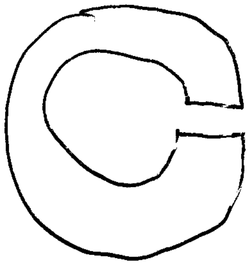
X.) i.) 2.) <u>Torus with gap:</u>

 d

linear material $\vec{B} = \mu \vec{H}$

$$\oint_C \vec{H} \cdot d\vec{l} = NI \quad + \underline{B.C.}$$

$\Rightarrow B =$ same everywhere

$$B = \frac{\mu \mu_0 N I}{\mu_0 2\pi r + d(\mu - \mu_0)}$$

End of Statics

XI.) Electromagnetic Induction: $\frac{\partial \vec{B}}{\partial t} \neq 0.$

a) $$\text{Emf} = \mathcal{E} = \oint_C \vec{E} \cdot d\vec{l} \neq 0 \text{ when } \frac{\partial \vec{B}}{\partial t} \neq 0.$$

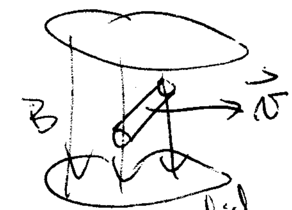
b.) <u>Faraday's law</u> new exp. law: <u>1831</u>

$$\mathcal{E} = -\frac{d\phi}{dt} \qquad \phi = \int_S \vec{B} \cdot d\vec{S} \quad \text{integral form.}$$

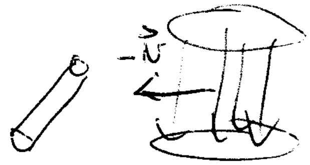
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{diff. form.}$$

(− sign = Lenz's law)

XI.c.) ex. 1.) Wire moving with $\vec{v}$ in $\vec{B}$

a) 

statics valid.

b.) 

need Faraday's law to get same result.

d.) Self- & Mutual- Inductance

1) $\phi = \phi(I)$ so $\frac{d\phi}{dt} = \frac{d\phi}{dI}\frac{dI}{dt}$,

rigid circuit.

Single circuit.

$$L = \frac{d\phi}{dI} \quad \text{self-ind.}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

ex. toroidal coil  $L = \frac{\mu_0 N^2 A}{l}$

2.) many circuits: $\phi_i = \sum_{j=1}^{n} \phi_{ij}$

$$\frac{d\phi_{ij}}{dt} = \frac{d\phi_{ij}}{dI_j}\frac{dI_j}{dt} \equiv M_{ij}\frac{dI_j}{dt} = -\mathcal{E}_i$$

mutual Ind.

$$M_{ii} = L_i$$

XI.) d.2) ex. twice wrapped toroid coil.

$$M_{12} = M_{21} = \frac{\mu_0 N_1 N_2 A}{l}$$

3.) Neumann Formula: $M_{ij} = M_{ji}$

B.-S. law $\Rightarrow$

$$M_{12} = \frac{\mu_0}{4\pi} \oint_{C_1} \oint_{C_2} \frac{d\vec{l}_1 \cdot d\vec{l}_2}{|\vec{r}_2 - \vec{r}_1|} .$$

---

e.) **Magnetic Energy**

1) Electrostatic: $U = \frac{1}{2} \int_V \vec{D} \cdot \vec{E} \, dv.$

2) $dU = -\mathcal{E} \, dq \quad \delta dq = \vec{J} \cdot d\vec{S} \, dt \, ; \quad dq = \int_S \vec{J} \cdot d\vec{S} \, dt$

$$dU = -\int_S \mathcal{E} \vec{J} \cdot d\vec{S} \, dt$$

$$= \left(\int_S \vec{J} \cdot d\vec{S} \; \frac{d\phi}{dt} \right) dt$$

Now $\quad \vec{J}(t) = \begin{cases} \frac{t}{T} \vec{J} & 0 \leq t \leq T \\ \vec{J} & t \geq T \end{cases}$

But $\phi \sim \vec{J}$ B-S law

$$\Rightarrow \quad \phi(t) = \begin{cases} \frac{t}{T} \phi & 0 \leq t \leq T \\ \phi & t \geq T \end{cases}$$

XI.) e.2.) So

$$U=\int_0^T\left(\int_S \frac{\partial\phi}{\partial t}\,\vec{J}\cdot d\vec{s}\right)dt$$

$$\boxed{U=\frac{1}{2}\int_S \phi\,\vec{J}\cdot d\vec{s}}=\frac{1}{2}\sum_i I_i\phi_i$$

But $\phi=\int_\Gamma \vec{B}\cdot d\vec{S}=\int_{\partial\Gamma}\vec{A}\cdot d\vec{l}$

So $\int_S \phi\,\vec{J}\cdot d\vec{s}=\int_S\int_{\partial\Gamma}\vec{A}\cdot d\vec{l}\,\vec{J}\cdot d\vec{s}=\int_V \vec{J}\cdot\vec{A}\,dv$

$$\Rightarrow\ \boxed{U=\frac{1}{2}\int_V \vec{J}\cdot\vec{A}\,dv}$$

finally $\vec{J}=\vec{\nabla}\times\vec{H}$ $\quad\vec{B}=\vec{\nabla}\times\vec{A}$

$$\Rightarrow\ \boxed{U=\frac{1}{2}\int_V \vec{B}\cdot\vec{H}\,dv.}$$

XII.) Maxwells equations

a.) Not only can $\frac{\partial\vec{B}}{\partial t}\neq 0$ but also $\vec{\nabla}\cdot\vec{J}\neq 0$.

Now $\vec{\nabla}\cdot\vec{J}=-\frac{\partial\rho}{\partial t}=-\vec{\nabla}\cdot\frac{\partial\vec{D}}{\partial t}\neq 0$,

since $\vec{\nabla}\cdot\vec{D}=\rho$.

XII.) a.) So Ampere's law needs to be modified consistent with charge conservation i.e continuity eq. $\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0$.

i.e. $\vec{\nabla}\times\vec{H} = \vec{J}$ take $\vec{\nabla}\cdot$; $\vec{\nabla}\cdot\vec{\nabla}\times\vec{H} \equiv 0$

$\Rightarrow \vec{\nabla}\cdot\vec{J} = 0$!

not true always.

So

Maxwell said

$$\boxed{\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}}$$

modified form of Ampere's law.

So

$$0 = \vec{\nabla}\cdot\vec{\nabla}\times\vec{H} = \vec{\nabla}\cdot\vec{J} + \frac{\partial}{\partial t}\vec{\nabla}\cdot\vec{D} = \vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t} \ !$$

Charge Conservation.

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b.) All E-M phenomena described by the 4 Maxwell's equations:

1) $\vec{\nabla}\cdot\vec{D} = \rho$: Gauss's law

2) $\vec{\nabla}\cdot\vec{B} = 0$: no magnetic monopoles

3) $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$: Faraday's law

4) $\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$: Ampere's law.

XII. b.) In addition we need def. of $\vec{D}$ & $\vec{H}$

$$\left.\begin{array}{l} 1)\ \vec{D} = \epsilon_0 \vec{E} + \vec{P} \\ 2)\ \vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M} \\ 3)\ \vec{J} = \vec{J}(\vec{E}) \end{array}\right\}$$

we further need $\vec{P}(\vec{E})$, $\vec{M}(\vec{B})$, $\vec{J}(\vec{E})$

For linear, isotropic material

$$\vec{D} = \epsilon \vec{E}$$
$$\vec{B} = \mu \vec{H}$$

$$\vec{J} = g \vec{E} \quad \text{(Ohmic)}$$

---

c.) Matter, i.e. charges, react to E-M fields via the Lorentz force law

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$