Initially let's consider such separable systems. $H = H_0 + H_s$

Further, since we have worked out many "spinless" problems—let's ignore the orbital motion. For the Schrödinger eq.

e.g., Suppose consider the hydrogen atom. For the pure Coulomb interaction we have $H = H_0$

$$= \frac{\hat{p}^2}{2m} + V(r)$$

$$= \frac{-e^2}{4\pi\varepsilon_0 r} \uparrow 1$$

Hence this is a separable (trivial) case since the interaction is completely independent of the spin. Hence

$$|\Psi\rangle = |\Psi_0\rangle \otimes |\Psi_s\rangle$$

& $\text{int}_s |\Psi_s\rangle = 0 \Rightarrow |\Psi_s\rangle$ is independent.

The wavefunctions are then just products of orbital & spin functions.
Choosing $E$, $L^2$, $L_z$, $S_z$, $S=\frac{3}{2}$ as our CSDO then the eigenfunctions are simply

$$\langle \vec{r}, \vec{s}, m_s | n_l, m_l, m_s, m_s \rangle$$

$$\langle n_l, m_l, m_s \rangle \rightarrow \frac{2}{\sqrt{2}} \chi_{n_l m_l} \left( \vec{r} \right) e^{i m_s \phi}$$

That is, the wavefunctions are just products of the constant wavefunctions $\chi_{n_l m_l}(\vec{r})$ and the $S_z$ eigenspinors $(\hat{s}) = e^{i \phi} \chi_0(\vec{r}) = (\vec{0})$

Since $H = H_0$, the energy levels remain unchanged — but we now have doubled the number of states with each energy level corresponding to the e- spin up or down along the $z$-axis!
Next, let the $H$-atom in a weak $B$-field $\mathbf{B} = B \mathbf{t}$. The proton and $e^-$ will interact with the $B$-field through their orbital and spin magnetic moments. Since $\frac{M_e}{mp} \ll 1$ ($\frac{1}{2000}$) we will neglect the proton's magnetic moment's interaction energies (NMR!) compared to the electron.

Now the dipole energy is $H_{\text{dipole}} = -\mathbf{\mu} \cdot \mathbf{B}$ for the $e^-$. $\mathbf{\mu}$ has 2-terms (more antislater)

$$\mathbf{\mu} = \frac{\mathbf{g}}{2m} \mathbf{L} + \frac{\mathbf{e}}{m} \mathbf{S} = \frac{-e}{2m} \mathbf{L} - \frac{e}{m} \mathbf{S}$$

$$\Rightarrow \quad H_{\text{dipole}} = \frac{eB}{2m} \left( L_z + 2S_z \right)$$

And the $H$-atom Hamiltonian becomes

$$H = H_{\text{coulomb}} + \frac{eB}{2m} \left( L_z + 2S_z \right) = H_{\text{coulomb}} + H_{\text{dipole}}.$$
Since $H$ still commutes with $\mathbf{L}^2$, $S_z$, $S_x$, $S_y$, it is diagonalized by the same eigenstates. 
\[ |n, l, m; s, \mu_s\rangle = |n, l, m; \frac{1}{2}, \mu_s\rangle \]

Since $s = \frac{1}{2}$ we drop it from list to save writing.

\[ \Rightarrow |n, l, m, \mu_s\rangle \]

Hence
\[ H|n, l, m, \mu_s\rangle = \frac{\hbar^2}{2} \left( \frac{e^2}{4\pi \varepsilon_0} \right)^2 l^2 n^2 \]
\[ + \frac{eB\hbar}{2m} (m + 2\mu_s) \left| n, l, m, \mu_s\rangle \right\]
\[ = \left[ -\frac{m e^2}{2} \alpha^2 \frac{1}{n^2} + \mu_B B (m + 2\mu_s) \right]|n, l, m, \mu_s\rangle \]

Bohr magneton
\[ \mu_B = 5.7 \times 10^{-5} \text{ eV} \]

Hence $E_n$ now depends on $m, \mu_s$ also. We have removed some of the degeneracy of the energy levels.
ex. The ground state was 2-fold degenerate due to spin $\uparrow \downarrow$ of e-
now it's "split" into 2 non-deg.
states
$n=1 \Rightarrow l=0 = m$ but $m_s = \pm 1/2$

$$E_{n=1} = -13.6\text{eV} \pm \mu_B B$$

Spin down is ground state
agrees with intuition
Hippie $= -\tilde{\mu} \cdot B \rightarrow \tilde{\mu}$ is opposite
/spin (neg. change)
So lower energy
$\tilde{\mu}$ aligns with $B \Rightarrow$
\text{5 anti-parallel with } B

For the first excited state we
had

$N=2 \ \ l=0, \ m=0, \ m_s = \pm \frac{1}{2}$

$N=1 \ l=1, \ m=-\frac{1}{2}; \ m_s = \pm \frac{1}{2}$

8 degenerate states

for $H_{\text{cont.}}$

Now
\[ E_{m=-2} = -\frac{13.6 \text{eV}}{4} + M_B B \]

\[
\begin{pmatrix}
  2 & 0 & 0 & 0 \\
  1 & \frac{1}{2} & \frac{1}{2} & 0 \alpha \pm 1 \\
  0 & 0 & 0 & 0 \alpha \pm 1 \\
  -1 & -\frac{1}{2} & -\frac{1}{2} & 1 \alpha \pm 1
\end{pmatrix}
\]

Some we split. The 8 degenerate states into 5 states, 3 of which are still doubly degenerate, etc.

In multi-electron atoms, add contributions from all \( e^- \) → the # of spectral lines increase and the spacing may be varied by varying \( B \).

This is called the (weak field) Zeeman Effect. Move on this later (strong field).

Since we have worked a lot of spinless problems let's focus just on some pure spin dynamics.

i.e. \[ i \hbar \frac{\partial}{\partial t} (X_{\infty}^s)_{ms} = \{H_s\} \ \ \ \ \ \ \ \ \ \ (X_{\infty}^s)_{ms}, \dot{ms} \]