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ELASTIC INTERFACE WAVES ALONG A FRACTURE

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Abstract. Non-welded interfaces can be treated as a displacement discontinuity characterized by elastic stiffnesses. Applying this boundary condition to a generalized Rayleigh wave, it is shown that a fast and a slow dispersive wave can propagate along the fracture, even when the seismic properties of the rock on each side are identical.

Introduction

The reflection and refraction of seismic waves by welded interfaces between lithologies, across which both stresses and displacements in the wave are continuous, are understood well and are used extensively. Under conditions of low and even moderate effective stress, which may prevail to crustal depths of several kilometers, contacts between lithologies may not be welded. Furthermore, discontinuities in the form of joints, fractures and faults are important crustal features. These discontinuites within the same lithology also may behave as non-welded contacts at low and moderate values of effective stress. There is considerable interest in locating and characterizing both natural discontinuites in rock masses and induced discontinuites such as hydrofractures.

In this paper, we develop the theory for a seismic interface-wave that can propagate along a fracture. This theory is based on the seismic properties of a non-welded interface or a displacement discontinuity. We have determined that the seismic response of a single fracture is modeled well by representing the fracture as a boundary between two elastic half-spaces subject to the following set of boundary conditions: continuous stress, but discontinuous displacements [Schoenberg, 1980]. The discontinunity in displacement is defined to be inversely proportional to the specific stiffness of the fracture. We applied this set of boundary conditions to a generalized Rayleigh wave and derived the dispersion relationship of elastic interface waves that can travel along a fracture. These waves are not Stoneley waves [Stoneley, 1924; Sewaza and Kanai, 1939] because the material properties of the half-spaces on either side of the fracture are the same. Two waves exist: a "slow" wave which exists at all frequencies and all fracture stiffnesses, and a "fast" wave which exists only for certain excitation frequencies.

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Existence of Elastic Interface-Waves

An interface wave between two media separated by a non-welded contact (or fracture) can be derived by finding the displacements at the surfaces of the media, subject to coupling through the fracture stiffness. The half space for $z > O$ is medium 1, while medium 2 is the half-space for $z < O$. The fracture consists of the x-y plane and has components of specific stiffness, κ_x, κ_y, and κ_z, in the x, y, and z directions. A solution for a generalized surface wave can be expressed as $u_i \exp\{ikx-rz-ikct\}$ where u_i is the component of displacement in the direction $i=x,y$, or z, k is the wavenumber and c the phase velocity. The assumed solution for medium 1 can be expressed as the linear combination of two components:

$$u_{i1} = u_{i1}^{'} + u_{i1}^{''} \qquad i = x,y,z \tag{1}$$

where $u_i^{'}$ is the longitudinal component and $u_i^{''}$ is the transverse component. The transverse component must satisfy div $u^{''} = 0$, while the longitudinal component must satisfy curl $u^{'} = 0$. The components are:

$$(u_{x1}^{'},u_{y1}^{'},u_{z1}^{'}) = \left(\frac{-\alpha_1^2}{c^2k^2}\right)(ik,0,-r_1)\, D \exp\{-r_1 z + ikx\} \tag{2}$$

$$(u_{x1}^{''},u_{y1}^{''},u_{z1}^{''}) = (s_1,a_1,ik)\, Q_1 \exp\{-s_1 z + ikx\} \tag{3}$$

where a_1, D and Q_1 are constants and

$$r_1^2 = k^2 - \left(\frac{c^2k^2}{\alpha_1^2}\right)$$

$$s_1^2 = k^2 - \left(\frac{c^2k^2}{\beta_1^2}\right)$$

In these equations, α is the compressional wave velocity and β is the shear wave velocity. A similar solution for medium 2 can be expressed as $[u_x,u_y,u_z] \exp\{ikx + rz - ikct\}$ and also consists of two components:

$$u_{i2} = u_{i2}^{'} + u_{i2}^{''} \qquad i = x,y,z \tag{4}$$

where

$$(u_{x2}^{'},u_{y2}^{'},u_{z2}^{'}) = \left(\frac{-\alpha_2^2}{c^2k^2}\right)(ik,0,r_2)\, E \exp\{r_2 z + ikx\} \tag{5}$$

$$(u_{x2}^{''},u_{y2}^{''}u_{z2}^{''}) = (s_2,a_2,-ik)\, Q_2 \exp\{s_2 z + ikx\} \tag{6}$$

where a_2, E and Q_2 are constants and

$$r_2^2 = k^2 - \left(\frac{c^2k^2}{\alpha_2^2}\right)$$

$$s_2^2 = k^2 - \left(\frac{c^2k^2}{\beta_2^2}\right)$$

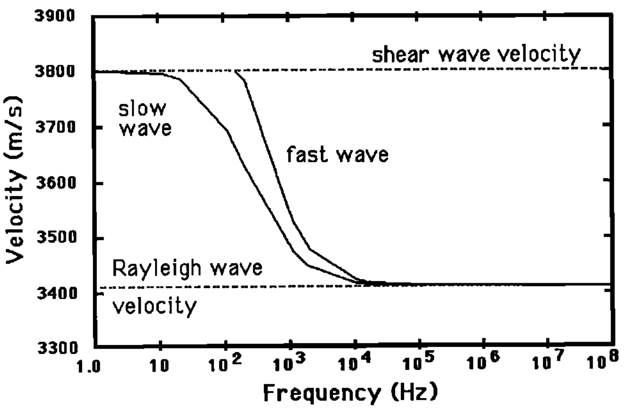

Fig. 1. A graph of the slow wave and fast wave velocity as a function of frequency. Upper asymptote is the shear wave velocity and the lower asymptote is the Rayleigh wave velocity. $\alpha_1 = \alpha_2 = 5800$ m/s, $\beta_1 = \beta_2 = 3800$ m/s, $\rho_1 = \rho_2 = 2600$ kg/m^3, $\kappa_x = \kappa_z = 10^9$ Pa/m.

To the assumed solutions for displacements in medium 1 and 2, the following displacement discontinuity boundary conditions were applied:

$$u_{z1} - u_{z2} = \frac{\tau_{zz1}}{\kappa_z} \tag{7}$$

$$\tau_{zz1} = \tau_{zz2} \tag{8}$$

$$u_{x1} - u_{x2} = \frac{\tau_{zx1}}{\kappa_x} \tag{9}$$

$$\tau_{zx1} = \tau_{zx2} \tag{10}$$

$$u_{y1} - u_{y2} = \frac{\tau_{zy1}}{\kappa_y} \tag{11}$$

$$\tau_{zy1} = \tau_{zy2} \tag{12}$$

where

$$\tau_{zz} = \lambda \left\{ \frac{\partial u_x}{\partial x} \right\} + (\lambda + 2\mu) \left\{ \frac{\partial u_z}{\partial z} \right\}$$

$$\tau_{zx} = \mu \left\{ \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} \right\}$$

$$\tau_{zy} = \mu \left\{ \frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z} \right\}.$$

These boundary conditions describe continuous stress across the interface, but discontinuous displacements. As in the case of the Stonely wave, there is no displacement component in the direction perpendicular to the direction of propagation (equations (11), (12), (2), (3), (5), (6)). Applying the remaining boundary conditions (7), (8), (9), (10) to equations (2), (3), (5), (6) and eliminating $(\alpha_1/c)^2 D$, $(\alpha_2/c)^2 E$, $ik^2 Q_1$, and $ik^2 Q_2$ from the resulting equations, leads to the following determinant:

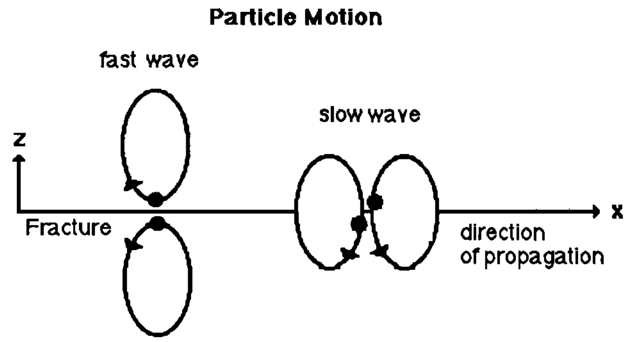

Fig. 2. Particle motion of the fast and slow interface-wave.

For a wave to exist, the velocity, c, and the wavenumber, k, must be such that this determinant vanishes. Furthermore, the velocity of this interface-wave must be real to be a traveling wave solution. The wave velocities were found numerically for the case where the properties of the elastic half-spaces are equal, namely: $\alpha_1 = \alpha_2 = 5800$ m/s, $\beta_1 = \beta_2 = 3800$ m/s, $\rho_1 = \rho_2 = 2600$ kg/m^3, for both $\kappa_x = \kappa_z$ and $\kappa_x \neq \kappa_z$. At most, two solutions to the determinant are found, corresponding to two distinct interface waves. For sufficiently low frequencies, however, only one wave is found to exist. Figure 1 displays the wave velocity vs. frequency for the two waves (referred to as the "fast" wave and the "slow" wave) that exist at a non-welded contact for the specific case where $\kappa_x = \kappa_z = 10^9$ Pa/m. At high frequency or low stiffness, both waves approach an asymptote defined by the Rayleigh wave velocity. At low frequencies or high stiffness, the slow wave approaches an asymptote defined by the shear wave velocity, while the fast wave approaches the shear velocity with a finite slope, and ceases to exist below a threshold frequency. Both wave velocities are functions of frequency and stiffness, and therefore these waves are weakly dispersive.

The particle motion of the waves is depicted in Figure 2. From direct calculation of the normal modes, it was determined that the x-components of displacement are in phase in the fast wave, and therefore the fast wave velocity is insensitive to κ_x. For the slow wave, the z-components of the wave are in phase and therefore the slow wave velocity is insensitive to κ_z. Together, the waves can be thought of as coupled Rayleigh waves. In the limit of low stiffness or high frequency, the fracture behaves as two free surfaces each of which supports a Rayleigh wave. As stiffness is increased, the coupling increases between the two Rayleigh waves, providing two new waves which contain different combinations of the original Rayleigh wave particle motions.

$$\begin{vmatrix} \rho_1(c^2-2\beta_1^2) & -\rho_2(c^2-2\beta_2^2) & -2\rho_1\beta_1^2(1-c^2/\beta_1^2)^{1/2} & 2\rho_2\beta_2^2(1-c^2/\beta_2^2)^{1/2} \\ 2\rho_1\beta_1^2(1-c^2/\alpha_1^2)^{1/2} & 2\rho_2\beta_2^2(1-c^2/\alpha_2^2)^{1/2} & \rho_1\beta_1^2(2-c^2/\beta_1^2) & \rho_2\beta_2^2(2-c^2/\beta_2^2) \\ \{(1-c^2/\alpha_1^2)^{1/2} - (k\rho_1/\kappa_z)(c^2-2\beta_1^2)\} & (1-c^2/\alpha_2^2)^{1/2} & \{1 + (2\beta_1^2 k\rho_1/\kappa_z)(1-c^2/\beta_1^2)^{1/2}\} & 1 \\ \{1 + (2\beta_1^2 k\rho_1/\kappa_x)(1-c^2/\alpha_1^2)^{1/2}\} & -1 & \{(1-c^2/\beta_1^2)^{1/2} + (k\beta_1^2\rho_1/\kappa_x)(2-c^2/\beta_1^2)\} & -(1-c^2/\beta_2^2)^{1/2} \end{vmatrix} = 0$$

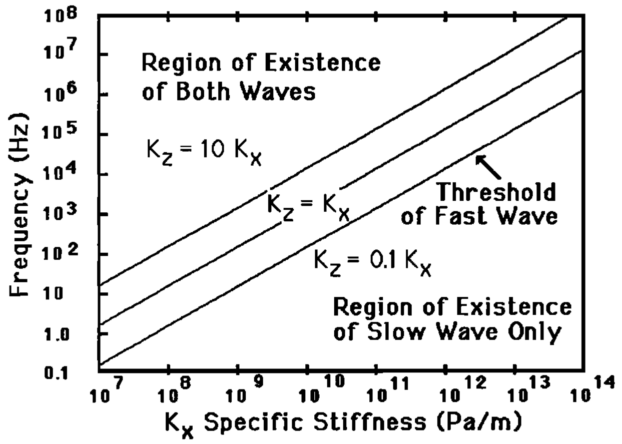

Fig. 3. A graph showing the regions for existence in terms of frequency and specific stiffness in the x-direction, and the ratio of κ_z/κ_x the specific stiffnesses in the x and z directions. The slow wave always exists. $\alpha_1 = \alpha_2 = 5800$ m/s, $\beta_1 = \beta_2 = 3800$ m/s, $\rho_1 = \rho_2 = 2600$ kg/m^3.

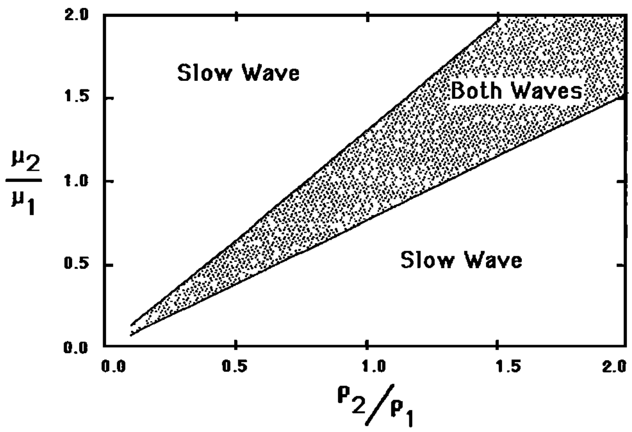

Fig. 5. A graph showing the regions of existence of the slow and fast waves for a wavenumber = 5600 in terms of the ratio of the shear moduli and the ratio of the densities. $\alpha_1 = 5358$ m/s, $\beta_1 = 3800$ m/s, $\rho_1 = 2600$ kg/m^3, $\kappa_x = \kappa_z = 10^9$ Pa/m. Poisson's ratio = 0.25.

Figure 3 shows the regions of existence for the waves in terms of frequency as a function of the specific stiffness in the x direction along the fracture. It was found that the slow wave exists for all frequencies and all stiffnesses. However, the region of existence of the fast wave depends on frequency and the relationship between κ_x and κ_z. In Figure 3, threshold curves are drawn for the cases where $\kappa_x = \kappa_z$, $\kappa_x = 0.1\ \kappa_z$, and $\kappa_x = 10\ \kappa_z$. As κ_z/κ_x increases, the threshold of existence for the fast waves moves to higher frequencies.

Existences curves were also determined for the case where the material properties of the elastic half-spaces are not equal. Curves are given for wavenumbers of k = 0.056 (Figure 4) and k = 5600 (Figure 5), which roughly corresponds to frequencies of 200 Hz and 20 MHz. Each material was assigned a Poisson's ratio of 0.25 ($\lambda = \mu$). Existence was investigated for different ratios of shear moduli (μ_2/μ_1) and density (ρ_2/ρ_1). When the material properties of the media are not equal, the existence of both waves depends on frequency. As the excitation frequency is decreased, the region of existence of both waves diminishes and a region where neither wave exists appears.

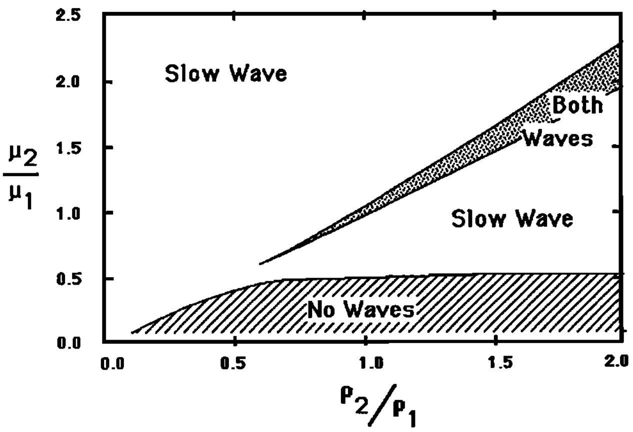

Fig. 4. A graph showing the regions of existence of the slow and fast waves for a wavenumber = 0.056 in terms of the ratio of the shear moduli and the ratio of the densities showing. $\alpha_1 = 5358$ m/s, $\beta_1 = 3800$ m/s, $\rho_1 = 2600$ kg/m^3, $\kappa_x = \kappa_z = 10^9$ Pa/m. Poisson's ratio = 0.25.

Conclusions

Elastic interface waves can exist along a non-welded contact between two media having the same material properties. The "slow" wave always exists, while, the "fast" wave has a threshold of existence which depends on the excitation frequency and the relationship between components of the specific stiffness parallel and perpendicular to the fracture. Both waves are weakly dispersive. When the material properties of the media on either side of the fracture are not equal, the existence of both waves is frequency dependent and depends on the ratio of the shear moduli.

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