

PHYSICS 271
ELECTRICITY AND MAGNETISM
FINAL EXAMINATION

14 December 2000

INSTRUCTIONS: Answer all questions on the answer sheet provided, it will be the only paper that is collected. This is a closed book exam.

1. Positive charge Q is distributed throughout an insulating sphere of radius R according to the charge density $\rho = \frac{Q}{\pi R^3} \frac{r}{R}$. The magnitude of the electric field at a point $R/2$ from the center is:

a) $E = \frac{Q}{4\pi\epsilon_0 \left(\frac{1}{2}R\right)^2}$

c) $E = \frac{Q}{16\pi\epsilon_0 R^2}$

b) $E = \frac{Q}{8\pi\epsilon_0 R^2}$

d) $E = \frac{Q}{64\pi\epsilon_0 R^2}$

2. For the sphere in problem 1, the electric potential has the highest value

a) $V = \frac{Q}{4\pi\epsilon_0 R}$

c) $V = \frac{Q}{2\pi\epsilon_0 R}$

b) $V = \frac{Q}{3\pi\epsilon_0 R}$

d) $V = \frac{Q}{12\pi\epsilon_0 R}$

3. A dielectric slab of thickness b is inserted between the plates of a parallel plate capacitor of plate separation d . The capacitance is given by

a) $C = \frac{\kappa_e \epsilon_0 A}{\kappa_e d + b(\kappa_e - 1)}$

c) $C = \frac{\kappa_e \epsilon_0 A}{\kappa_e d - b(\kappa_e - 1)}$

b) $C = \frac{\kappa_e \epsilon_0 A}{\kappa_e b + d(\kappa_e - 1)}$

d) $C = \frac{\kappa_e \epsilon_0 A}{\kappa_e b - d(\kappa_e - 1)}$

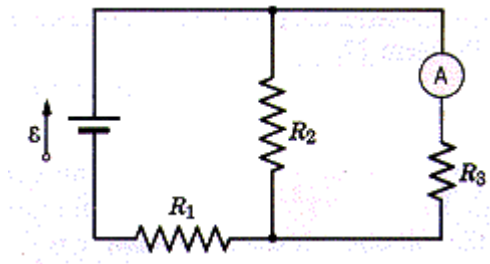
4. In the figure below imagine an ammeter inserted in the branch containing R_3 . What will it read, assuming $E = 5.0V$, $R_1 = 2.0 \text{ } \Omega$, $R_2 = 4.0 \text{ } \Omega$, and $R_3 = 6.0 \text{ } \Omega$?

a) $\frac{3}{22} \text{ A}$

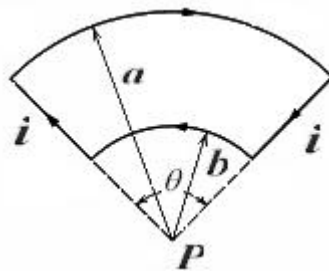
c) $\frac{1}{11} \text{ A}$

b) $\frac{5}{22} \text{ A}$

d) $\frac{5}{11} \text{ A}$



5-6. Consider the circuit drawn below. The curved segments are arcs of circles of radii a and b . The straight segments are along the radii. Assume current i flows in the circuit.



5. The direction of the magnetic field at P is

a) B points up

c) B points to the right

e) B points into the page

b) B points down

d) B points to the left

f) B points out of the page

g) B = 0

6. The magnitude of the magnetic field at P is

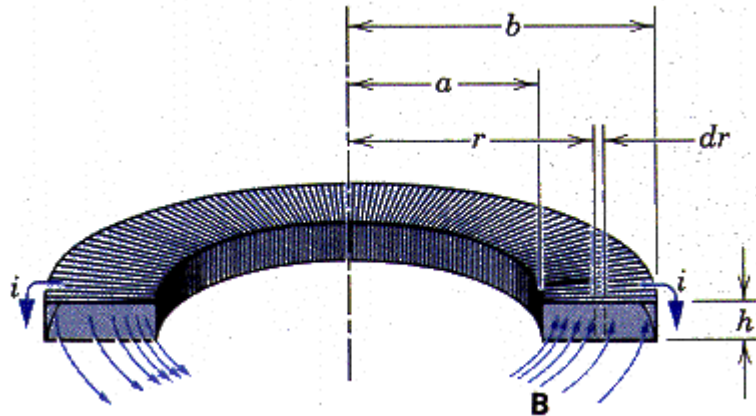
a) $B = \frac{\mu_0}{4\pi} i \frac{1}{2} \left(\frac{1}{a} + \frac{1}{b} \right)$

c) $B = 0$, since P is outside the loop

b) $B = \frac{\mu_0}{4\pi} i q \left(\frac{1}{b} - \frac{1}{a} \right)$

d) $B = \frac{\mu_0 i}{[(a-b) + q(a+b)]}$

For questions 7-12 a rectangular toroid, whose cross section is shown below, is considered. It has inner radius a and outer radius b with a thickness h with a total of N turns of wire carrying current i .



7. A toroid with a rectangular cross section carries current i . The magnetic field has its largest magnitude

- a) just inside the toroid at its inner surface
- b) at the center of the hole
- c) just inside the toroid at its outer surface
- d) at any point inside (the field is uniform)

8. The value of the magnitude of the magnetic field at radius r inside the toroid is

- a) $\frac{\mu_0 i N}{h}$
- b) $\frac{\mu_0 i N}{\rho r^2}$
- c) $\frac{\mu_0 i N}{4\pi h} \frac{b}{a}$
- d) $\frac{\mu_0 i N}{2\pi r}$

9. The inductance of the toroid is given by

- a) $L = \mu_0 n^2 h \pi (b^2 - a^2)$
- b) $L = \frac{\mu_0 N^2 a b}{2\pi h}$
- c) $L = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right)$
- d) none of the above

10. If the toroid is filled with magnetic material of permeability \mathbf{m} , its inductance L as compared to its inductance L_0 without the material is

a) $L = \mathbf{m} L_0$

c) $L = \frac{1}{\mathbf{m}} L_0$

b) $L = \frac{\mathbf{m}}{\mathbf{m}_0} L_0$

d) $L = \mathbf{c}_M L_0$

11. The energy density as a function of radial distance r for the unfilled (vacuum) toroid is

a) $u_B = \frac{\mathbf{m}_0 i^2 N^2}{8\mathbf{p}^2 r^2}$

c) $u_B = \left(\frac{\mathbf{m}_0 i N h}{8\mathbf{p}} \right)^2$

b) $u_B = \frac{\mathbf{m}_0 i N}{4\mathbf{p}} \ln\left(\frac{r}{h}\right)$

d) $u_B = (\mathbf{m}_0 i n)^2$

12. The total energy stored in the magnetic field of the unfilled (vacuum) toroid is

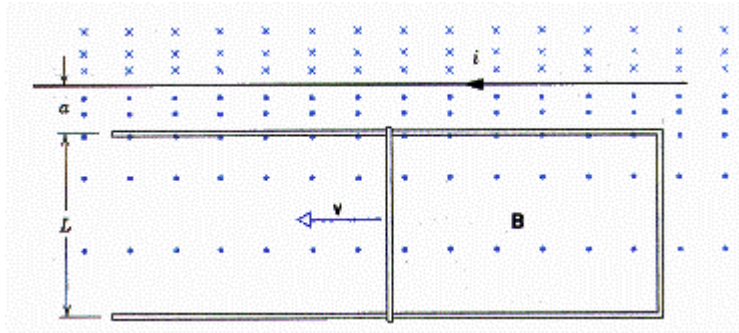
a) $U_B = i^2 R h \mathbf{p} (b^2 - a^2)$

c) $U_B = (\mathbf{m}_0 i n)^2 h \mathbf{p} (b^2 - a^2)$

b) $U_B = \frac{1}{2} L i^2$

d) none of the above

13. A rod with length L and resistance R moves along horizontal parallel conducting rails of negligible resistance at constant speed v , as shown below. The magnetic field \mathbf{B} in which the rod moves is not uniform but is provided by a current i in a long parallel wire.



13. The current I in the conducting loop is

a) $I = \frac{\mathbf{m}_0 i v}{2\mathbf{p} R} \left[a + \frac{1}{2} L \right]$

c) $I = \frac{\mathbf{m}_0 i v L}{2\mathbf{p} R a}$

e) $I = \mathbf{E}/R$

b) $I = \frac{\mathbf{m}_0 i v}{2\mathbf{p} R}$

d) $I = \frac{\mathbf{m}_0 i v}{2\mathbf{p} R} \ln \left(1 + \frac{L}{a} \right)$

14. Referring to problem 13 above, the force F that must be applied to the rod by an external agent in order to maintain its motion is given by

a) $F = \frac{P}{v}$

c) $F = \left(\frac{\mathbf{m}_0 i L}{2\mathbf{p} a} \right)^2 \frac{v}{R}$

e) $F = \left(\frac{\mathbf{m}_0 i}{2\mathbf{p}} \left[a + \frac{1}{2} L \right] \right)^2 \frac{v}{R}$

b) $F = \left(\frac{\mathbf{m}_0 i}{2\mathbf{p}} \ln \left(1 + \frac{L}{a} \right) \right)^2 \frac{v}{R}$

d) $F = \left(\frac{\mathbf{m}_0 i}{2\mathbf{p}} \right)^2 \frac{v}{R}$

15. In a certain RLC circuit, operating at 60 Hz, the maximum voltage across the inductor is twice the maximum voltage across the resistor, while the maximum voltage across the capacitor is the same as the maximum voltage across the resistor. By what phase angle does the current lag the generator emf?

a) $\mathbf{f} = 0^\circ$

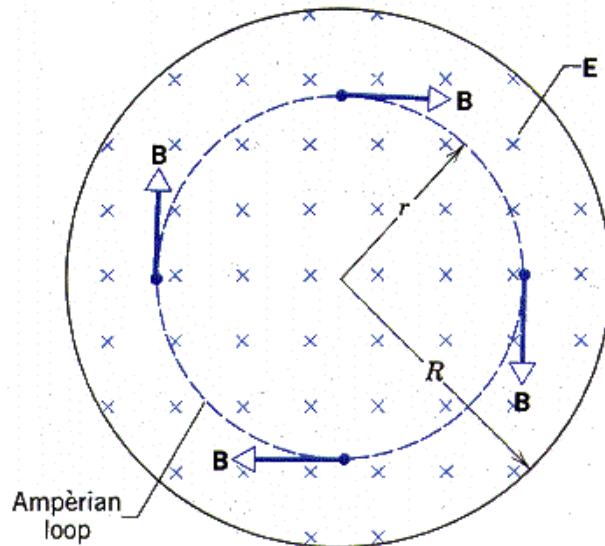
c) $\mathbf{f} = 45^\circ$

e) $\mathbf{f} = 90^\circ$

b) $\mathbf{f} = 30^\circ$

d) $\mathbf{f} = 60^\circ$

16-17. A parallel plate capacitor with circular plates is being charged. Determine the induced magnetic field at various radii r .



16. In the region between the plates, $r \leq R$, the magnetic field is

a) $B = \frac{1}{2} \epsilon_0 \mu_0 r \frac{dE}{dt}$

c) $B = \frac{\mu_0 i}{8\pi^2 r}$

b) $B = \frac{\mu_0 i}{2\pi r}$

d) $B = -\frac{1}{2\pi r} \frac{dE}{dt}$

17. In the region outside the plates, $r \geq R$, the magnetic field is

a) $B = \frac{\epsilon_0 \mu_0 R^2}{2r} \frac{dE}{dt}$

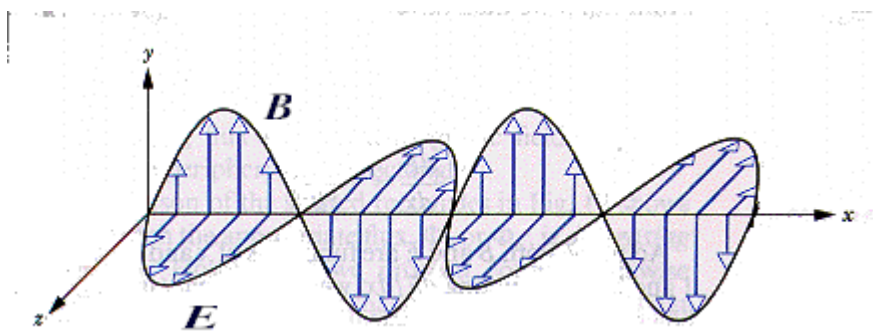
c) $B = \frac{\mu_0 i}{8\pi^2 R}$

b) $B = \frac{\mu_0 i}{2\pi R}$

d) $B = -\frac{1}{2\pi R} \frac{dE}{dt}$

18. For the electromagnetic wave pictured below, in which direction is the wave traveling:

- a) +x direction c) +y direction e) +z direction
 b) -x direction d) -y direction f) -z direction
 g) it is a standing wave



19. An electromagnetic wave is travelling in the positive z -direction with its electric field along the x -axis and its magnetic field along the y -axis, the fields are related by

- a) $\frac{\partial E}{\partial z} = \epsilon_0 \mu_0 \frac{\partial B}{\partial t}$ c) $\frac{\partial B}{\partial z} = \epsilon_0 \mu_0 \frac{\partial E}{\partial x}$ e) $\frac{\partial B}{\partial z} = \epsilon_0 \mu_0 \frac{\partial E}{\partial t}$
 b) $\frac{\partial E}{\partial z} = \epsilon_0 \mu_0 \frac{\partial B}{\partial x}$ d) $\frac{\partial B}{\partial z} = -\epsilon_0 \mu_0 \frac{\partial E}{\partial t}$

20. The dimension of $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ is:

- a) J/m^2 c) W/m^2 e) J/m^3
 b) J/s d) W/s