 Gluon vertex)

$\frac{Z_\alpha^{C'}}{\alpha} = \frac{Z_C}{\alpha_0}$ with $Z_\alpha^C = 1+\delta_\alpha$ ($\phi\pi$ KE)

$Z_\theta g_3 \theta \equiv g_3^0 \theta^0 Z_3$

$Z_\theta^{3G} g_3^2 \theta \equiv (g_3^0)^2 \theta^0 Z_3^{3/2}$

S_0

$$\mathcal{L} = \sum_m Z_2 \bar{q}_m^a i\not{\partial} q_m^a + \sum_m Z_1^F g_3 \bar{q}_m^a T^i_{ab} \not{G}^i q_m^b$$

$$- \sum_m (m_{(m)} + \delta m_{(m)}) \bar{q}_m^a q_m^a$$

$$- \frac{Z_3}{4}\left[\partial_\mu G_\nu^i - \partial_\nu G_\mu^i\right]^2 - \frac{2 Z_1^{3G}}{4} g_3 \left[\partial_\mu G_\nu^i - \partial_\nu G_\mu^i\right] \times$$

$$\times f_{ijk} G^{j\mu} G^{k\nu}$$

$$- \frac{Z_1^{4G}}{4} g_3^2 \left[f_{ijk} G_\mu^j G_\nu^k\right]^2 - \frac{Z_\alpha^G}{\alpha}(\partial_\mu G^{i\mu})^2$$

I.B.) $- Z_\alpha^c \frac{1}{\alpha} \partial^\mu \bar{c}_i \partial_\mu c_i$

$$- \frac{Z_1^c g_3}{\alpha} \partial^\mu \bar{c}_i f_{ikj} G^k_\mu c_j$$

$+ \theta$-terms

Now BRS invariance (gauge invariance + ΦΠ) implies relations among counter-terms which must hold for any BRS-gauge invariant regulation-Renormalization procedure. Also there are more counterterms than invariant bare Lagrangian terms so ⇒
All the same g_3^0 ⇒

$$\frac{Z_1^F}{Z_2 Z_3^{1/2}} = \frac{Z_1^c Z_3^{1/2}}{Z_c Z_\alpha} = \frac{Z_1^{3G}}{Z_3^{3/2}} = \frac{(Z_1^{4G})^{1/2}}{Z_3}$$

and gauge fixing & Φ-Π have same coeff. by BRS

i.e. same α_0 ⇒

$$Z_\alpha^c = \frac{Z_c Z_\alpha^G}{Z_3}$$

i.e. there are "3" types of fields = 3 parameters Z's
1 mass = 1 " m_0
1 coupling const = 1 " g_3^0
1 gauge fixing-ΦΠ = 1 parameter α^0

6 parameters

(ignore θ for now)

but we have 10 counterterms ⇒ 4 relations

(Could just say 9 monomials so 9 counter-terms with 3 relations $\Rightarrow$ 6 parameters as above i.e. $\frac{Z_\alpha^c}{\alpha} = \frac{Z_c}{\alpha_0} = \frac{Z_c Z_\alpha^G}{\alpha Z_3}$, no need for Z_α^c then)

I.B.) This is what we found above. So we need to specify <u>6 normalization conditions</u> to define 3-Z's, m, g_3 & α (Many choices this is not mass indep. scheme)

1) $\langle 0 | T \tilde{q}^a_m(p)\, \bar{q}^b_n(0) | 0 \rangle^{1PI} \Big|_{\not p = m} \equiv \tilde{S}^{\,/-1\,ab}_{F\,mn}(p) \Big|_{\not p = m} = 0$ (on-shell) $\left(\text{or } \not p = 0\ (\text{intermediate})\ \text{or } \not p = 0 \equiv -im\right)$

2) $\frac{\partial}{\partial \not p} \tilde{S}^{\,/-1\,ab}_{F\,mn}(p) \Big|_{\not p = \mu\ (p^2 = -\mu^2)} = i$

$\hat{3}$) $\tilde{\Delta}^{/-1}_{F\,\mu\nu}(p) = \Delta_T^{-1} P_{T\mu\nu}(p) + \Delta_L^{-1} P_{L\mu\nu}(p)$

$\Delta_T^{-1}(p^2) \Big|_{p^2 = 0} = 0$ massless guaranteed

3) $\frac{\partial}{\partial p^2} \Delta_T^{-1}(p^2) \Big|_{-p^2 = \mu^2} = i$

$\hat{4}$) $\Delta_L^{-1}(p^2) \Big|_{p^2 = 0} = 0$ massless guaranteed

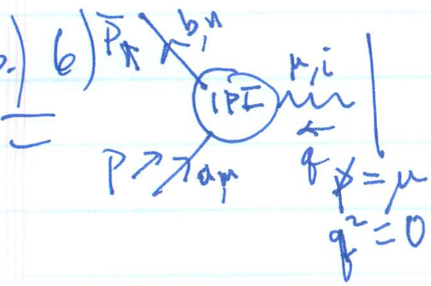
4) $\frac{\partial}{\partial p^2} \Delta_L^{-1}(p^2) \Big|_{-p^2 = \mu^2} = \frac{i}{\alpha}$

$\hat{5}$) $\tilde{\Delta}^{/-1}_c(p^2) \Big|_{p^2 = 0} = 0$ massless guaranteed

5) $\frac{\partial}{\partial p^2} \tilde{\Delta}^{/-1}_c(p^2) \Big|_{p^2 = \mu^2} = i$

$$\langle 0 | T\, \tilde{q}^a_m(p)\, \bar{\tilde{q}}^b_n(\bar p)\, G^i_\mu(0) |0\rangle = \Gamma^{(1,1,1,0,0)\,ab\,k}_{mn\,\mu}(p, \bar p, q = \bar p - p) =$$

I.B.) 6)



$$= i g_3 T^i_{ab}\, \delta_{mn}\, \gamma^\mu = \Gamma^{(1,1,1,0,0)}(p, \bar p, q)\Big|_{\text{n.P.}}$$

n.P. $\begin{cases} p^2 = -\mu^2 \\ \bar p^2 = -\mu^2 \\ q^2 = 0 \end{cases}$
$\Rightarrow p \cdot q = 0$
$\bar p \cdot q = 0$
$\bar p = p + q$

These conditions fix the theory (along with gauge invariant regulation/renormalization)

So we can write the $\mathcal{L}$ as the bilinear$_0$ plus $\mathcal{L}_{int}$.

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$$

$$\mathcal{L}_0 = \sum_m{}' \bar q^a_m \left(i\not\partial - m_{(m)} \right) q^a_m$$
$$- \frac{1}{4}\left[\partial_\mu G^i_\nu - \partial_\nu G^i_\mu\right]^2 - \frac{1}{\alpha}\left(\partial_\mu G^{i\mu}\right)^2$$
$$- \frac{1}{\alpha}\partial^\mu \bar c_i \partial_\mu c_i$$

and

$$\mathcal{L}_{int} = (Z_2 - 1)\sum_m{}' \bar q^a_m\, i\not\partial\, q^a_m - \sum_m{}' \delta m_{(m)} \bar q^a_m q^a_m$$
$$- \frac{(Z_3 - 1)}{4}\left[\partial_\mu G^i_\nu - \partial_\nu G^i_\mu\right]^2 - \frac{(Z_\alpha - 1)}{\alpha}\left(\partial_\mu G^{i\mu}\right)^2$$
$$- \frac{(Z^c_\alpha - 1)}{\alpha}\partial^\mu \bar c_i \partial_\mu c_i - \frac{Z^c_1 g_3}{\alpha}\partial^\mu \bar c_i f_{ikj} G^k_\mu c_j$$
$$+ \sum_m{}' Z^F_1 g_3 \bar q^a_m T^i_{ab} \not G^i q^b_m - \frac{1}{2} Z^{3G}_1 g_3 \left[\partial_\mu G^i_\nu - \partial_\nu G^i_\mu\right] f_{ijk} G^{j\mu} G^{k\nu}$$
$$- \frac{Z^{4G}_1}{4} g_3^2 \left[f_{ijk} G^j_\mu G^k_\nu\right]^2.$$

IB.) Hence the Green functions are given by

$$\langle 0| T\, q^{a_1}_{m_1}(x_1)\cdots q^{a_m}_{m_m}(x_m)\,\bar{q}^{b_1}_{n_1}(y_1)\cdots \bar{q}^{b_n}_{n_n}(y_n)\,\times$$
$$\times\, G^{i_1}_{\mu_1}(z_1)\cdots G^{i_\ell}_{\mu_\ell}(z_\ell)\, C^{j_1}(u_1)\cdots C^{j_c}(u_c)\,\bar{C}^{k_1}(v_1)\cdots \bar{C}^{k_{\bar{c}}}(v_{\bar{c}})\,|0\rangle$$

$$= \int \frac{d^4p_1}{(2\pi)^4}\cdots\frac{d^4\bar{p}_1}{(2\pi)^4}\cdots\frac{d^4q_1}{(2\pi)^4}\cdots\frac{d^4r_1}{(2\pi)^4}\cdots\frac{d^4\bar{r}_1}{(2\pi)^4}\cdots\frac{d^4\bar{r}_{\bar{c}}}{(2\pi)^4}\,\times$$
$$\times\, e^{-ip_ix_i}\, e^{+i\bar{p}_jy_j}\, e^{-iq_kz_k}\, e^{-ir_\ell u_\ell}\, e^{+i\bar{r}_h v_h}\,\times$$
$$\times \sum_{\Gamma\in G_1^{(m,n,\ell,c,\bar{c})}} \alpha(\Gamma)\,\delta_\Gamma \int \frac{d^4k_1}{(2\pi)^4}\cdots\frac{d^4k_{m(\Gamma)}}{(2\pi)^4}\, I_\Gamma(p,\bar{p},q,r,\bar{r},k)$$

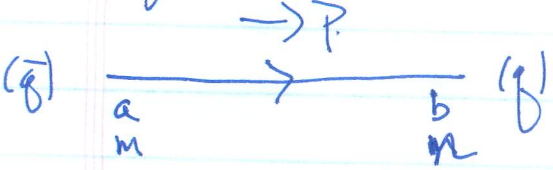
where $\alpha(\Gamma)$ = symmetry #

$\delta_\Gamma = (2\pi)^4\delta^4(p+\cdots+q+\cdots-\bar{p}\cdots)$ for each connected factor of Γ; $m(\Gamma)$ = # of loops

$I_\Gamma = I_\Gamma(p,\bar{p},q,r,\bar{r},k)$ Feynman integrand made from graphical rules:

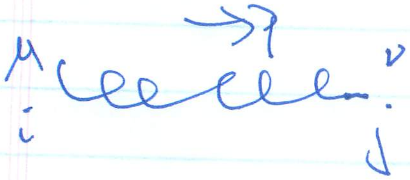
I.B.) A) Propagators in I Factors in I_Γ

1) quark lines



$$\left(\frac{\delta^{ab}\delta^{mn}\, i}{\not p - m}\right)$$

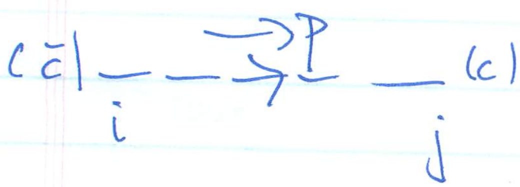
2) Gluon lines



$$\left[\frac{-i\delta^{ij}}{p^2}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) - \frac{i\alpha\delta^{ij}}{p^2}\,\frac{p^\mu p^\nu}{p^2}\right].$$

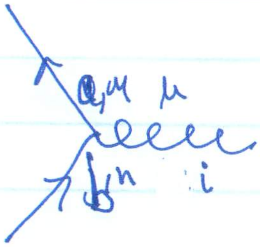
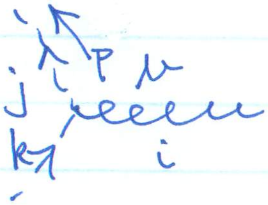
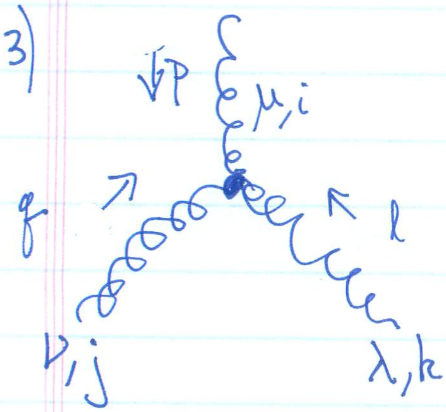
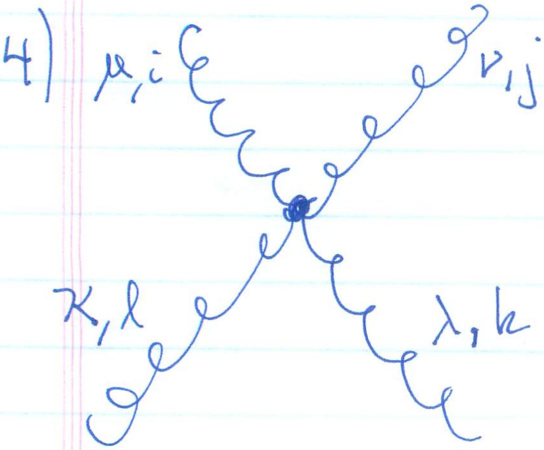
$$= \left[\frac{-i\delta^{ij}g^{\mu\nu}}{p^2} - \frac{i\delta^{ij}}{p^2}(\alpha-1)\frac{p^\mu p^\nu}{p^2}\right]$$

3) Φ-Π ghost lines

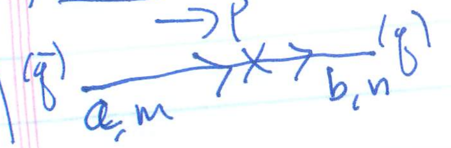


$$\left(\frac{-i\delta^{ij}\alpha}{p^2}\right)$$

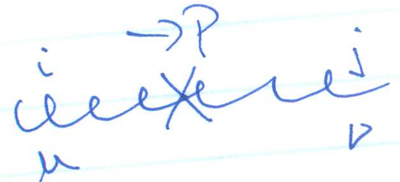
I.B.) B.) Vertices

Vertex in Γ	Vertex Factor in I_Γ
1) 	$iZ_1^F g_3 T^i_{ab} \delta_{mn} \gamma^\mu$
2) 	$\frac{Z_1^c g_3}{\alpha} p^\mu f_{jik}$
3) 	$+ Z_1^{3G} g_3 f_{ijk}\left[(p-q)^\lambda g^{\mu\nu} + (q-l)^\mu g^{\nu\lambda} + (l-p)^\nu g^{\mu\lambda}\right]$
4) 	$-iZ_1^{4G} g_3^2 \times \left[f_{hij}f_{hkl}(g_{\mu\lambda}g_{\nu\kappa} - g_{\mu\kappa}g_{\nu\lambda}) + f_{hik}f_{hlj}(g_{\mu\kappa}g_{\lambda\nu} - g_{\mu\nu}g_{\lambda\kappa}) + f_{hil}f_{hjk}(g_{\mu\nu}g_{\lambda\kappa} - g_{\mu\lambda}g_{\nu\kappa})\right]$

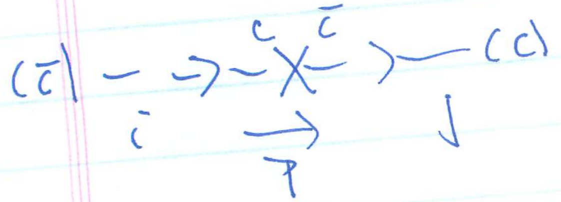
I.B.B) Vertices Cont'd.:

5) 

$i(Z_2-1)\not{p}\,\delta^{ab}\delta_{mn}$
$-i\,\delta m_{(m)}\,\delta_{mn}\,\delta^{ab}$

6) 

$-i(Z_3-1)\left[g^{\mu\nu}p^2-p^\mu p^\nu\right]\delta^{ij}$
$-i\,\dfrac{(Z_\alpha-1)}{\alpha}\,p^\mu p^\nu\,\delta^{ij}$

7) 

$-i\,\dfrac{(Z_\alpha^c-1)}{\alpha}\,p^2\,\delta_{ij}$

(Recall BRS transformations:

$$\delta_{BRS}\,q^a_m(x) = -i\left(\frac{\lambda^i}{2}\right)_{ab} C^i(x)\,q^b_m(x)$$

$$\delta_{BRS}\,\bar{q}^a_m(x) = +i\,\bar{q}^b_m(x)\,C^i(x)\left(\frac{\lambda^i}{2}\right)_{ba}$$

$$\delta_{BRS}\,G^i_\mu(x) = Z_3^{-1/2}\,\partial_\mu C^i(x) - g_3^0\, f_{ijk}\, G^j_\mu(x)\, C^k(x)$$

$$\delta_{BRS}\,C^i(x) = -\,f_{ijk}\,C^j(x)\,C^k(x)$$

$$\delta_{BRS}\,\bar{C}^i(x) = \partial^\mu G^i_\mu(x)\left(\frac{Z_3^{1/2}}{Z_C}\right)\quad)$$

I.B.) Consider BRS variation of $\langle 0|T\, G^i_{0\mu}(x)\, \bar{C}^j_0(y)|0\rangle = 0$

$\Rightarrow$

$$\delta_{BRS}\langle G_0 \bar{C}_0\rangle = 0$$

$$\Rightarrow\; 0 = \langle 0|T\, G^i_{0\mu}(x)\, \partial^\nu G^j_{0\nu}(y)|0\rangle$$

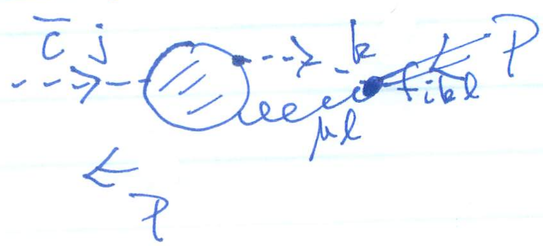
$$+ \langle 0|T\left(\partial_\mu C^i(x) - g_3^0 f_{ikl} G^k_{0\mu}(x)\, C^l_0(x)\right)\bar{C}^j_0(y)|0\rangle$$

(drop "0") $\Rightarrow$

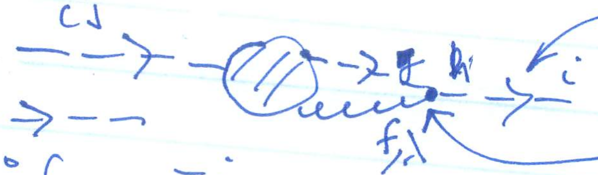
$$\boxed{\begin{aligned} i p^\nu \langle 0|T\, \tilde{G}^i_\mu(p)\, G^j_\nu(0)|0\rangle &= i p_\mu \langle 0|T\, \tilde{C}^i(p)\, \bar{C}^j_0(0)|0\rangle \\ &\quad - g_3^0 f_{ikl}\langle 0|T\, \widetilde{[G^k_\mu C^l]}(p)\, \bar{C}^j(0)|0\rangle \end{aligned}}$$

Consider RHS

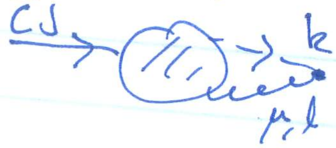
$= i p_\mu$ 

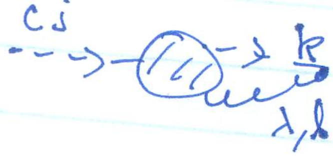
$- g_3^0 f_{ikl}$ 

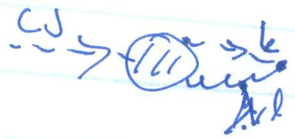
I.B.) Only $\bar{c}$ interaction is the 3 vertex

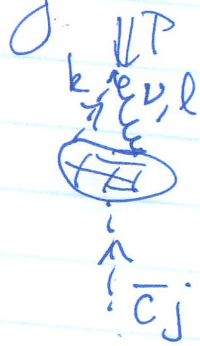
$= i p_\mu$  $\dfrac{\delta^{ih}\,(-i\alpha_0)}{p^2}$

$+ i p_\mu$ $- \dfrac{g_3^0}{\alpha_0} f_{hfg}\, p^\lambda$

$- g_3^0 f_{ikl}$ 

$= + \dfrac{p^\mu p^\lambda}{p^2}\, g_3^0 f_{ikl}$  $+ \xrightarrow{i p_\mu}$

$- g_3^0 f_{ikl}$  $g^{\mu\lambda}$

$= \left[g^{\mu\nu} - \dfrac{p^\mu p^\nu}{p^2} \right] (- g_3^0 f_{ikl})$ 

$+ i p_\mu \dashrightarrow$

$= i p^\nu \tilde{\Delta}^{ij}_{\mu\nu}(p) = i p_\mu \Delta_L$

$\Rightarrow$

$i p^\mu p^\nu \tilde{\Delta}^{ij}_{\mu\nu} = i \Delta_L p^2$

$\Rightarrow i \Delta_L p^2 = i p^2 \dfrac{(-i)\alpha_0}{p^2}$

$= \alpha_0$

$\Rightarrow$

$\Delta_L = \dfrac{\alpha_0}{p^2}$ $\Rightarrow$ No Renormalization for $\alpha = \alpha_0 / Z_3$ only that of G_μ

So rescale G_μ

I.B.| Also $\Rightarrow P_T^{\mu\nu}(p)$  $= 0$

Renormalization Group: The Green's functions depend on the arbitrary scale μ at which the normalization conditions were given. Hence

$$\Gamma^{(m,n,l,c,\bar{c})}(p, m, g_3, \alpha; \mu)$$

Now suppose we change $\mu \to \mu'$ as the normalization pt. and $m \to m'$, $g_3 \to g_3'$, $\alpha \to \alpha'$ at this norm. pt. The effects of such a change can be absorbed by a wave function re-scaling

$$\phi'(x, m', g_3', \alpha'; \mu') = Z^{-1/2}(\mu', m, g_3, \alpha; \mu)\, \phi(x, m, g_3, \alpha; \mu)$$

hence

$$\Gamma^{(m,n,l,c,\bar{c})}(p, m', g_3', \alpha'; \mu') = Z_2^{\frac{m+n}{2}} Z_3^{l/2} Z_c^{\frac{c+\bar{c}}{2}}\, \Gamma^{(m,n,l,c,\bar{c})}(p, m, g_3, \alpha; \mu)$$

[This applies to the bare $\to$ renormalized case as well

$$\Gamma^{(m,n,l,c,\bar{c})}(p, m, g_3, \alpha; \mu) = Z_2^{\frac{m+n}{2}} Z_3^{l/2} Z_c^{c+\bar{c}/2}\, \Gamma^{(m,l,l,c,\bar{c})}(p, m_0, g_3^0, \alpha_0; \Lambda)$$

Λ: cutoff

$$Z_i = Z_i(\mu, m_0, g_3^0, \alpha_0; \Lambda)$$
]