Lecture 2: Research Results

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Purdue University

Image 27

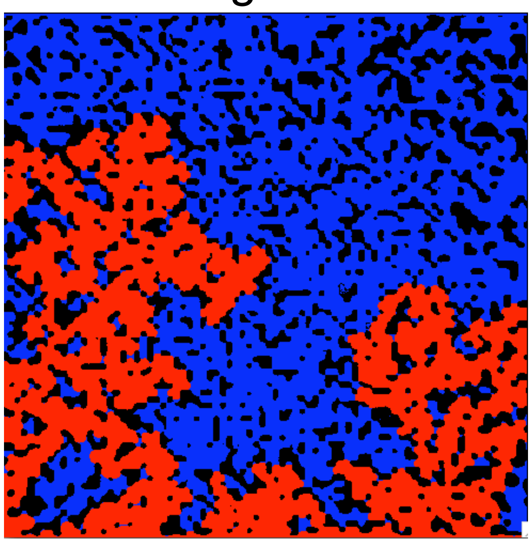


Image 42

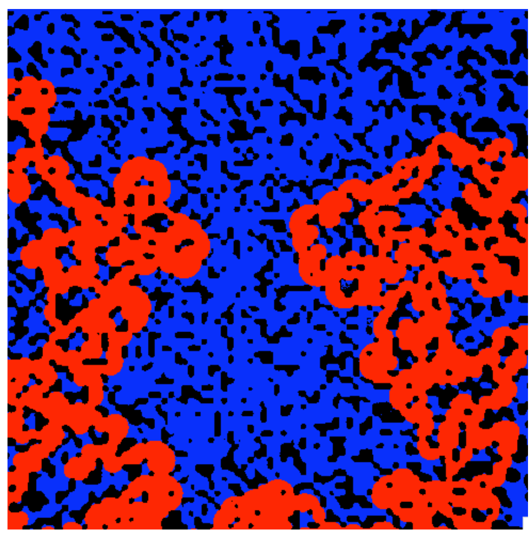
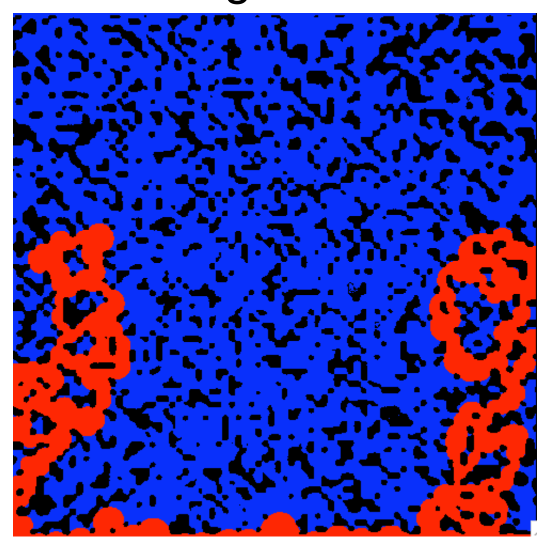
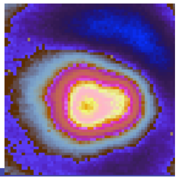


Image 91



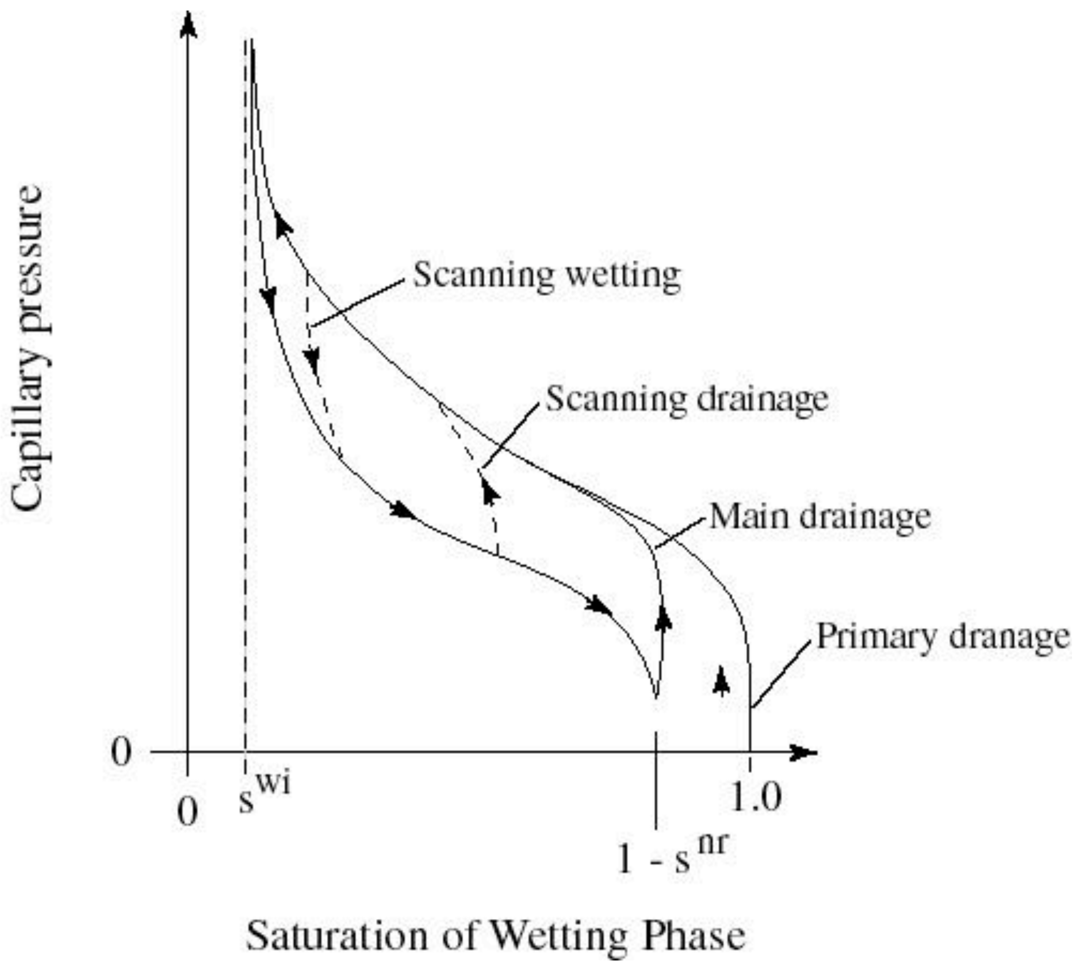
Objective

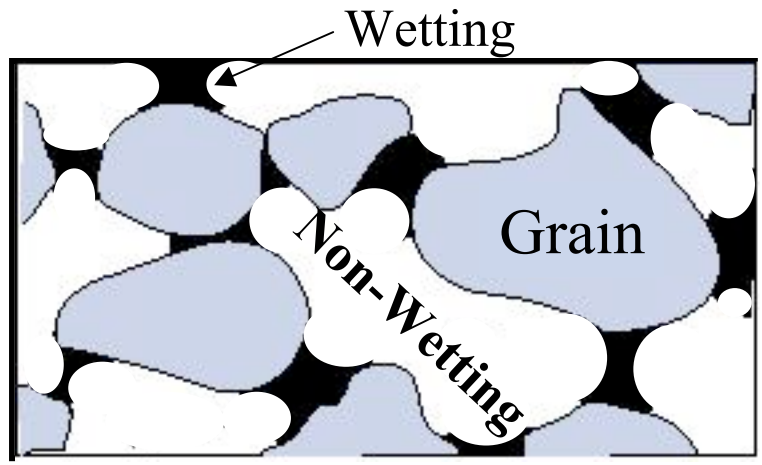


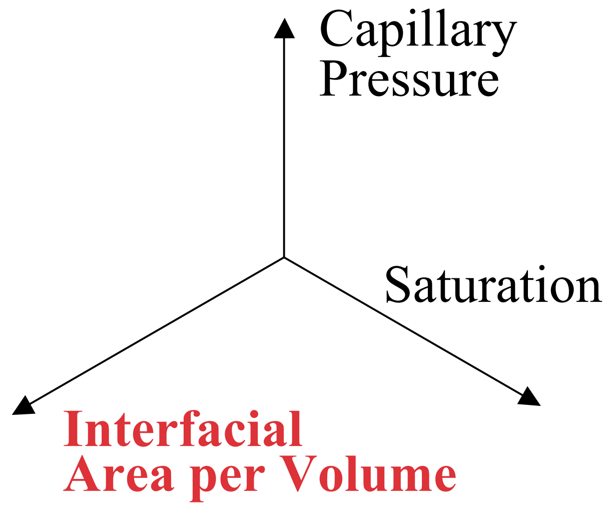
To test theoretical hypotheses on the relationships among capillary pressure, saturation and interfacial area per volume…

… using laboratory data from micro-models of porous media.

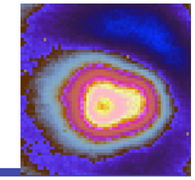
Fluid Distributions







Topics:



*Does IAV lift the ambiguity in the P_{cap} - S relationship?

* How is the P_{cap}- *S*- *IAV* surface affected by porosity?

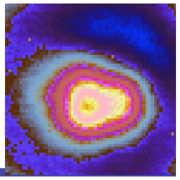
* Do points collected from drainage scans and points collected from imbibition scans lie on the same surface?

* Will the data satisfy the following equation:

$$P^c = -\frac{1}{\phi}\left[\gamma^{wn} \left(\frac{\partial a^{wn}}{\partial s^w} \right) \Bigg|_{T,\phi,\Gamma^{\alpha\beta},A^{\alpha\beta}} + \gamma^{ns} \left(\frac{\partial a^{ns}}{\partial s^w} \right) \Bigg|_{T,\phi,\Gamma^{\alpha\beta},A^{\alpha\beta}} + \gamma^{ws} \left(\frac{\partial a^{ws}}{\partial s^w} \right) \Bigg|_{T,\phi,\Gamma^{\alpha\beta},A^{\alpha\beta}} \right]$$

* Current Directions: Simple Wedge, Micro-capacitors, 3D

Topics:



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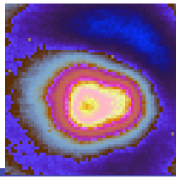
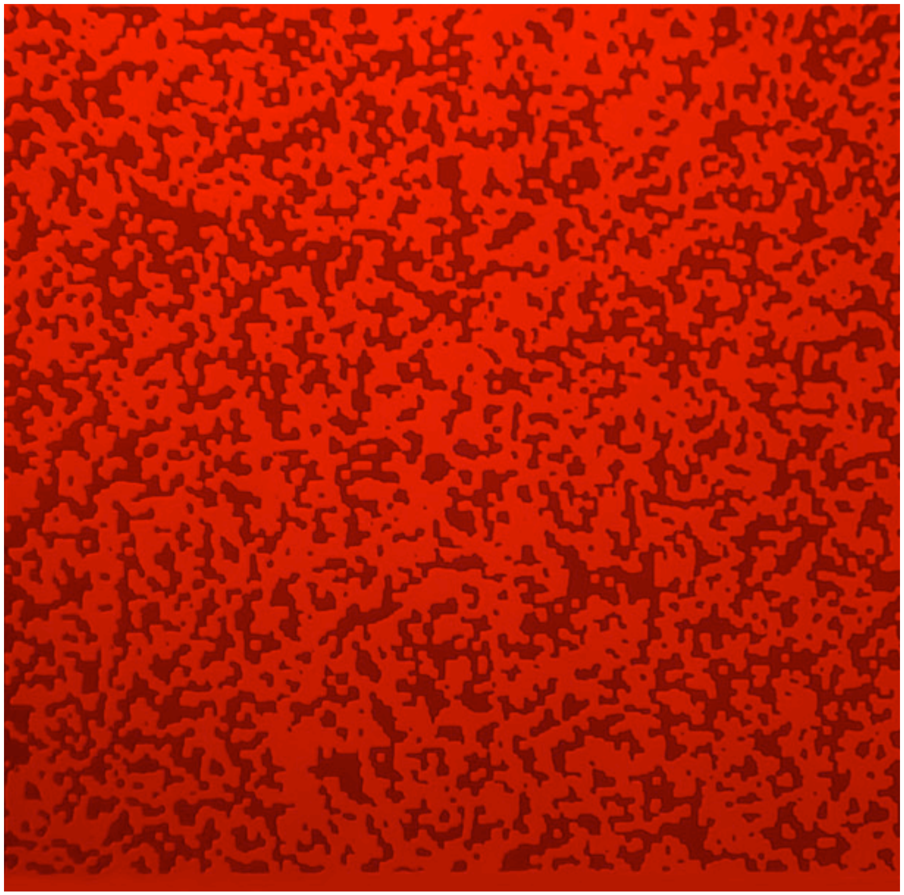
Experimental Approach



- ❖Photo-projection lithography
- ❖Displacement measurements
- ❖Image Analysis

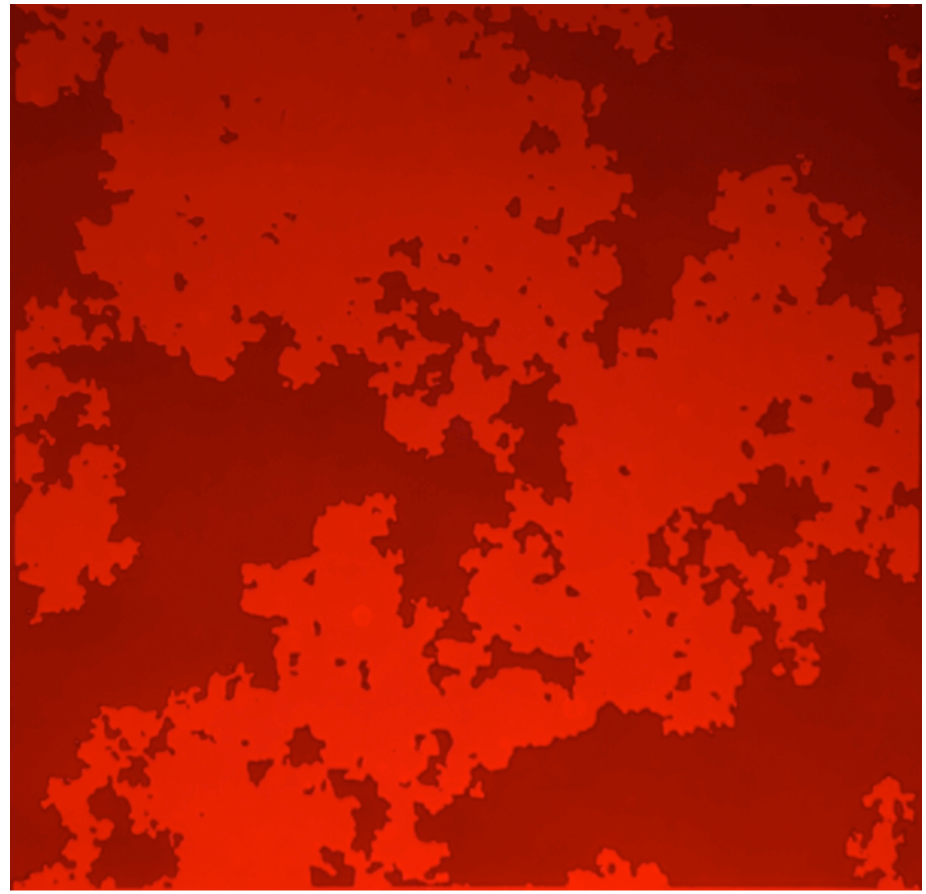
Simulated Pore Structures

Spatially Uncorrelated



600 μm

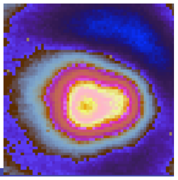
Spatially Correlated



Aperture ~ 1.08 μm

Porosity ~ 64%

Fluid Invasion: Nitrogen into Decane



Decane-Nitrogen - Micro-model System

Surface Tension:

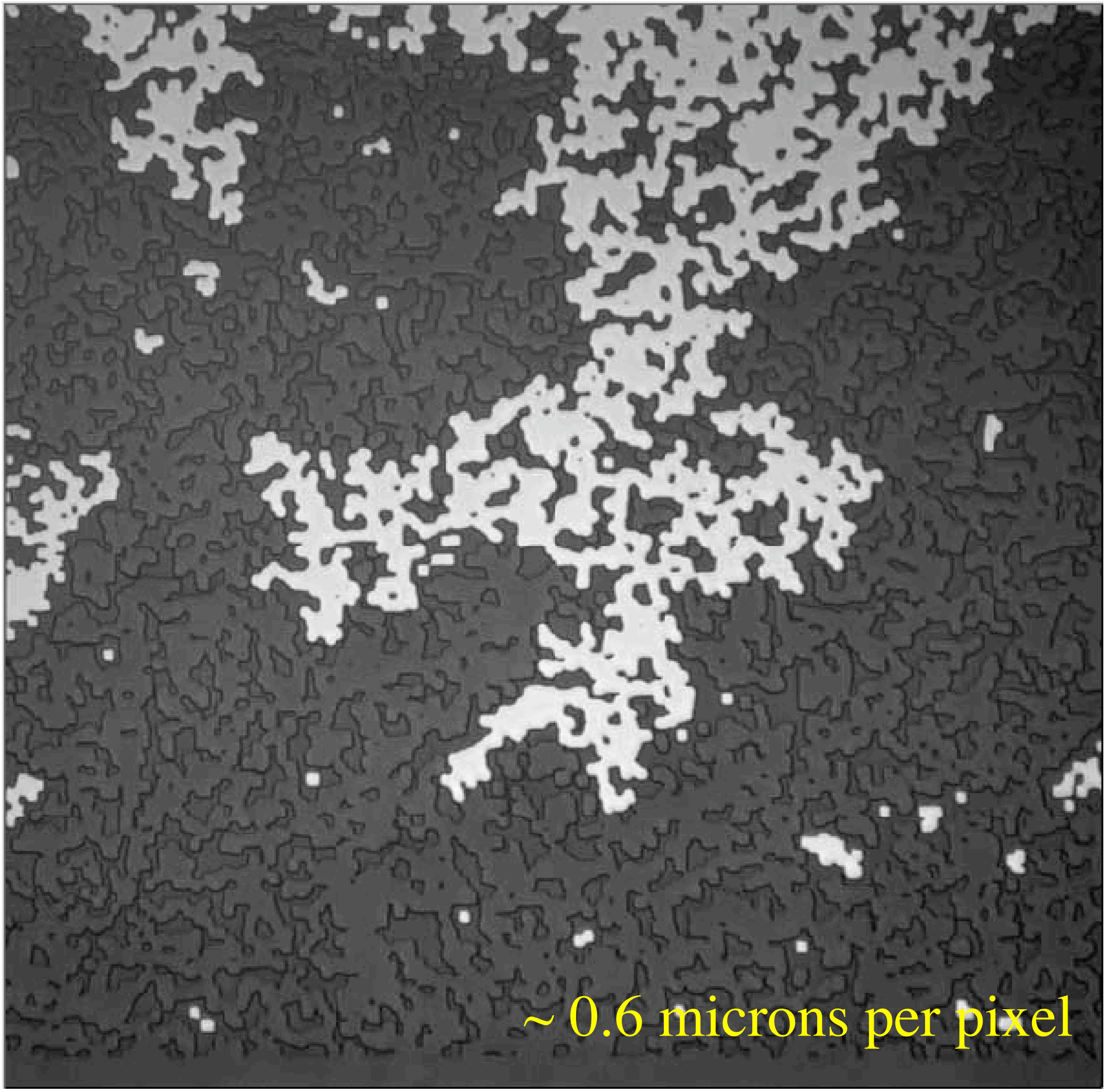
24.7 dynes/cm

Contact Angle:

Glass: 4.4^{o}

Photoresist: 4.1^{o}

Fluid Invasion: Nitrogen into Decane



Data: Fluid Invasion of Nitrogen into Decane

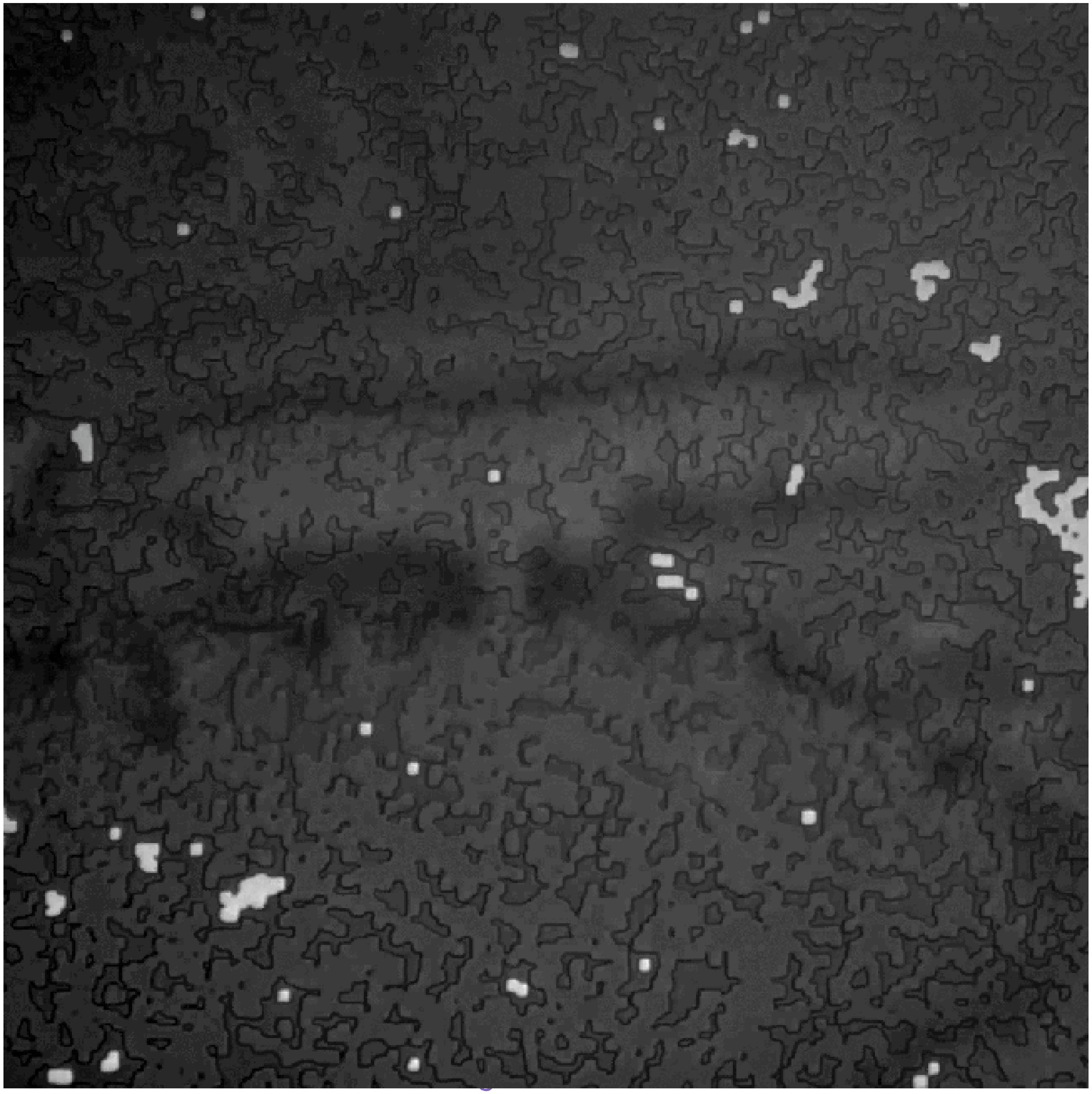
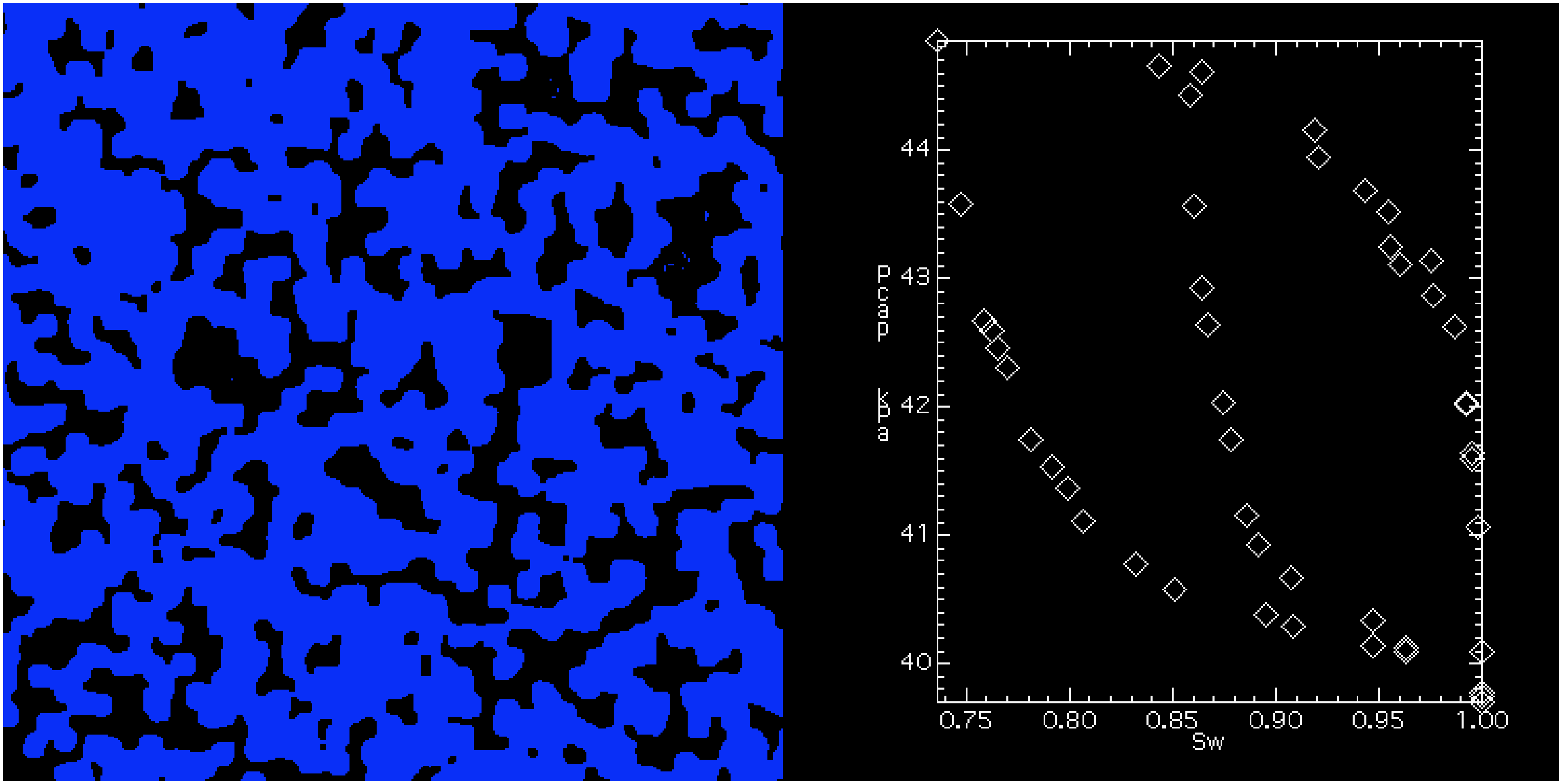


Image Analysis

Image Analysis:

- Phase Identification
- Interface Identification
- Curvature

Capillary Pressure - Wetting Phase Saturation



Black - Photoresist **Maroon - Nitrogen** **Blue - Decane**

White - Capillary Dominated **Bright Red - All pressures**

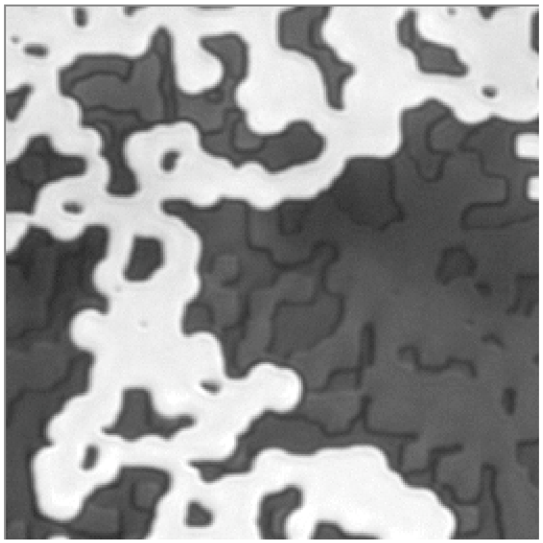
Methods for Analyzing IAV

Method 1: Assumes that wetting fluid leaves a thin film. There is no contact between non-wetting fluid and solid.

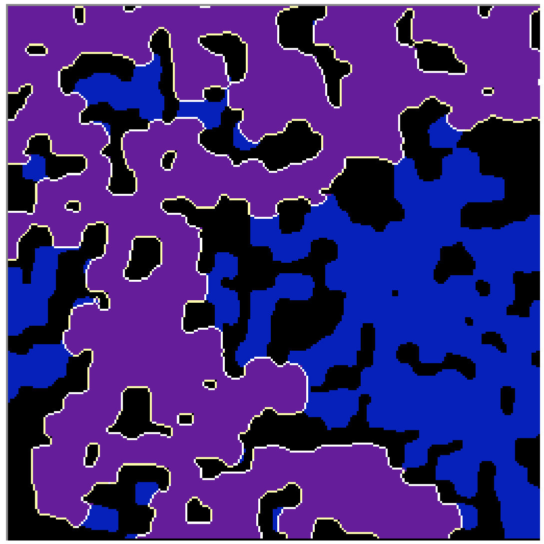
Method 2: Wetting phase film thinner than 2 microns (optical resolution scale) is discounted. The non-wetting fluid can be in contact with the solid.

Wetting - Non-wetting Interfaces

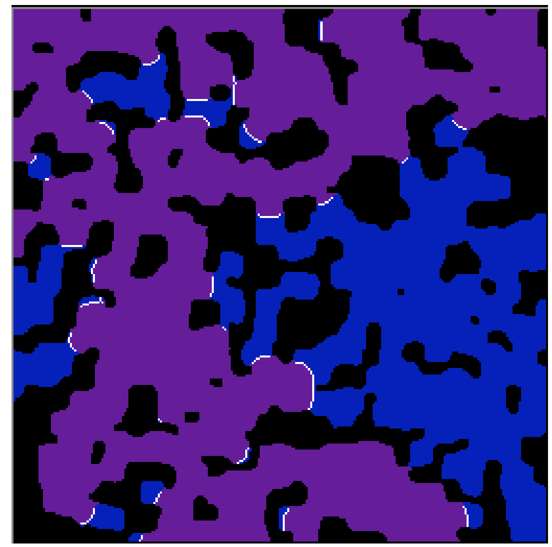
Raw Image



Method 1
Including Films



Method 2



Dark Gray - Photoresist

White - Nitrogen

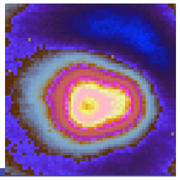
Gray - Decane

Black - Photoresist

Purple - Nitrogen

Blue - Decane

White - Interface Decane-Nitrogen



From Image Analysis:

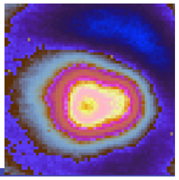
- Saturation
- Interfacial Length
- Curvature

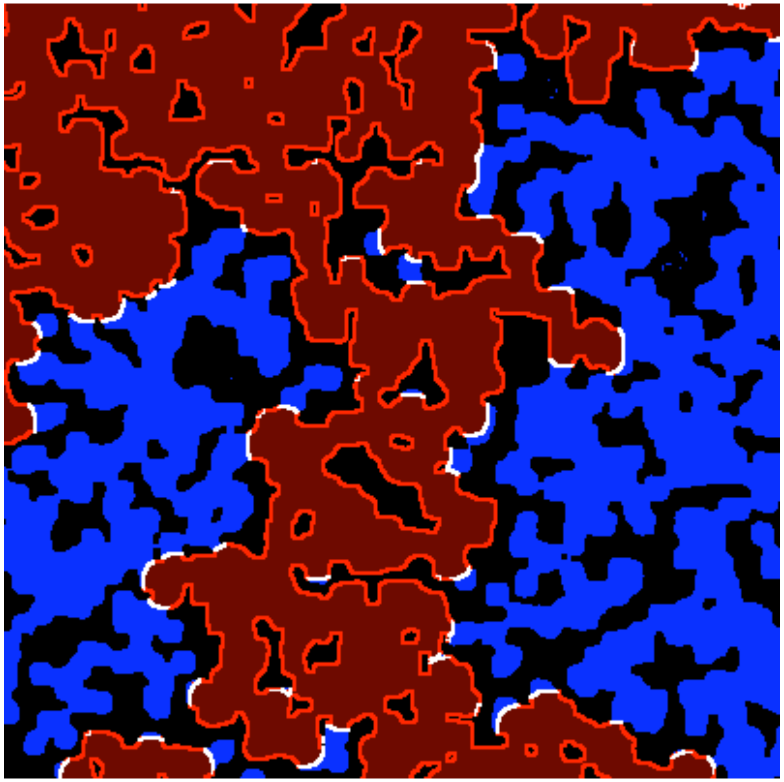
Curvature:

$$K = \frac{\Phi_y^2 \Phi_{xx} - 2\Phi_x \Phi_y \Phi_{xy} + \Phi_x^2 \Phi_{yy}}{\sqrt[3]{\Phi_x^2 + \Phi_y^2}}$$

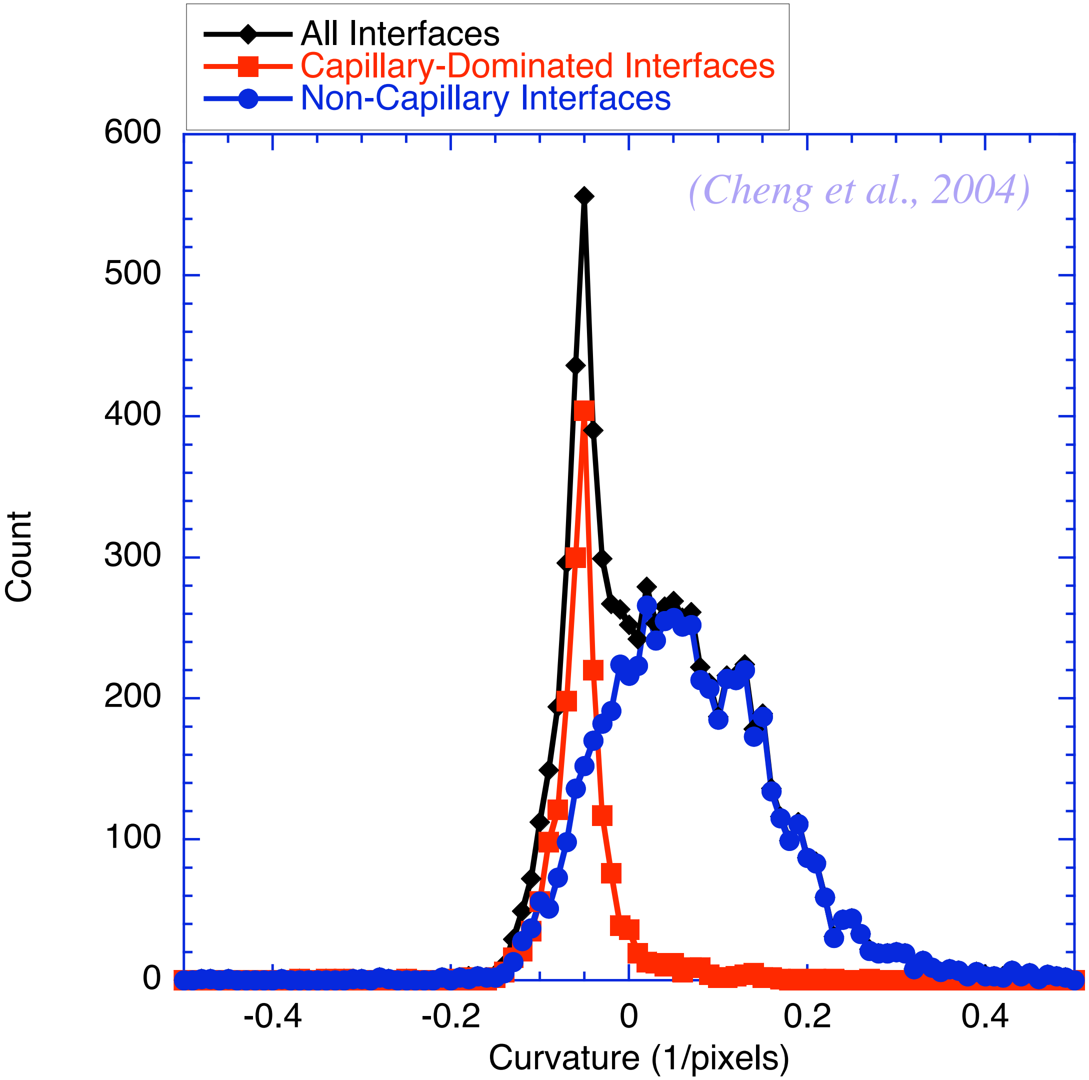
(Sethian, 1985)

Image Analysis: Curvature Analysis





Black - Photoresist
Maroon - Nitrogen
Blue - Decane
White - Capillary-Dominated Interfaces
Bright Red - Other Interfaces

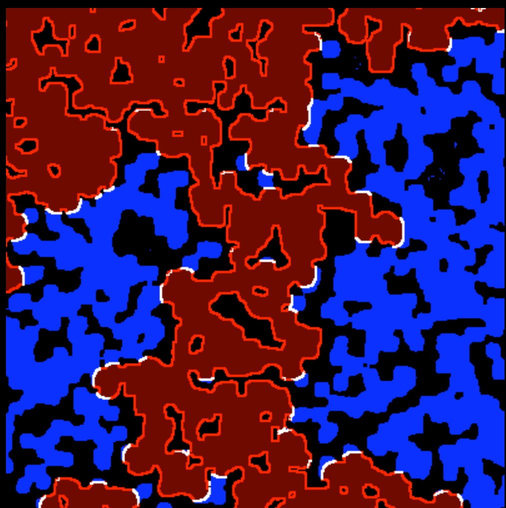


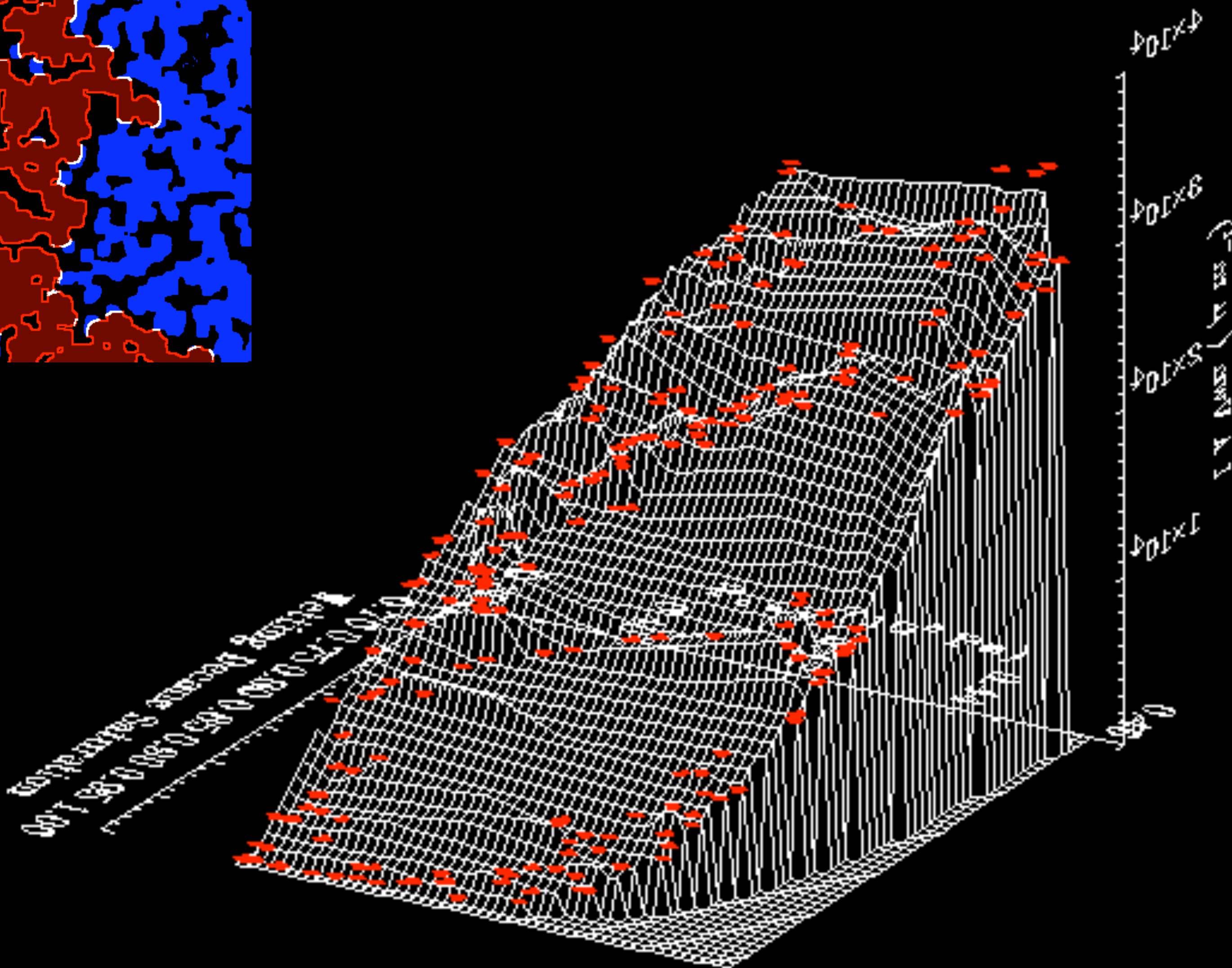
S8c.histogram.kld

S8.c.18

Spatially Uncorrelated: For All Interfaces: Includes Films

Capillary Pressure - Wetting Saturation - IAVwn relationship





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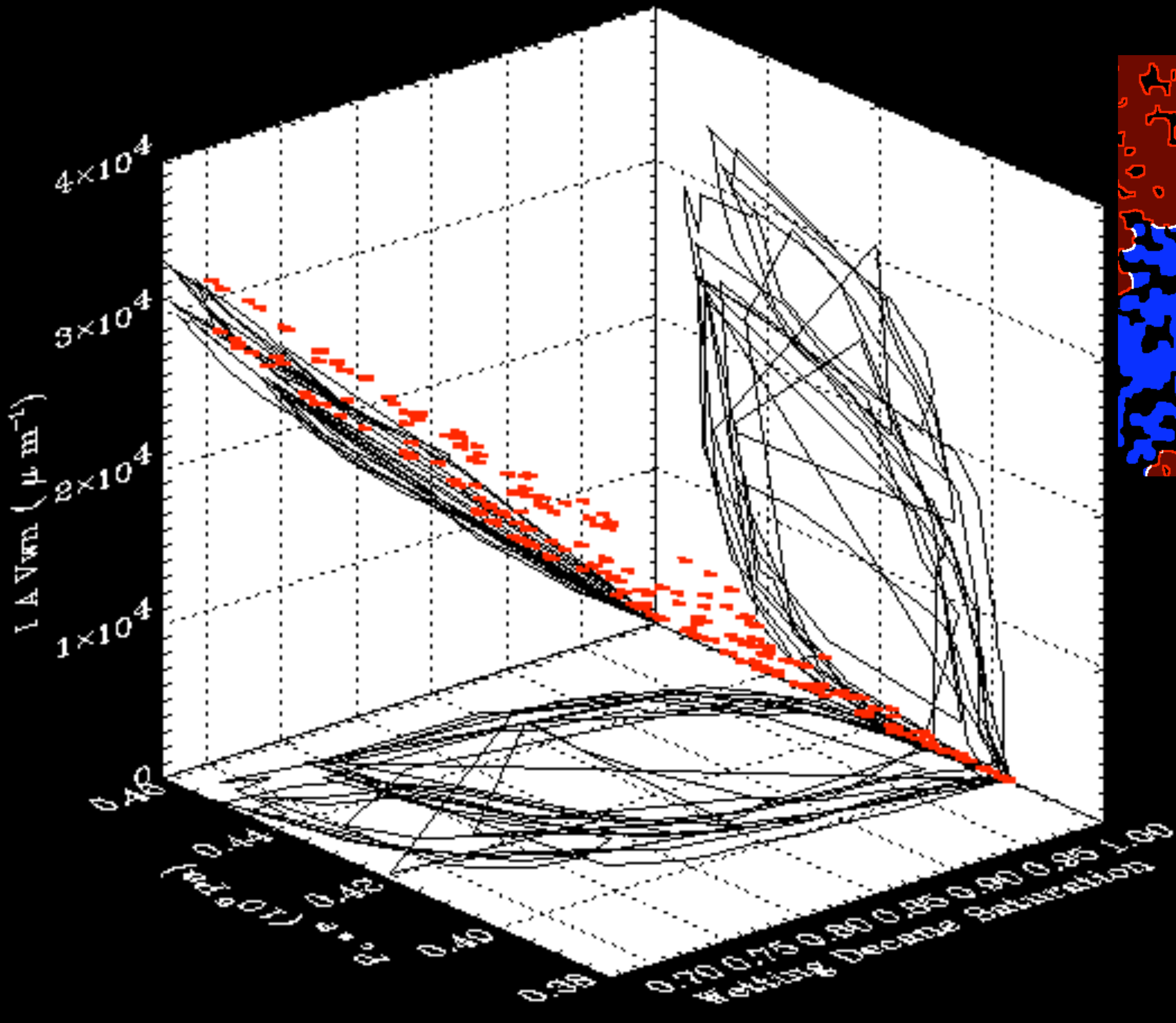
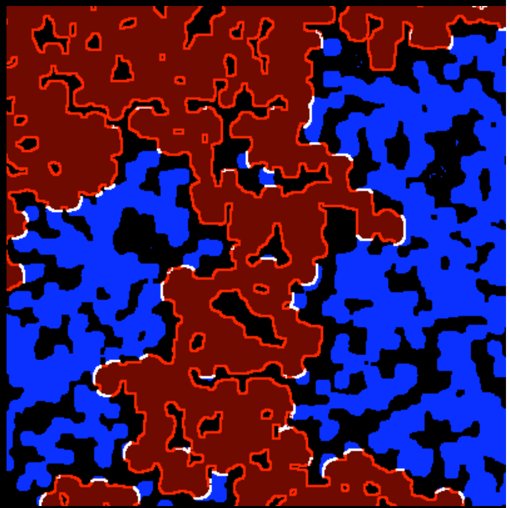
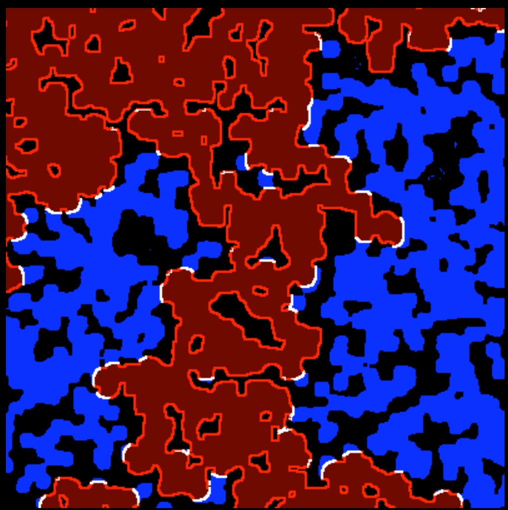
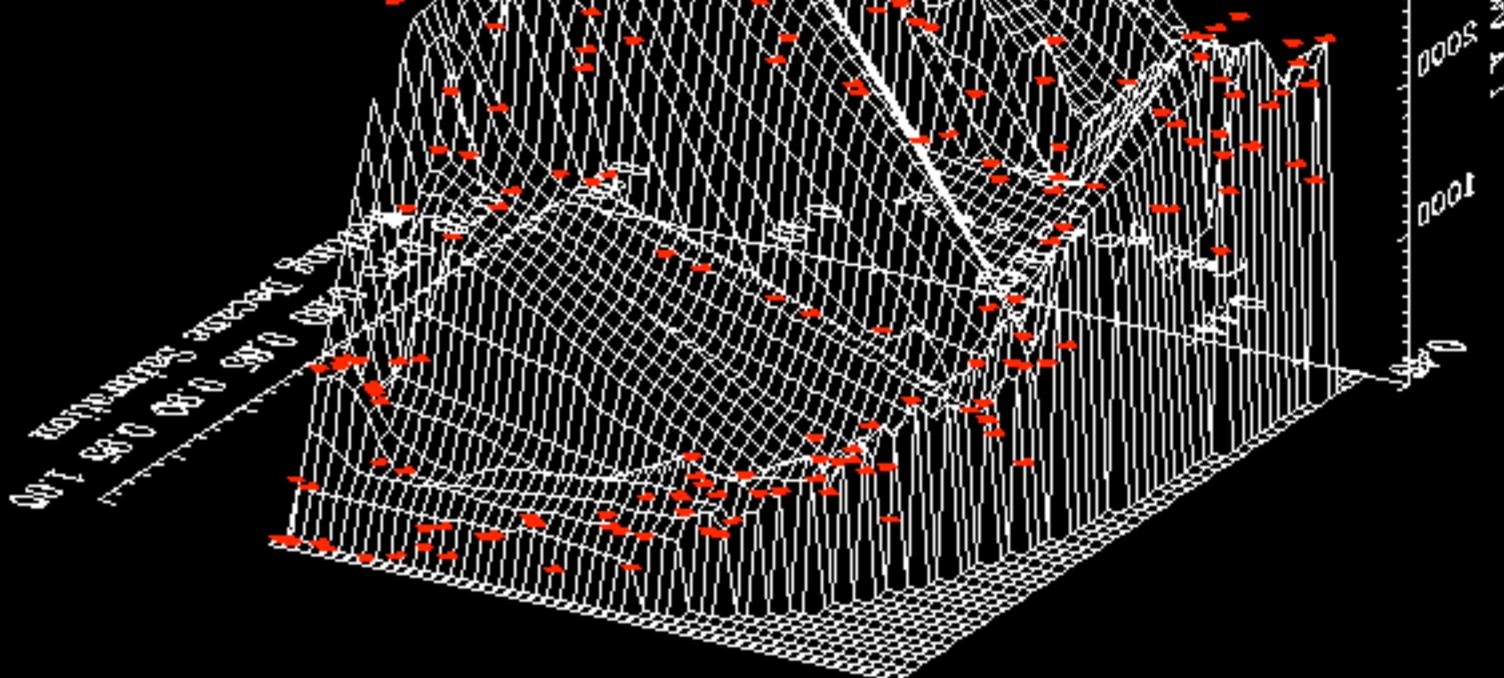




Spatially Uncorrelated: For Capillary Dominated Interfaces:

Capillary Pressure - Wetting Saturation - IAVwn relationship





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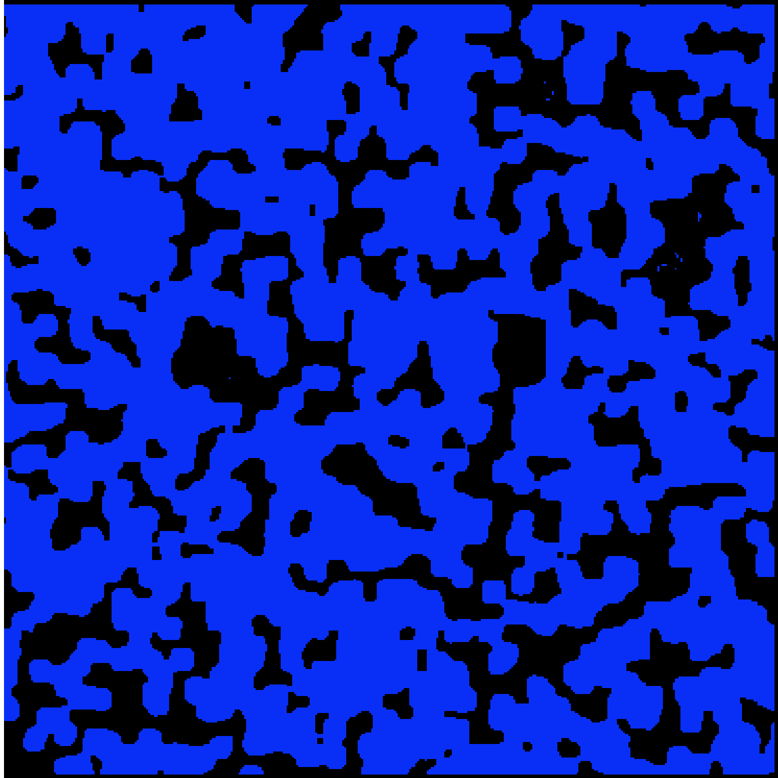

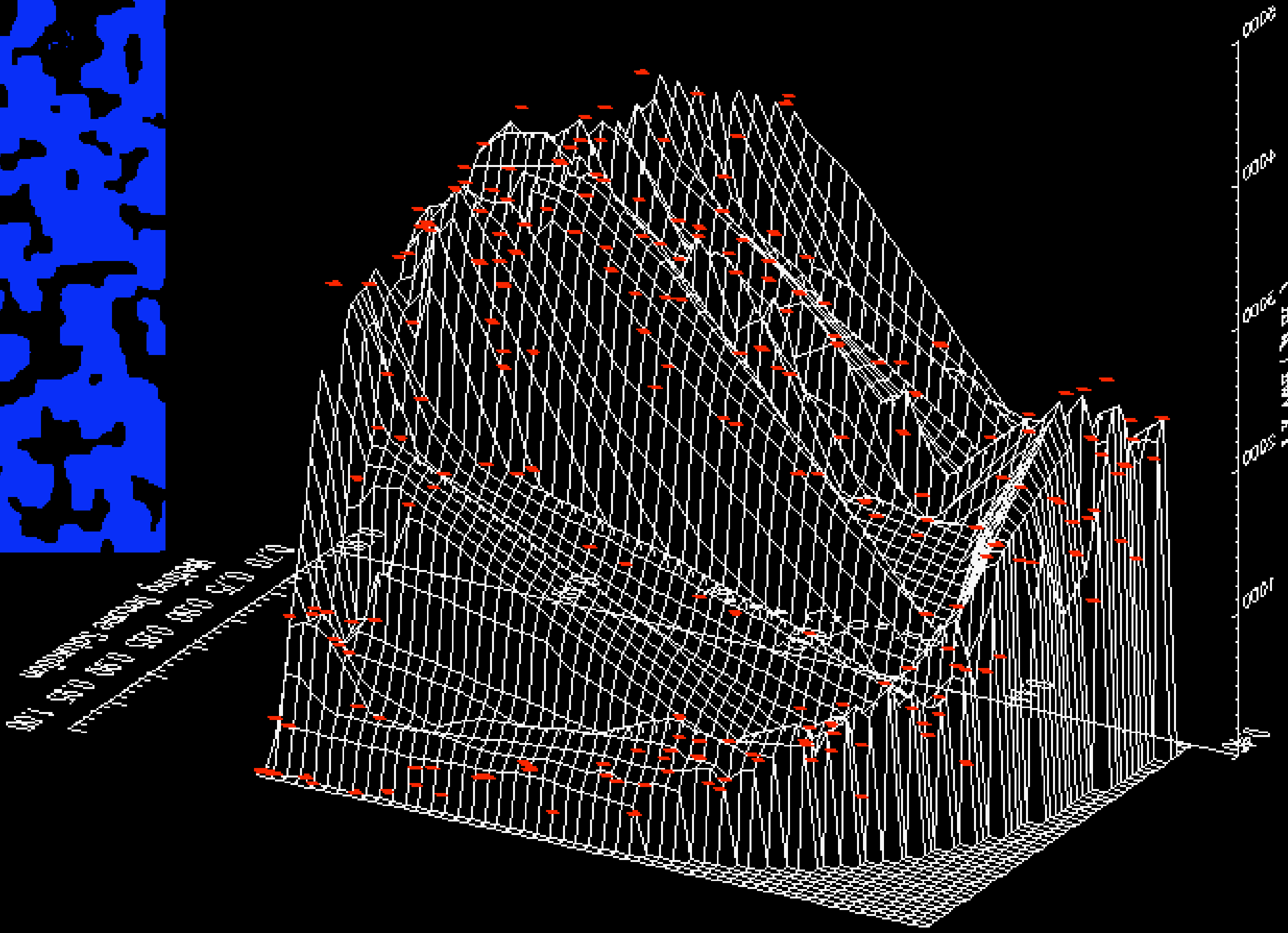

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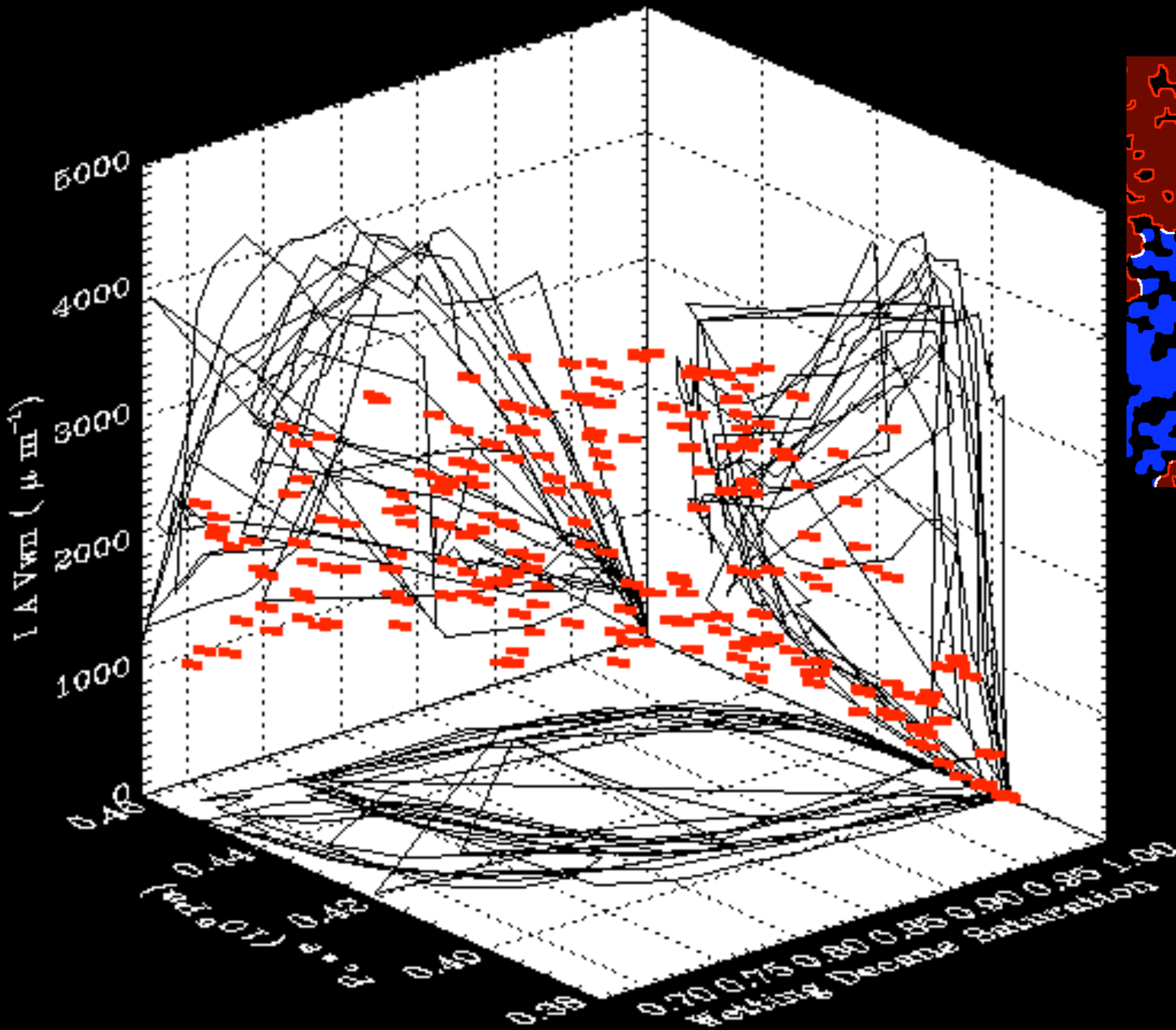
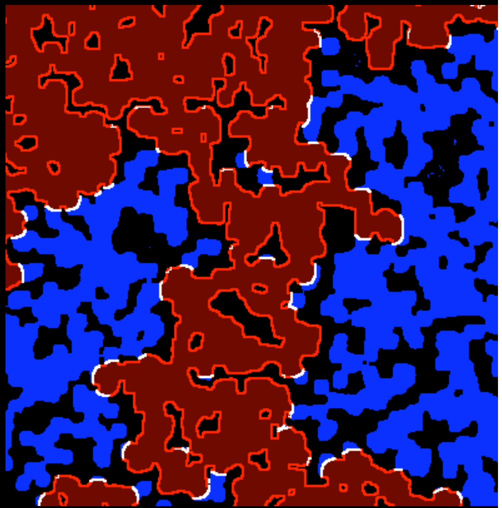
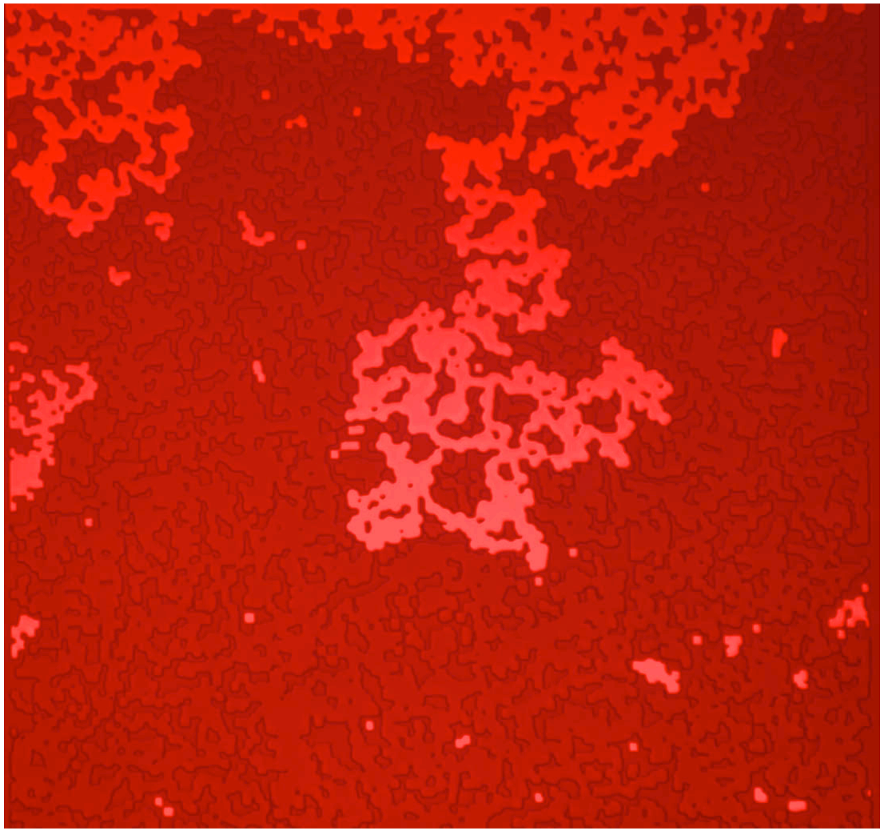




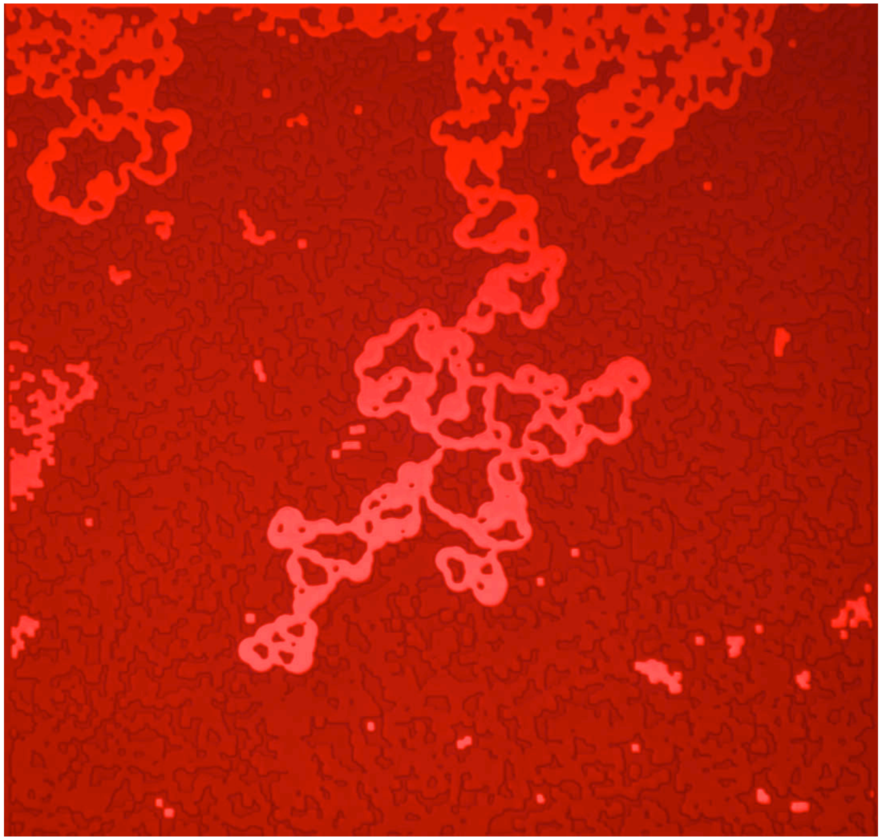
Same Saturation



Saturation: 0.75

Pressure: 43.8 kPa

IAV: 1727 m $^{-1}$



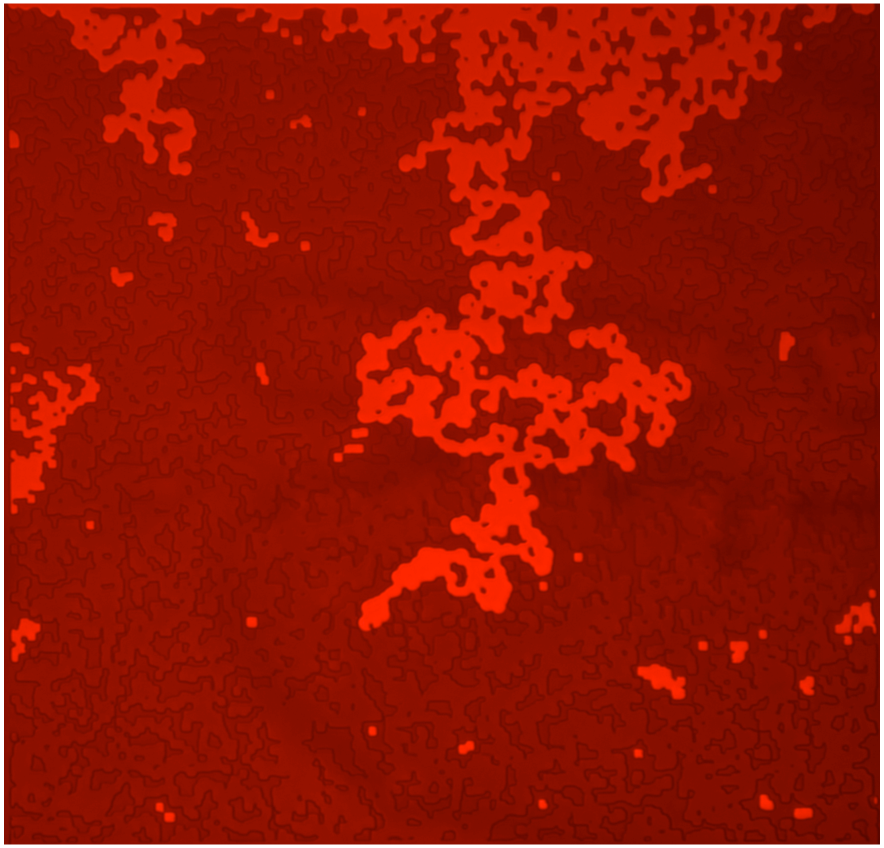
Saturation: 0.75

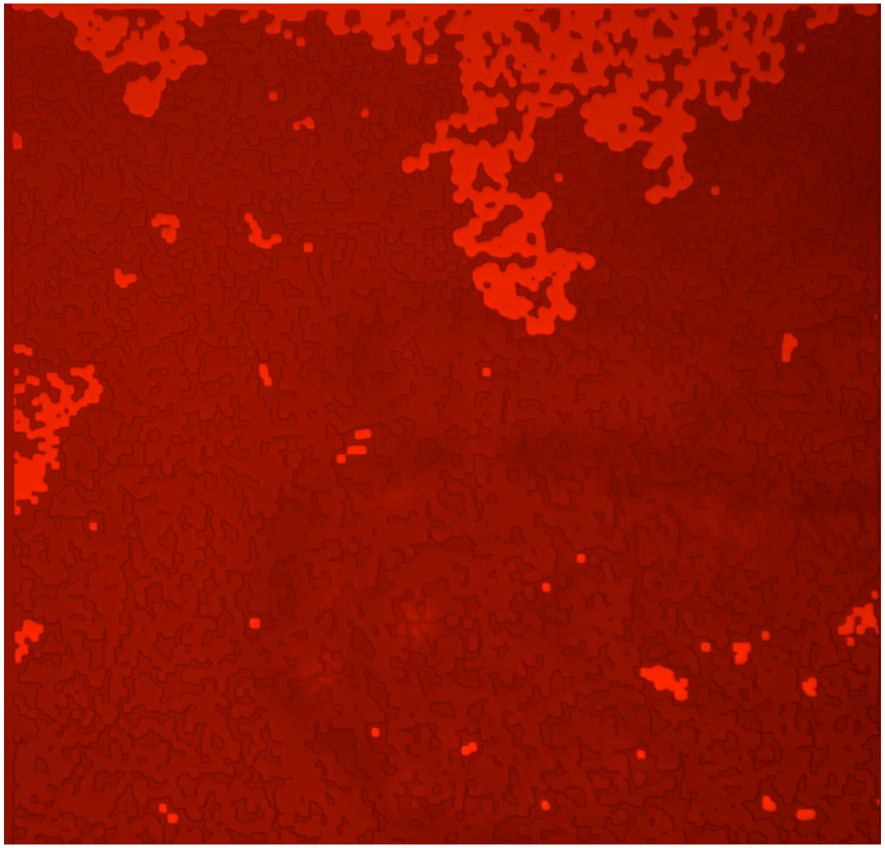
Pressure: 41.6 kPa

IAV: 3247 m^{-1}

Sample S8g

Same Pressures





Saturation: 0.76

Pressure: 42.67 kPa

IAV: 2393 m^{-1}

Saturation: 0.88

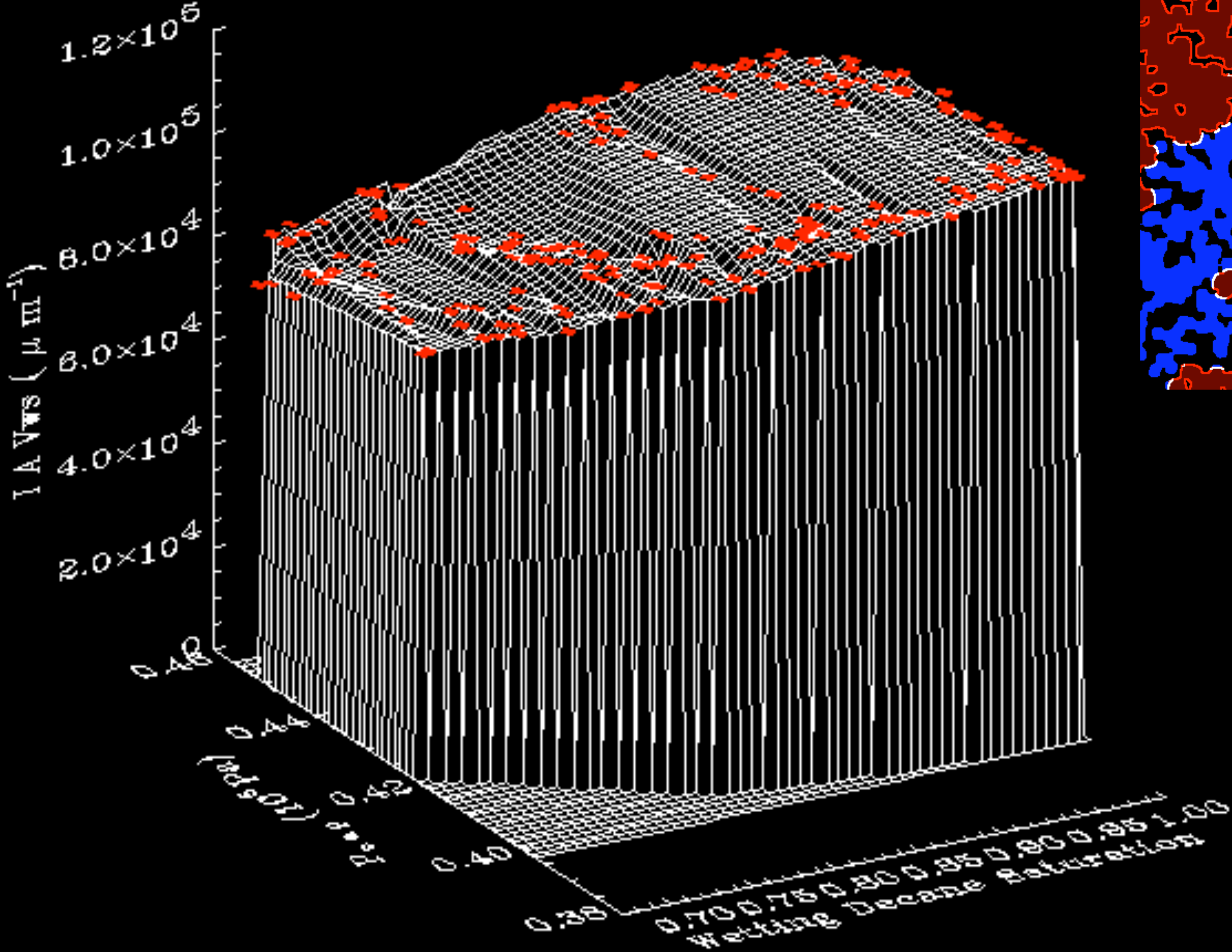
Pressure: 42.64 kPa

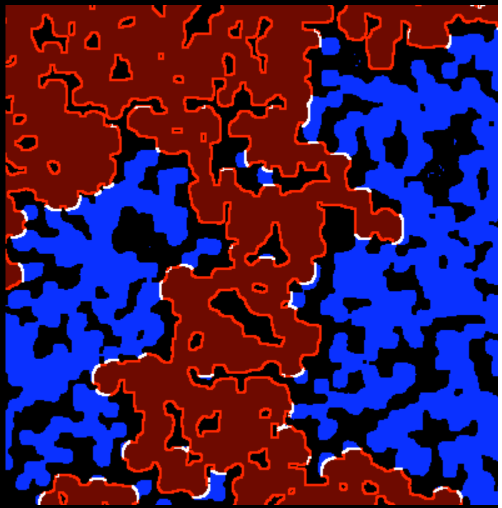
IAV: 1240 m^{-1}

Sample S8c

Spatially Uncorrelated: For Capillary Dominated Interfaces:

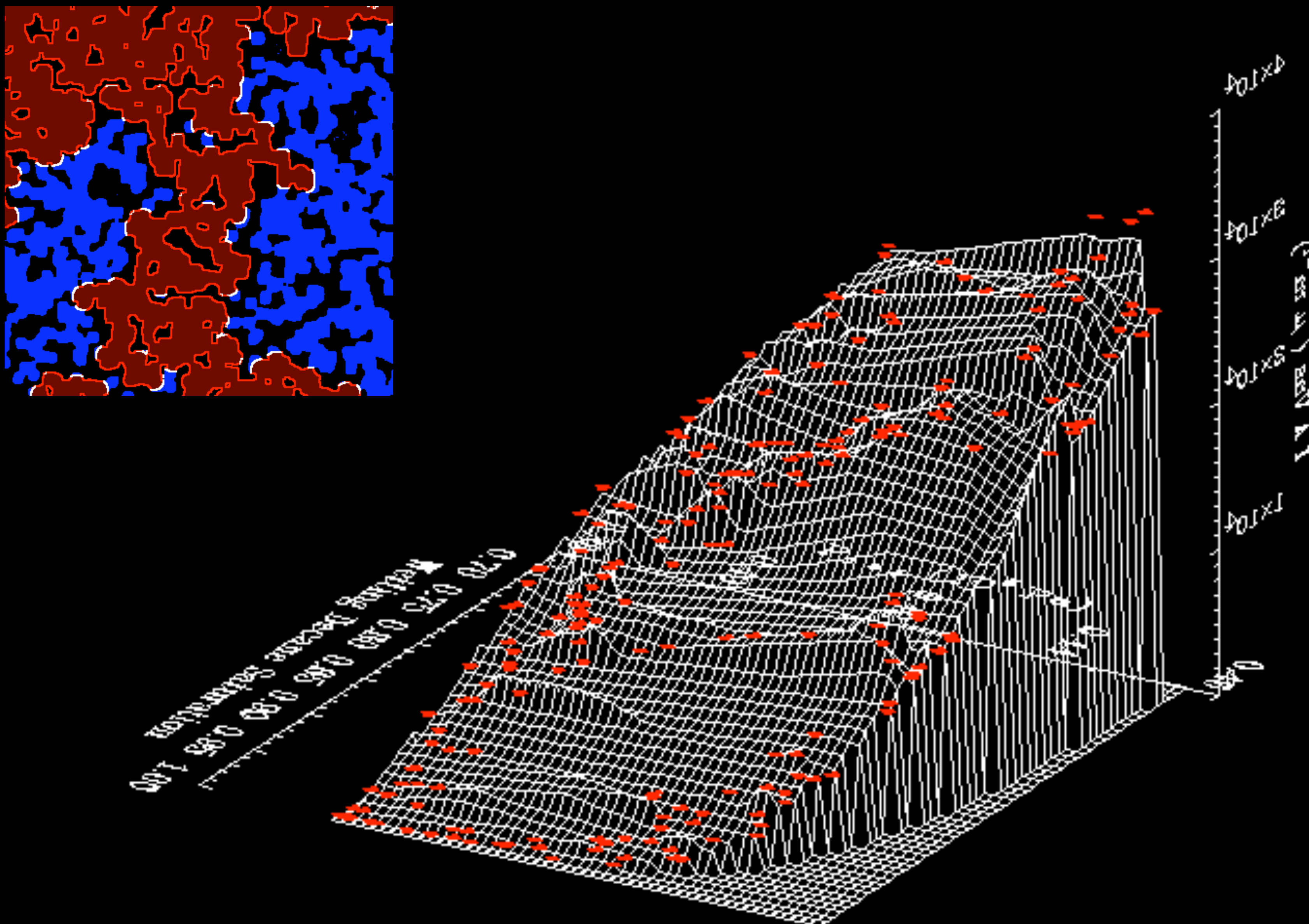
Capillary Pressure - Wetting Saturation - IAVws relationship





Spatially Uncorrelated: For Capillary Dominated Interfaces:

Capillary Pressure - Wetting Saturation - IAVns relationship



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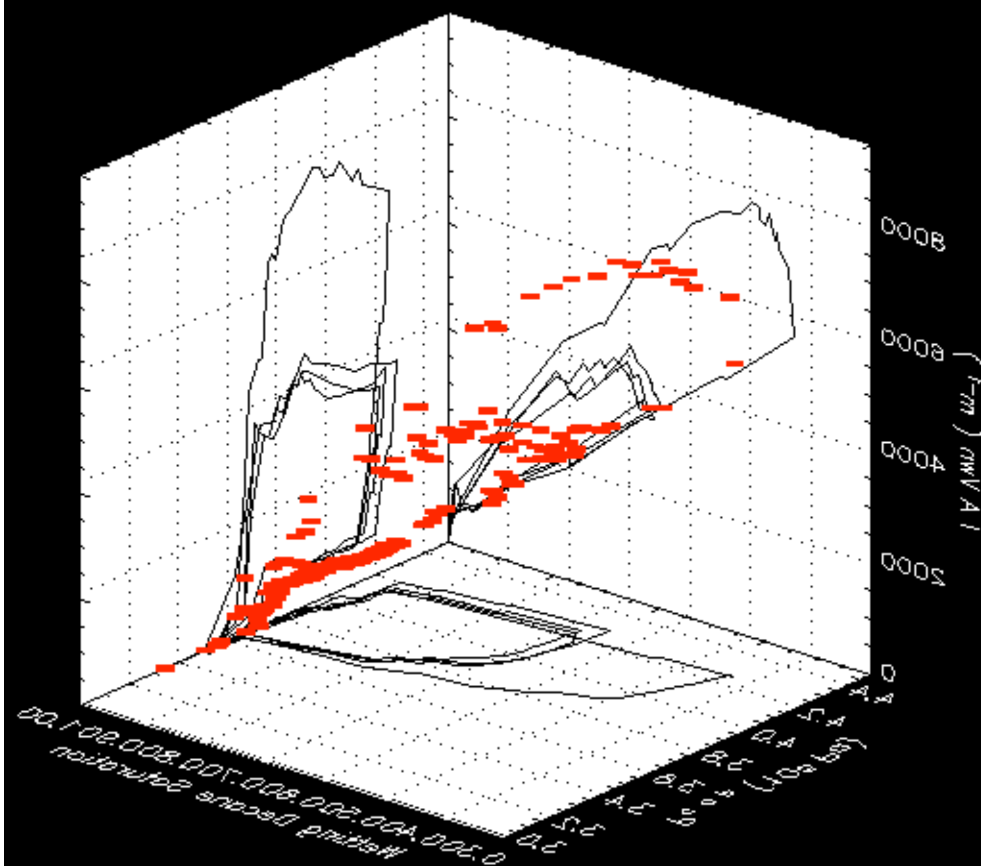
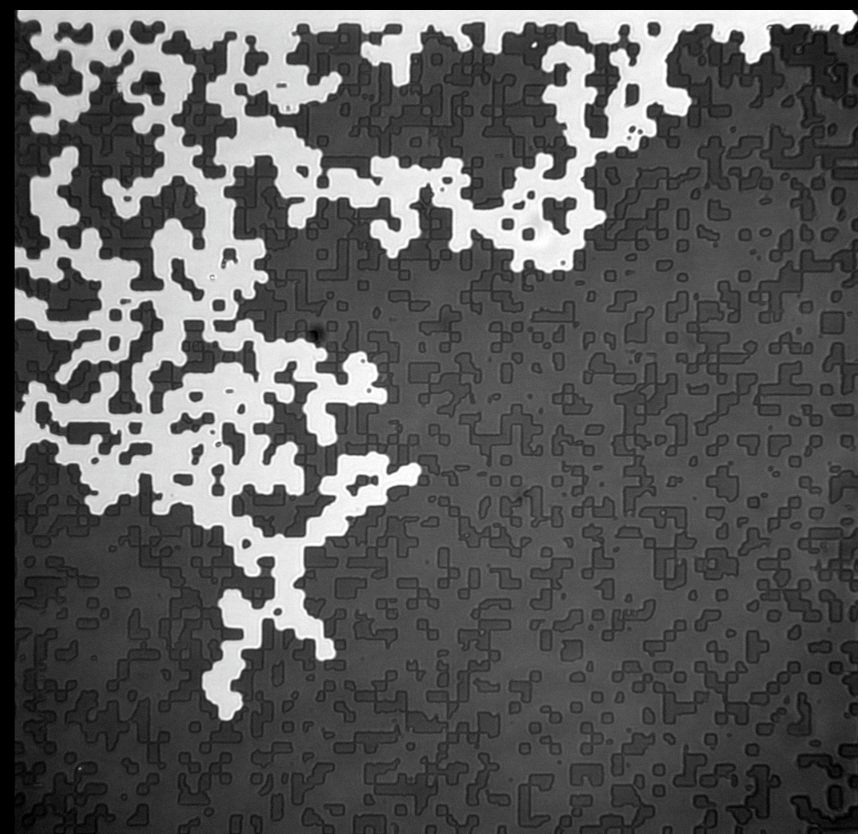
* Current Directions: Simple Wedge, Micro-capacitors, 3D

Samples

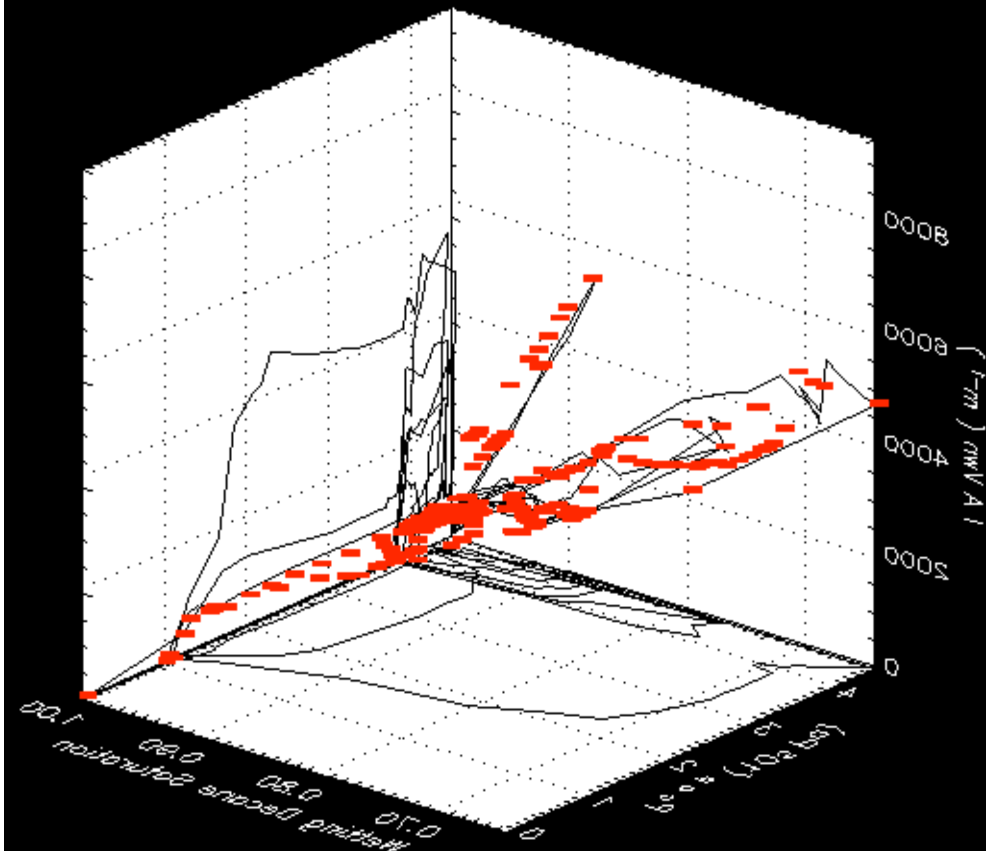
Sample ID	Desired Porosity	Real Porosity
S70	0.70	0.67
S75	0.75	0.70
S80	0.80	0.78
S85	0.85	0.80
S90	0.90	0.90
S95	0.95	0.91

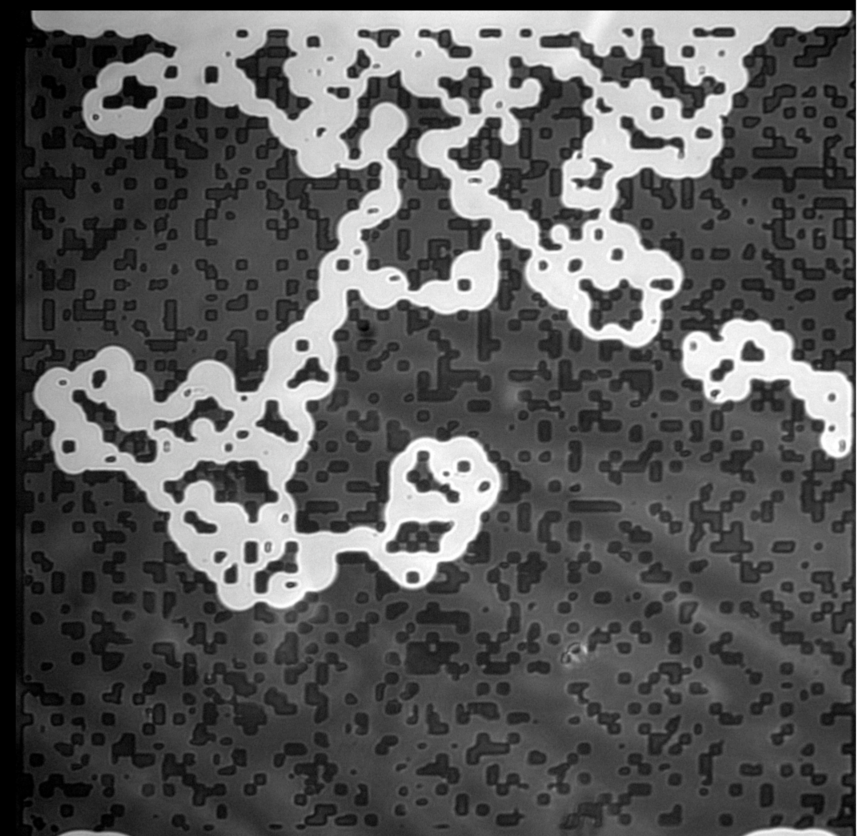
Sample: s70 with porosity = 67%



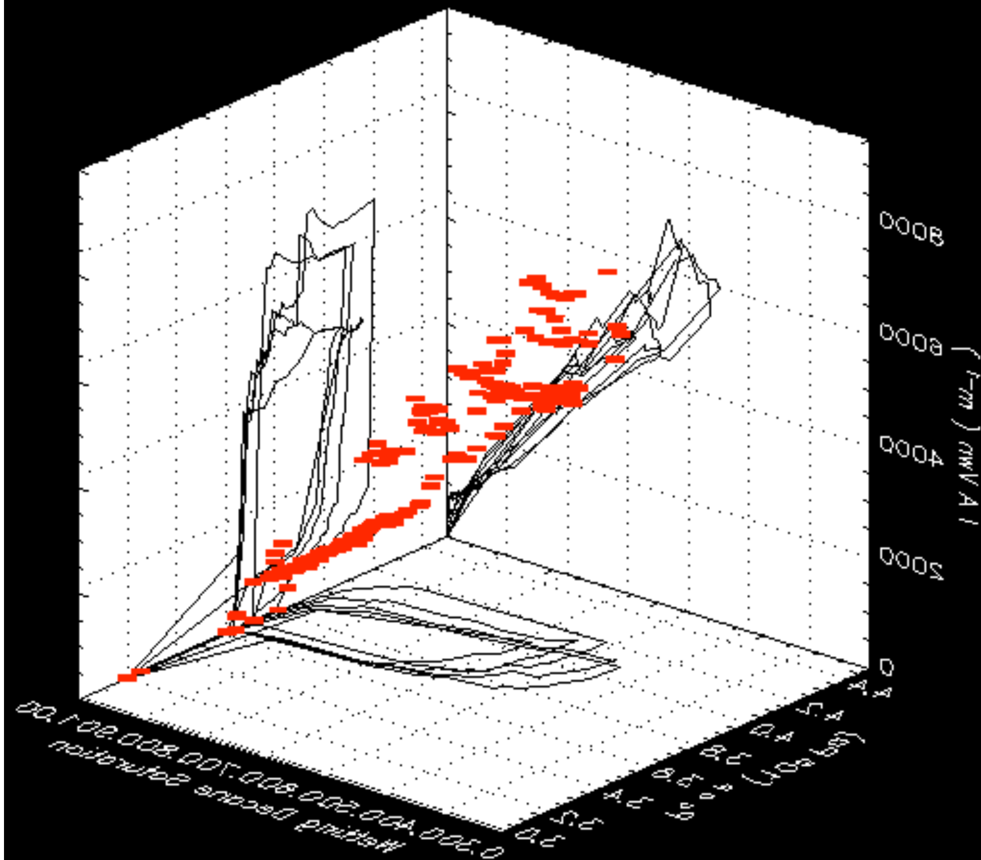


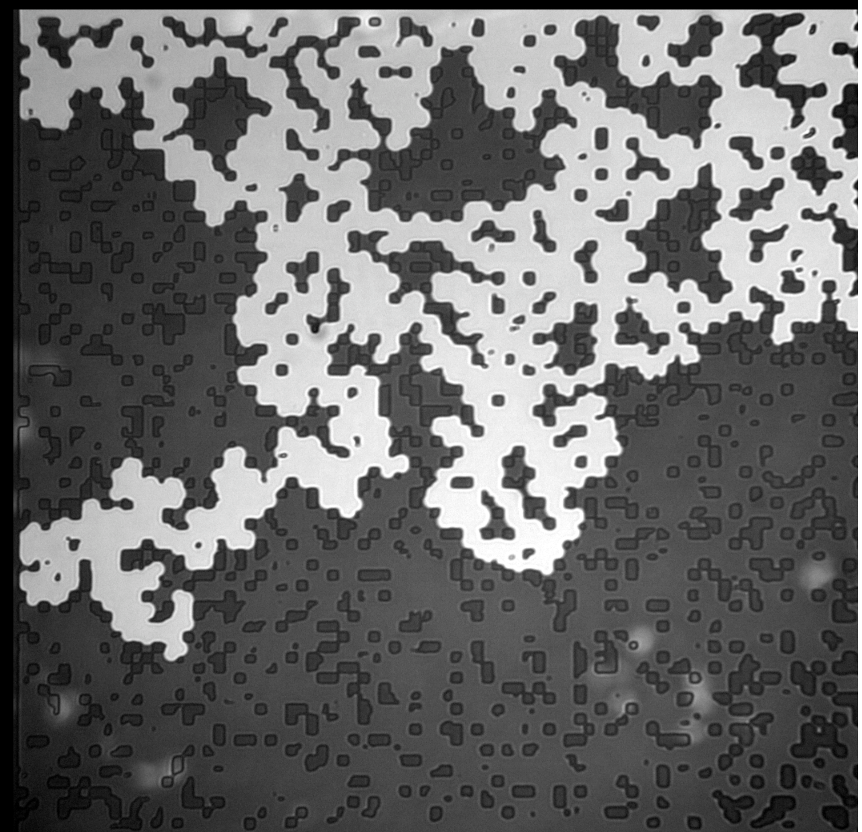
Sample: s75 with porosity = 70%



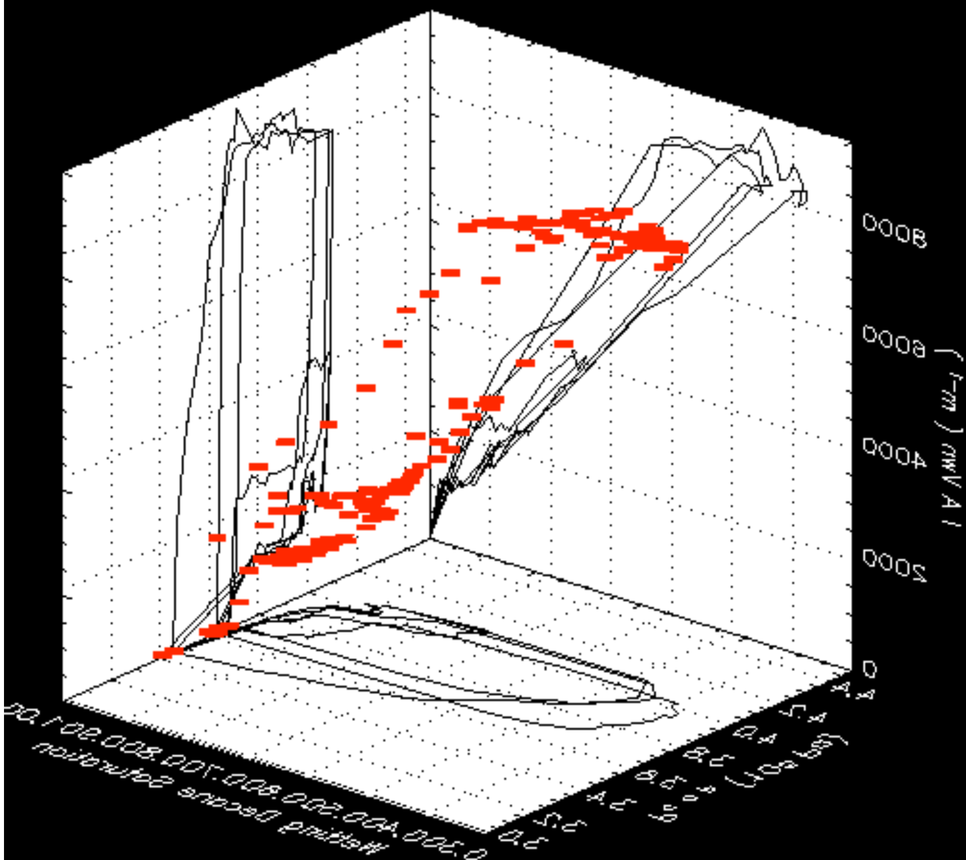


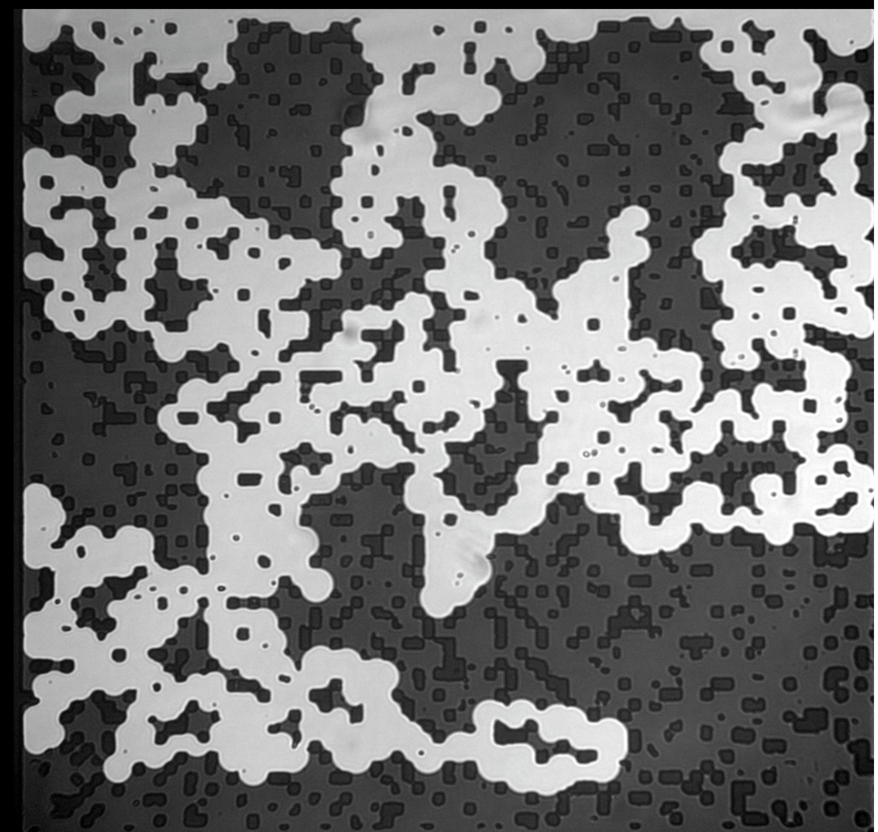
Sample: s80 with porosity = 78%



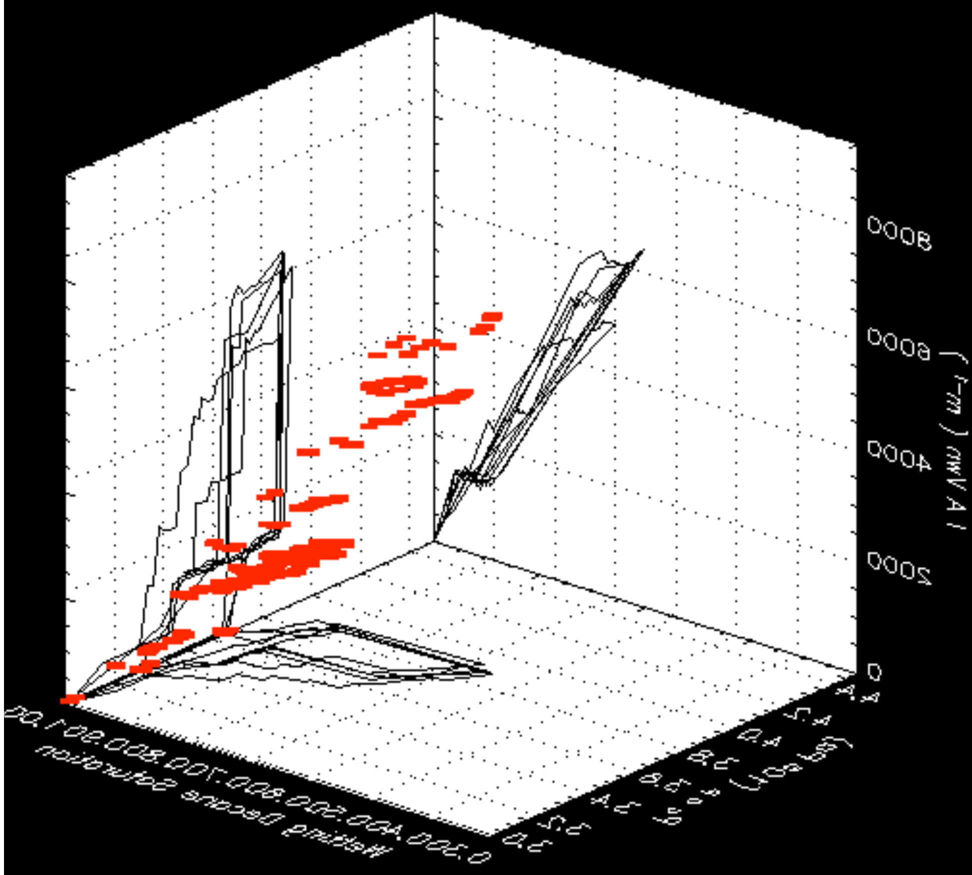


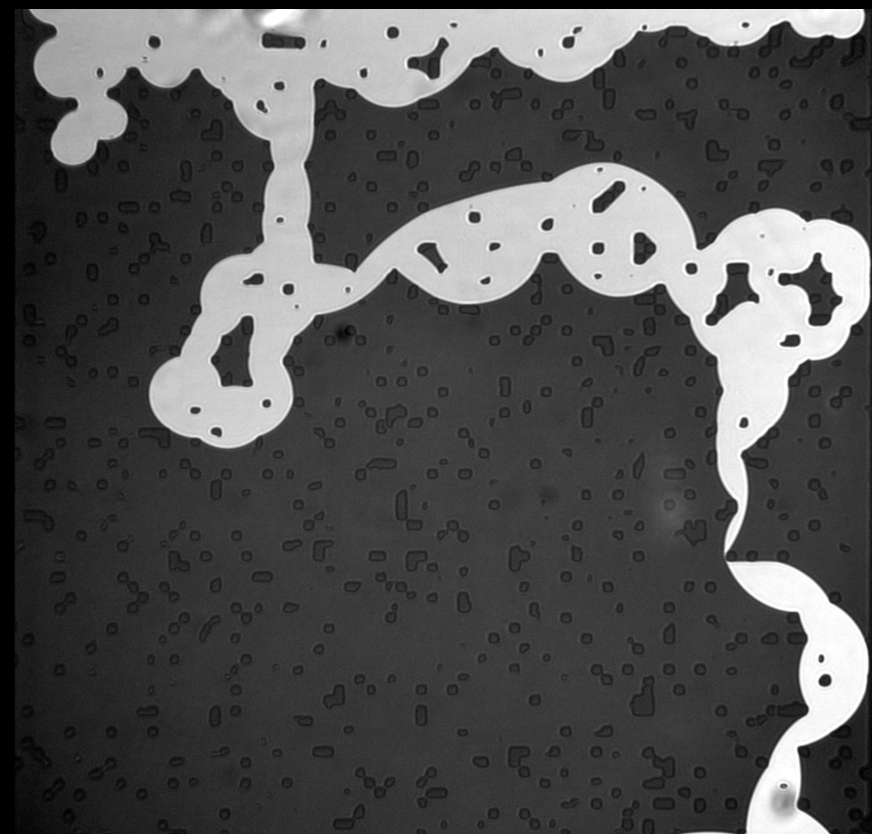
Sample: s85 with porosity = 80%



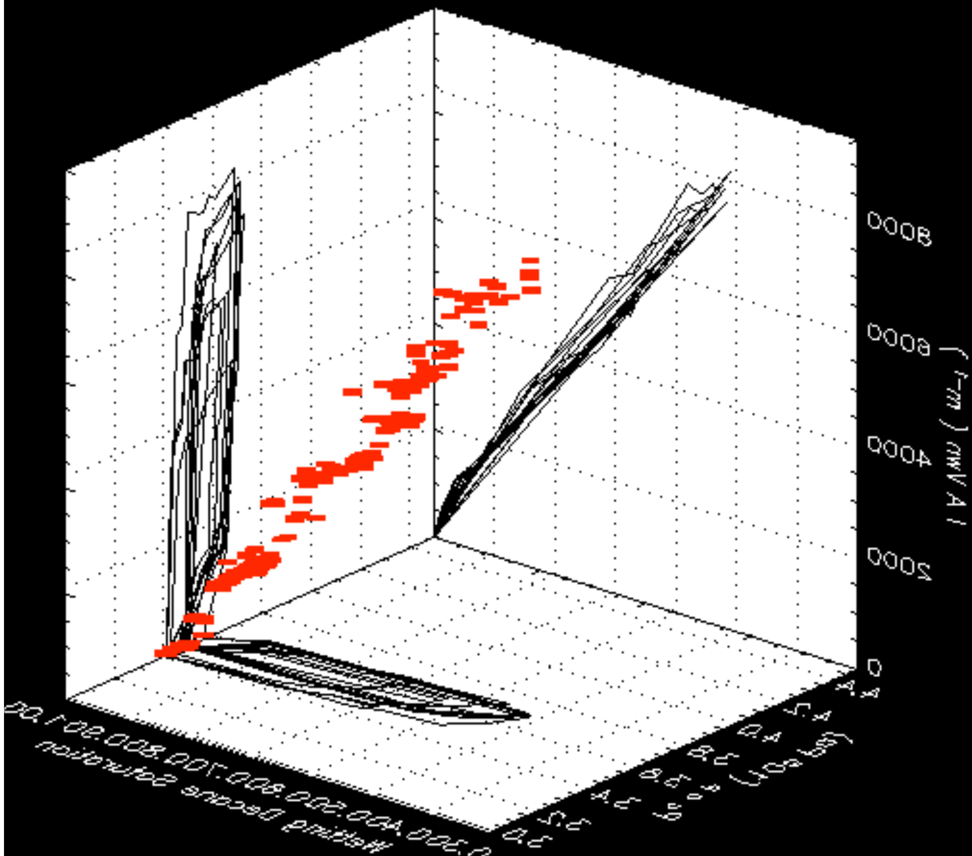


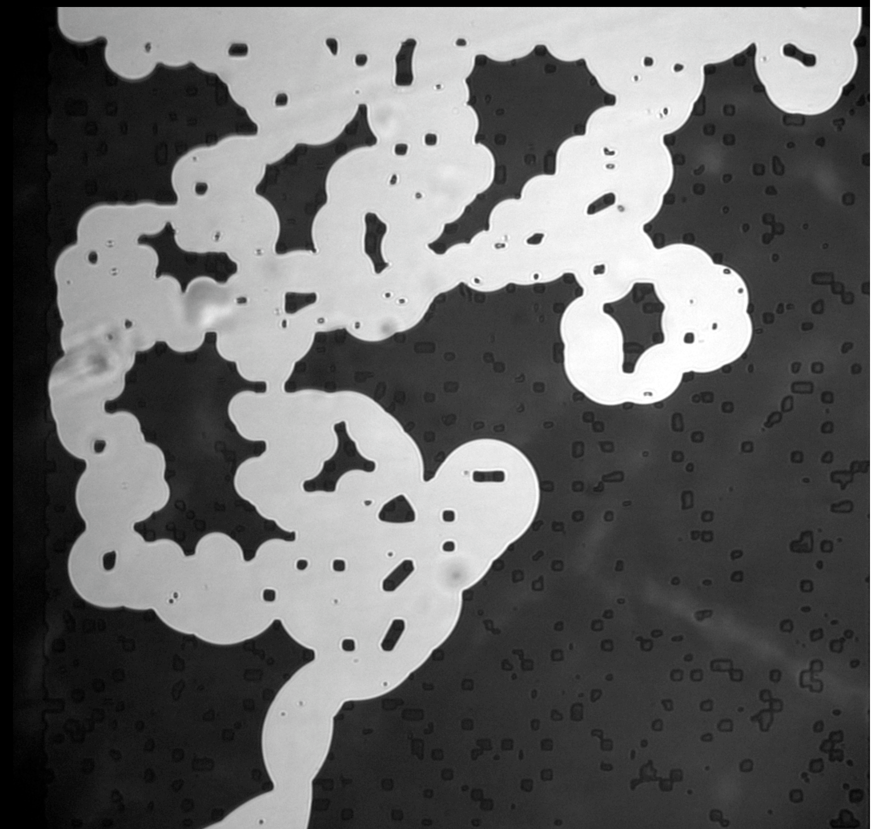
Sample: s90 with porosity = 90%



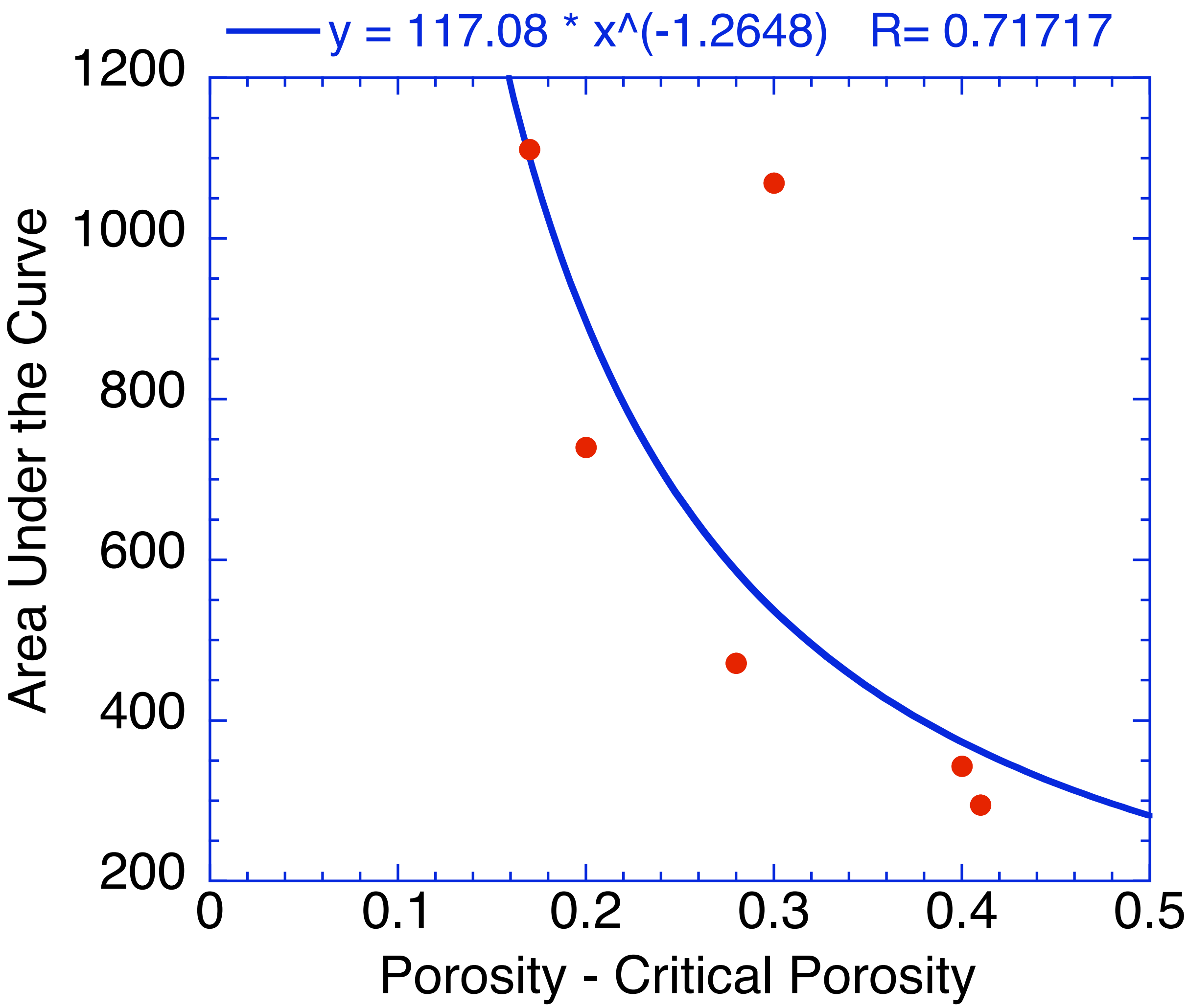


Sample: s95 with porosity = 91%



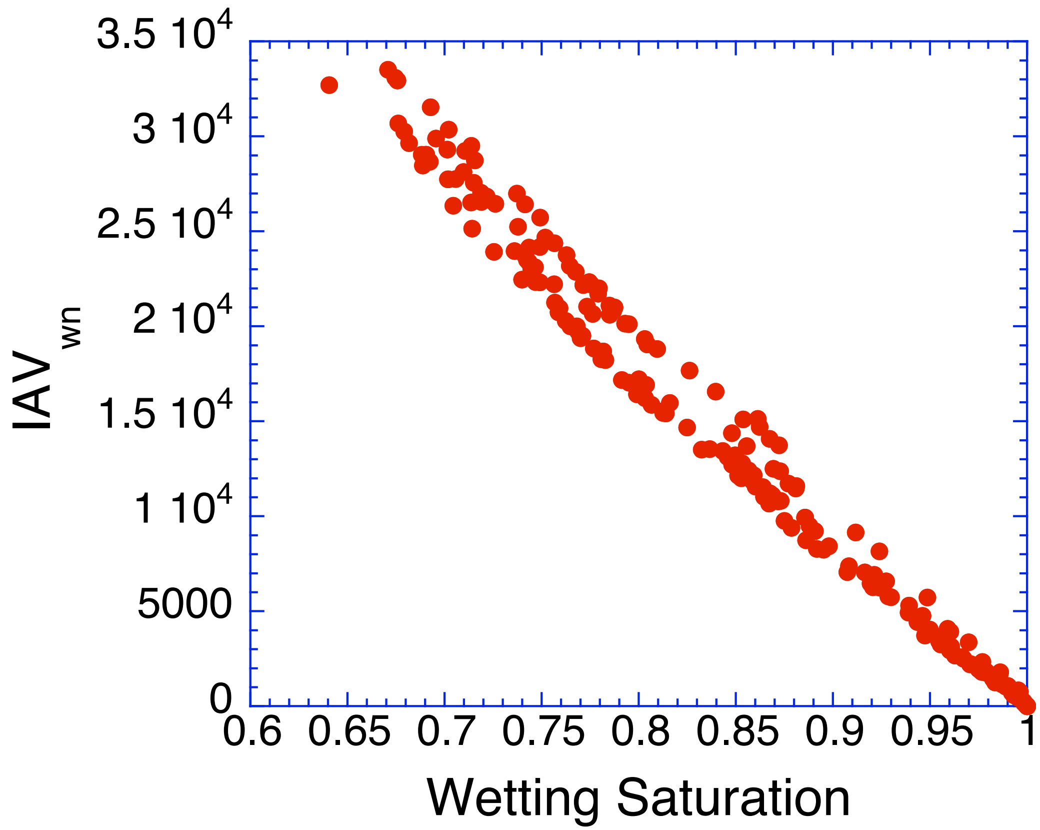


Hysteresis in IAVwn - Saturation Relationship as a Function of Distance from Percolation Threshold

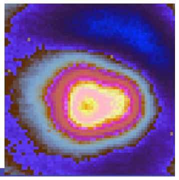


Midpoint Summary

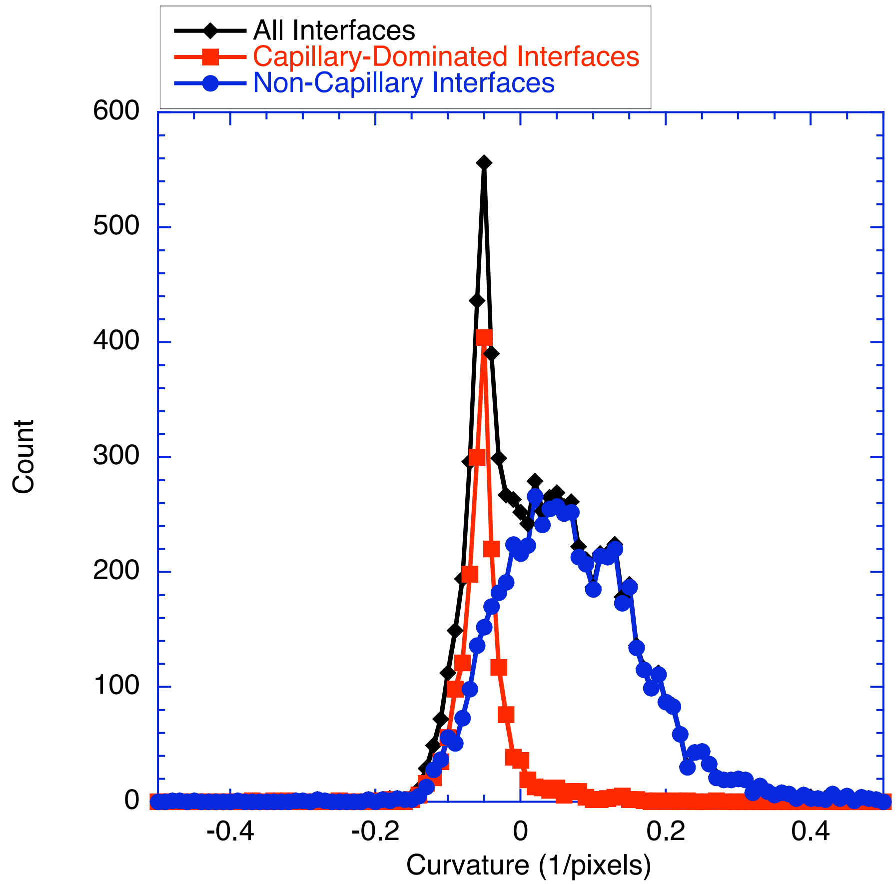
*If interfaces from all pressures are considered, IAV_{nw} exhibits a linear dependence on wetting-phase saturation. The relationship is non-invertible.



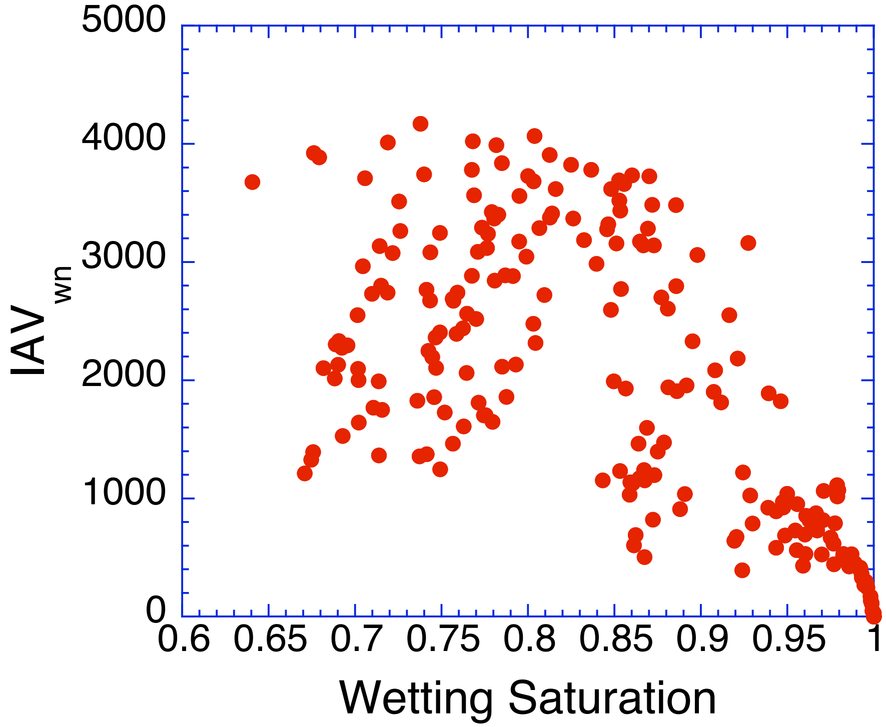
Midpoint Summary



* If only interfaces dominated by capillary forces are considered, a hysteretic relationship between IAV_{nw} - Saturation is formed.

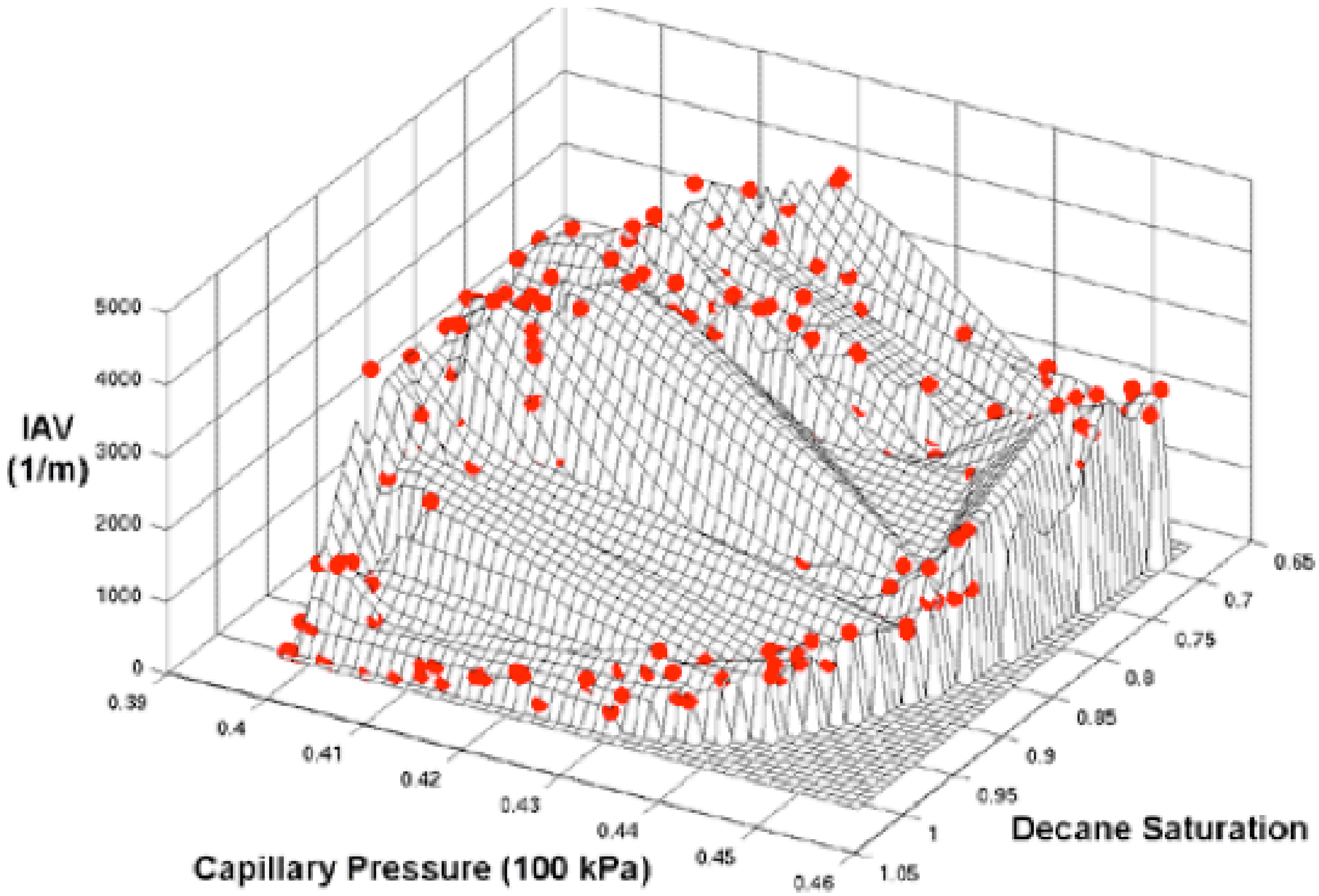


S8c.histogram.kld

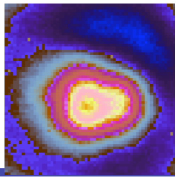


Midpoint Summary

* If only interfaces dominated by capillary forces are considered, IAV_{nw} is a single valued function of capillary pressure and saturation to within 4% and is now invertible.



Topics:



*Does IAV lift the ambiguity in the P_{cap} - S relationship?

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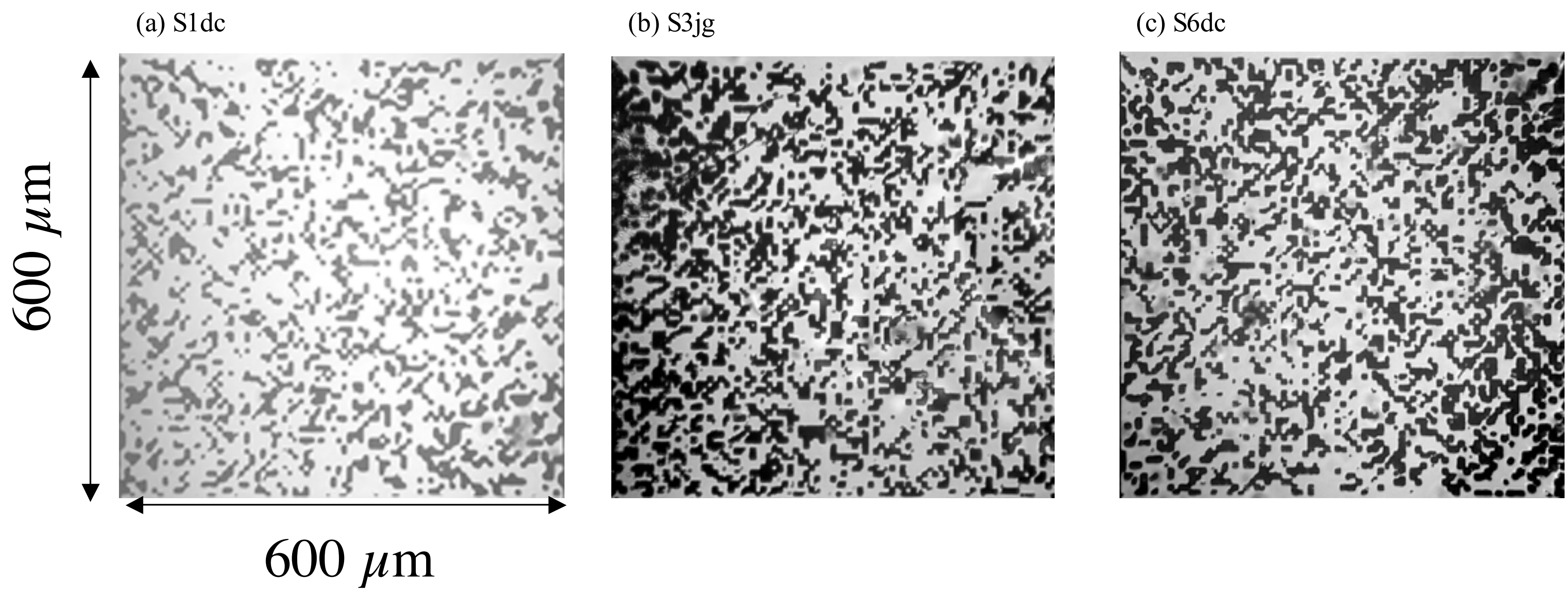
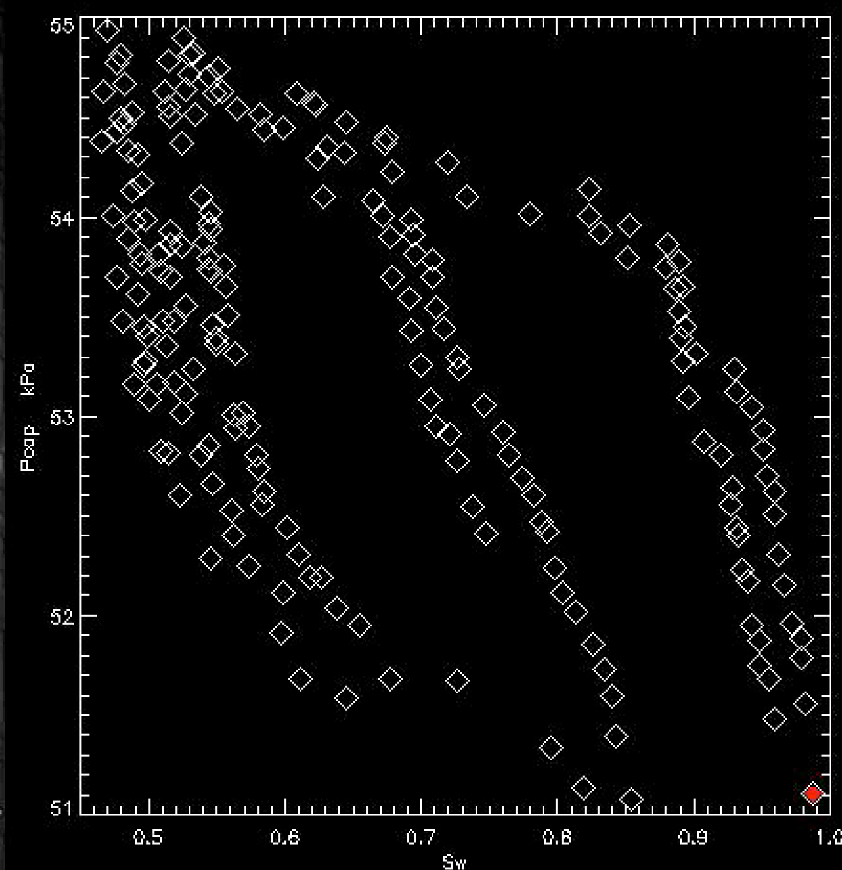
* Current Directions: Simple Wedge, Micro-capacitors, 3D

Micro-Models in this study



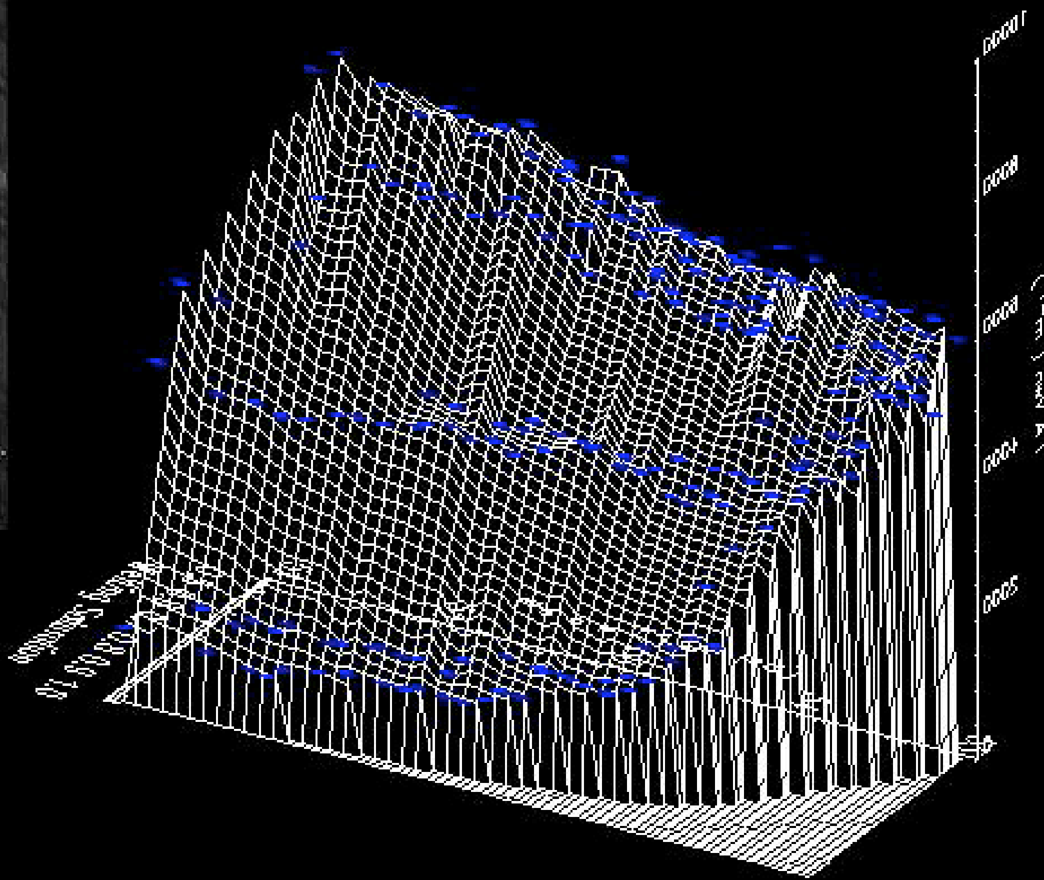
Porosity ~ 70.3%	Porosity ~ 51.4%	Porosity ~ 55.6%
Depth ~ 1 μm	Depth ~ 2 μm	Depth ~ 1 μm

Data Set:

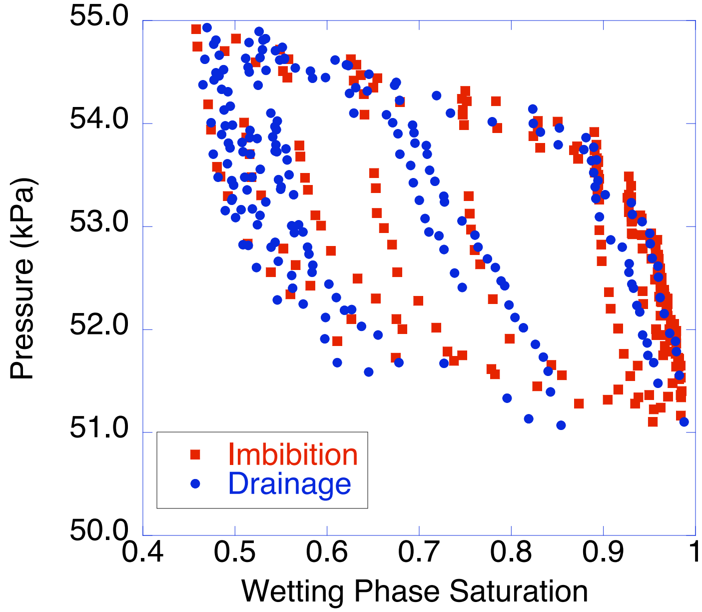


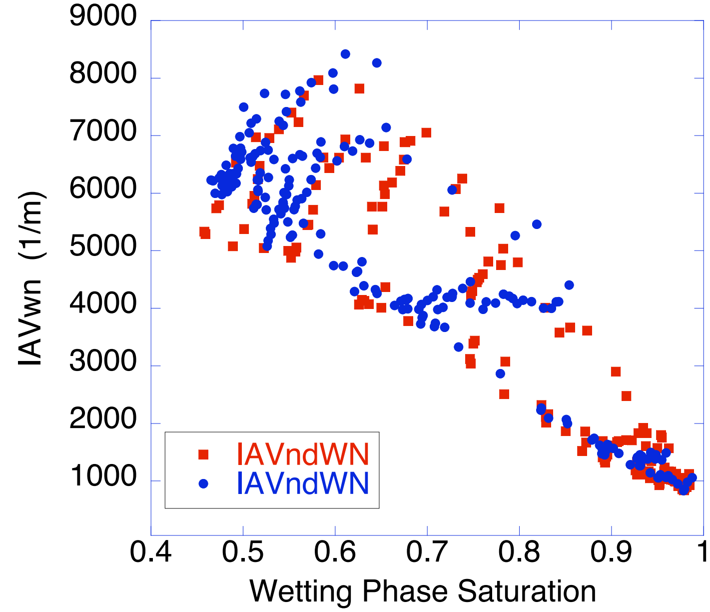
for Sample S1_sum05_Q

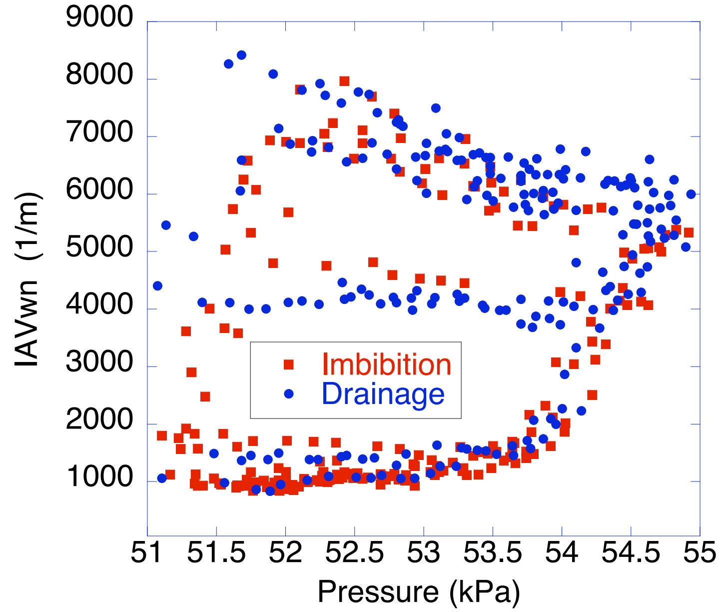
Relationship among Pcap, Wetting Saturation & IAVnw for Drainage Condition



Relationships

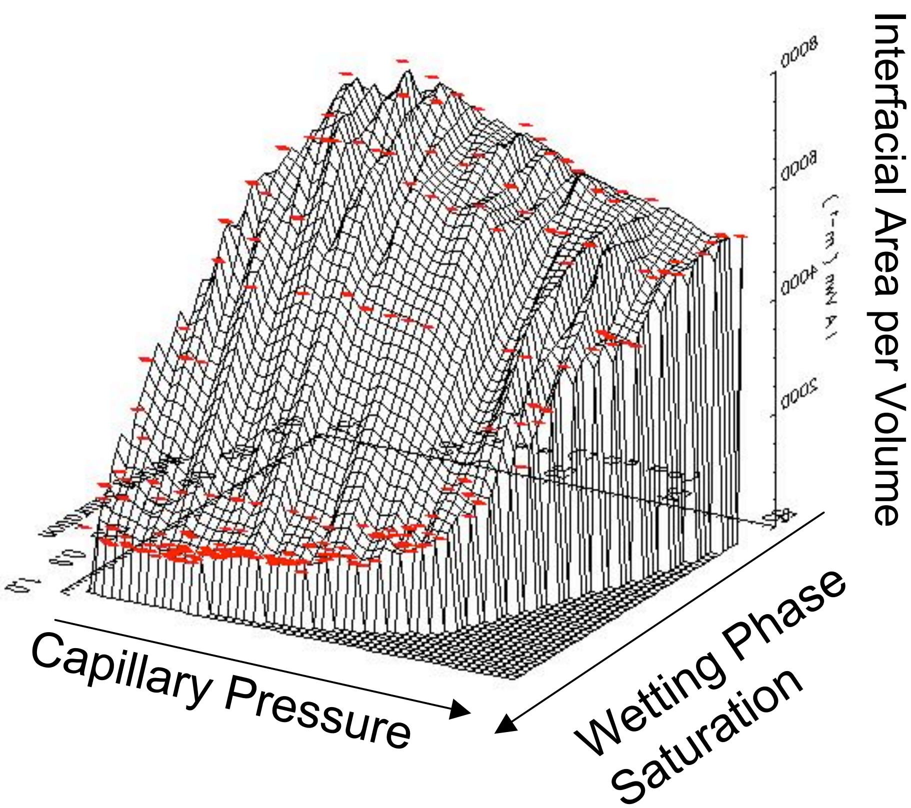




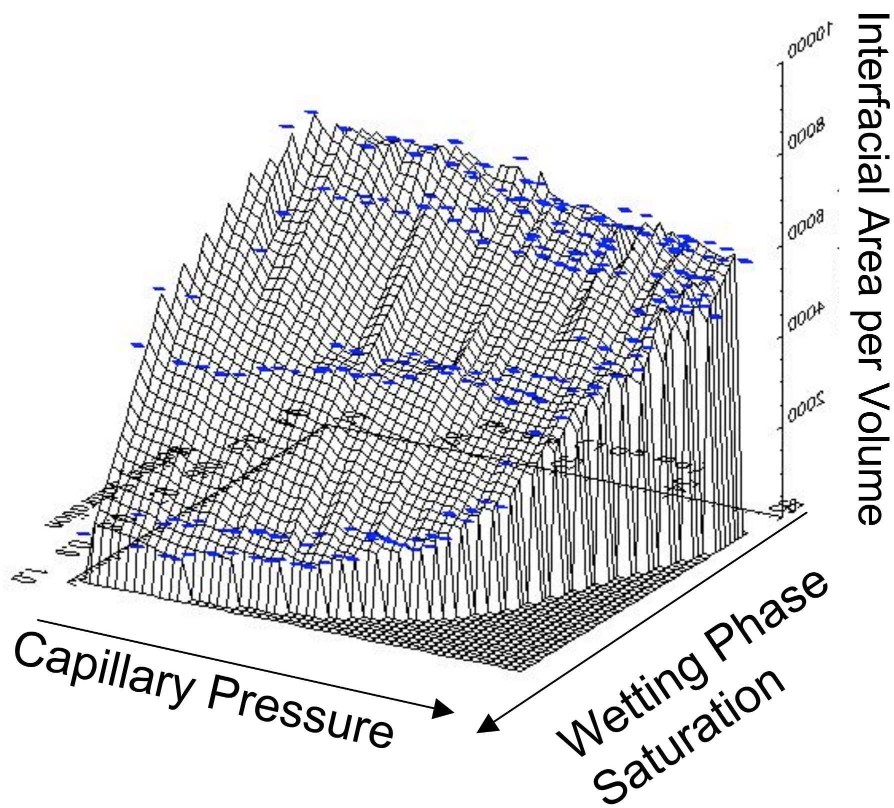


Comparison of Imbibition & Drainage Surfaces

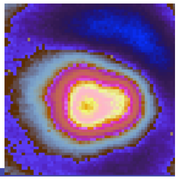
Imbibition



Drainage



Comparison of Imbibition & Drainage Surfaces



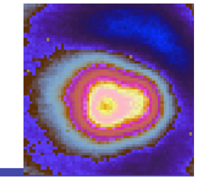
Method of comparison

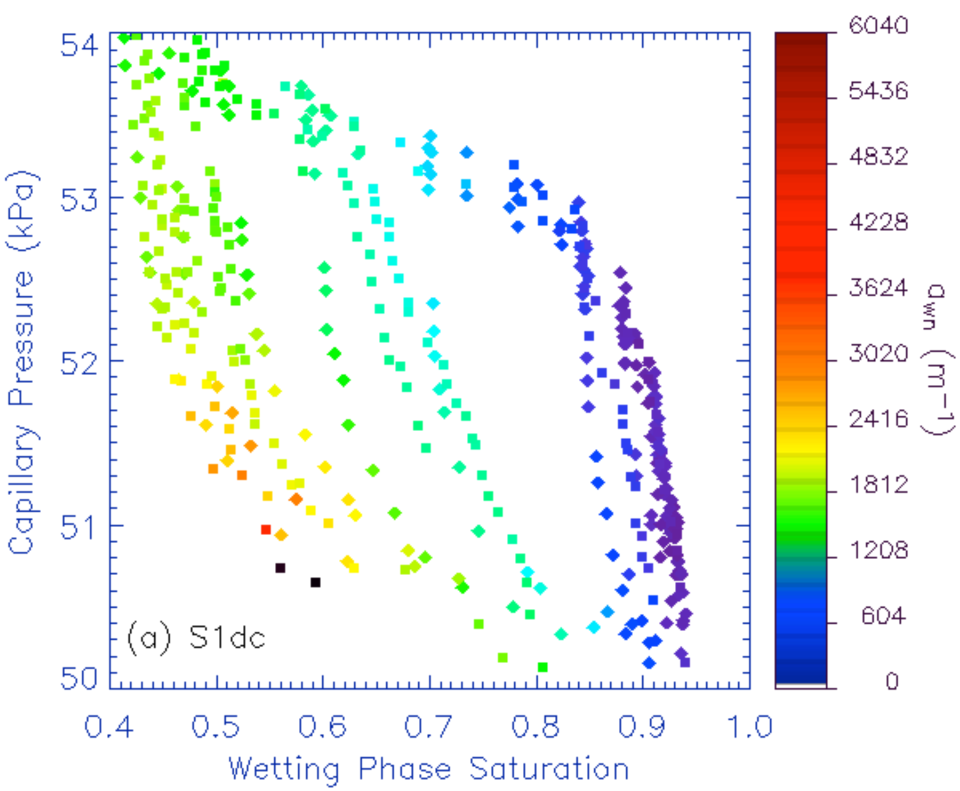
$$R = \frac{a_{data}(s,p)}{a_{\text{interpolated}}(s,p)}$$

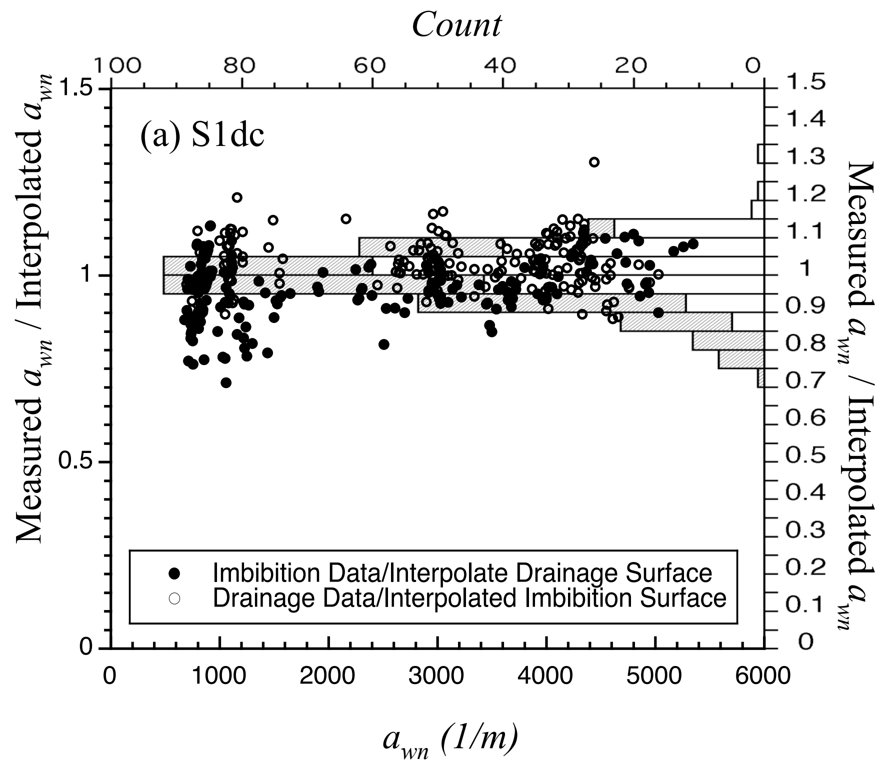
$$R_{imbibition} = \frac{a_{data,imbibition}(s,p)}{a_{\text{interpolated,drainage}}(s,p)}$$

$$R_{drainage} = \frac{a_{data,drainage}(s,p)}{a_{\text{interpolated,imbibition}}(s,p)}$$

Comparison of Imbibition & Drainage Surfaces



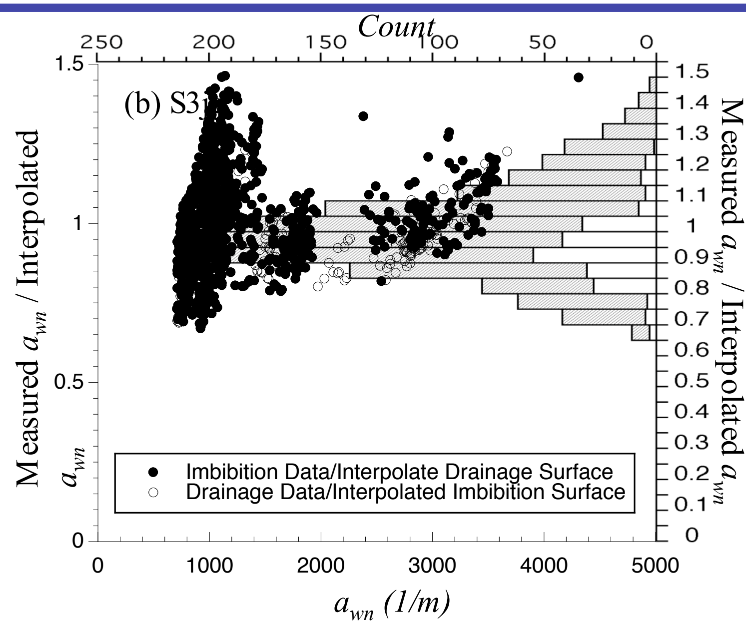


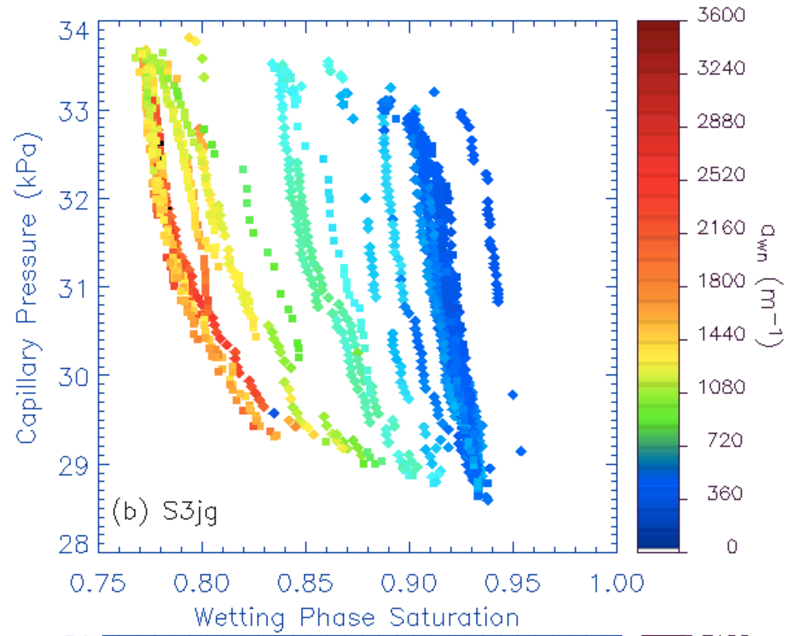


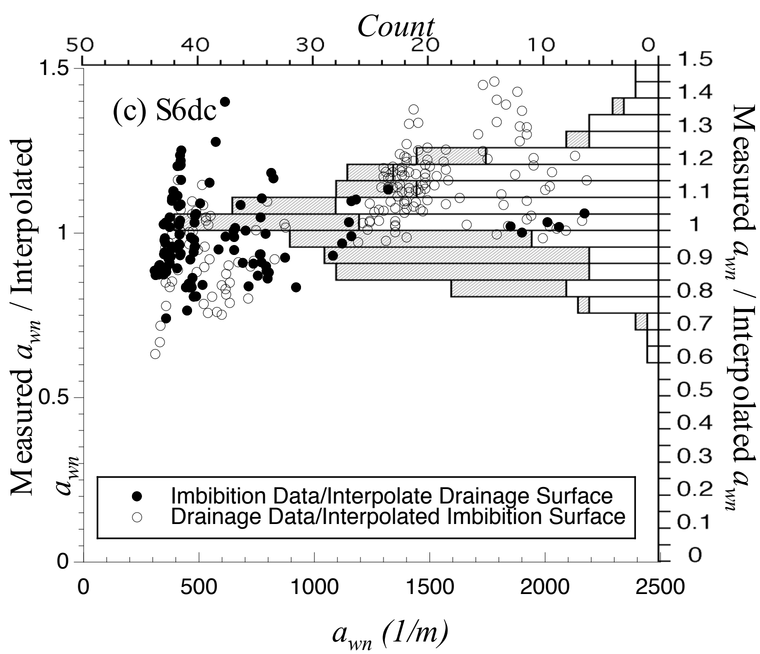
Squares: imbibition

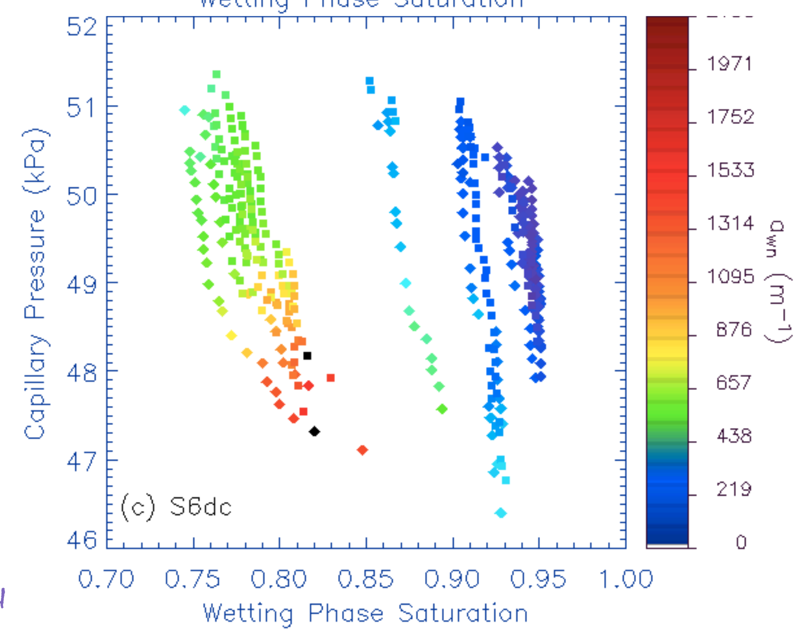
Diamonds: drainage

Comparison of Imbibition & Drainage Surfaces

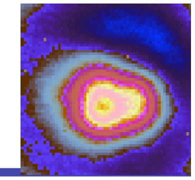








Topics:



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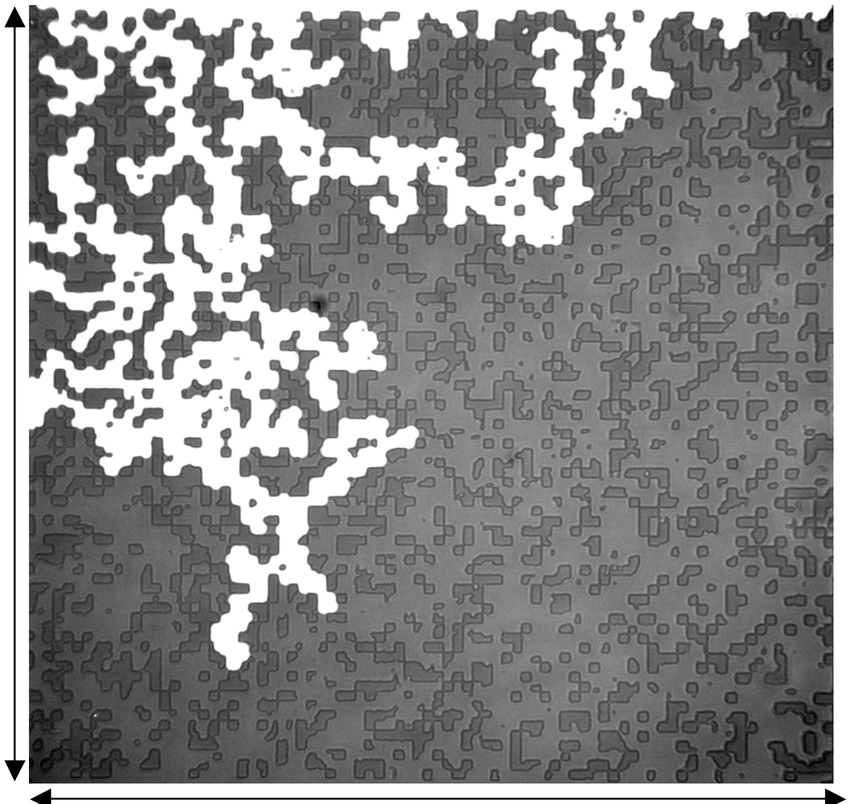
* Current Directions: Simple Wedge, Micro-capacitors, 3D

Micro-Models in this study

Sample s70

Channel Depth ~ 2 micron



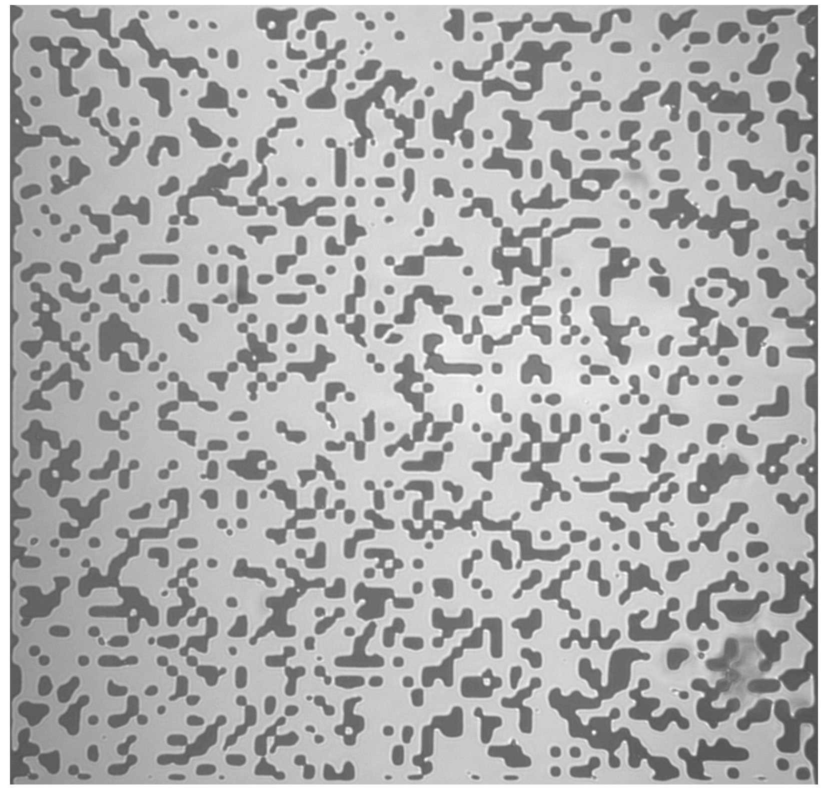
600 microns

600 microns

Porosity ~ 62.3%

Sample s1

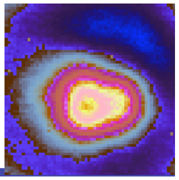
Channel Depth ~ 1 micron



Porosity ~ 72.9%

0.6 microns per pixel edge length

Capillary Pressure from Curvature



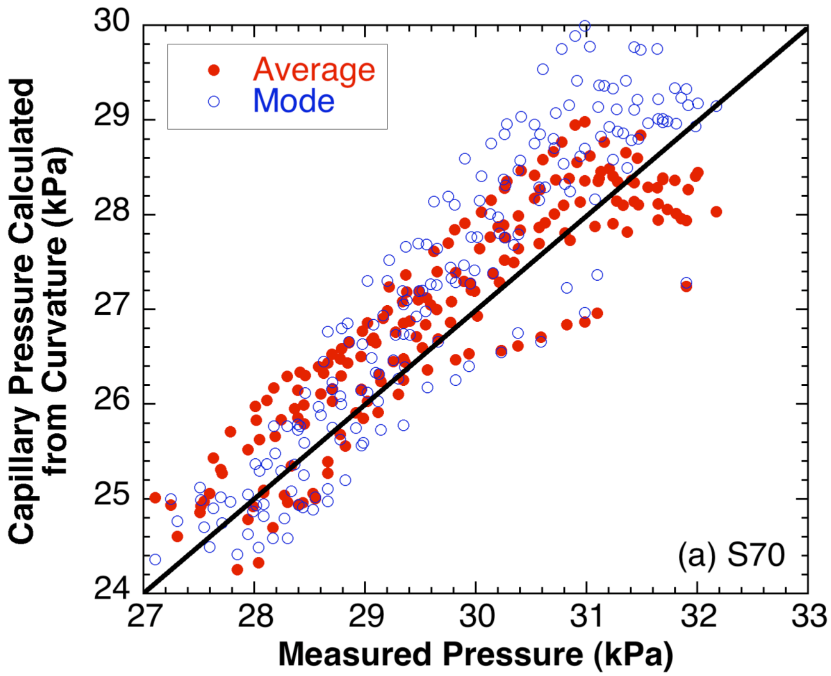
$P_{cap} = \gamma^{wn}\kappa$ where k is curvature of the interface between phases α & β

$\gamma^{\alpha\beta}$ - interfacial tension between phases α & β

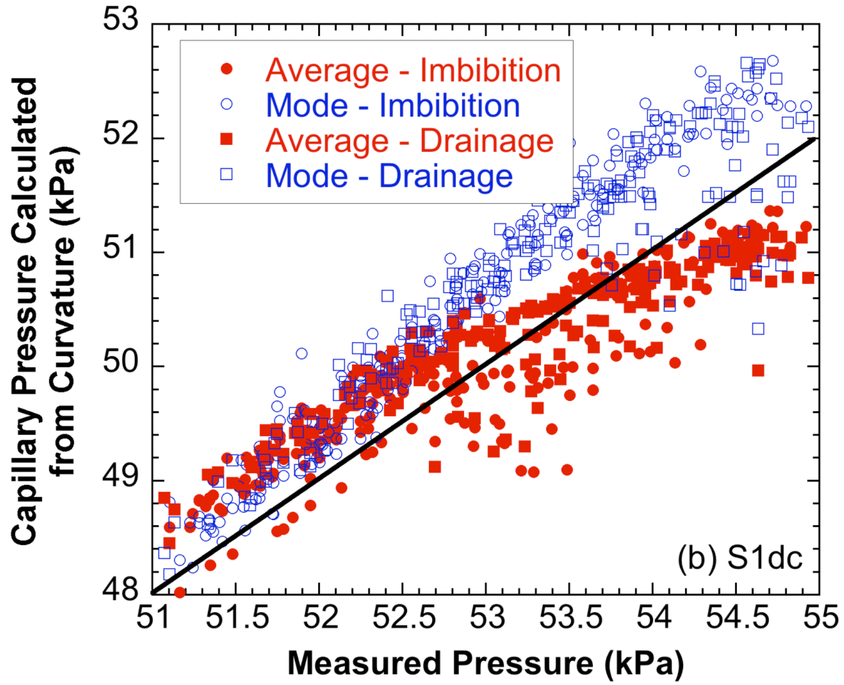
w - wetting phase

n - nonwetting phase

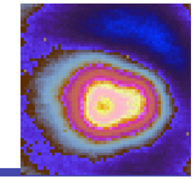
Sample s70



Sample s1dc



Capillary Pressure from Interfacial Area per Volume



$$p^c = -s^w \rho^w \frac{\partial A^w}{\partial s^w} - s^n \rho^n \frac{\partial A^n}{\partial s^w} - \sum_{\alpha\beta} \frac{\gamma^{\alpha\beta}}{\phi} \left(\frac{\partial a^{\alpha\beta}}{\partial s^w} \right) \Bigg|_{T,\phi,\Gamma^{\alpha\beta},A^{\alpha\beta}}$$

(Hassanizadeh & Gray, Water Resources Research, 29(10), 3389-3405, 1993)

$p^c =$ capillary pressure

$A =$ Helmholtz free energy

$\gamma^{\alpha\beta} =$ interfacial tension between α & β

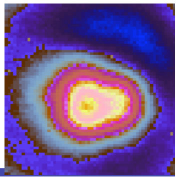
$a^{\alpha\beta} =$ interfacial area per volume α & β

$\phi =$ porosity

$s^w =$ saturation of the wetting phase

$w, n, s =$ wetting, nonwetting, solid

Calculating Pcap from IAV surfaces



$$P^c = -\frac{1}{\phi}\left[\gamma^{wn}\left(\frac{\partial a^{wn}}{\partial s^w}\right)\bigg|_{T,\phi,\Gamma^{\alpha\beta},A^{\alpha\beta}} + \gamma^{ns}\left(\frac{\partial a^{ns}}{\partial s^w}\right)\bigg|_{T,\phi,\Gamma^{\alpha\beta},A^{\alpha\beta}} + \gamma^{ws}\left(\frac{\partial a^{ws}}{\partial s^w}\right)\bigg|_{T,\phi,\Gamma^{\alpha\beta},A^{\alpha\beta}} \right]$$

Measured

$$\theta = 4.1^o$$

$$\gamma^{wn} = 0.0247 \text{ mN/m}$$

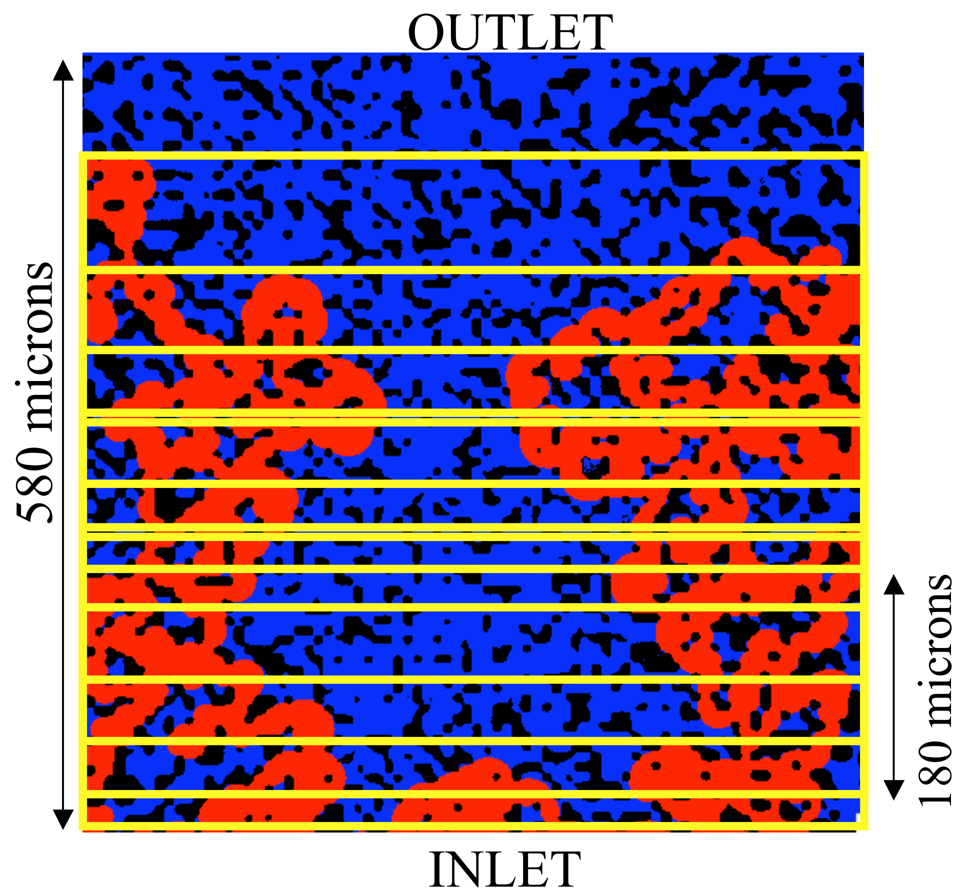
$$\gamma^{ws} = 0.0406 \text{ mN/m}$$

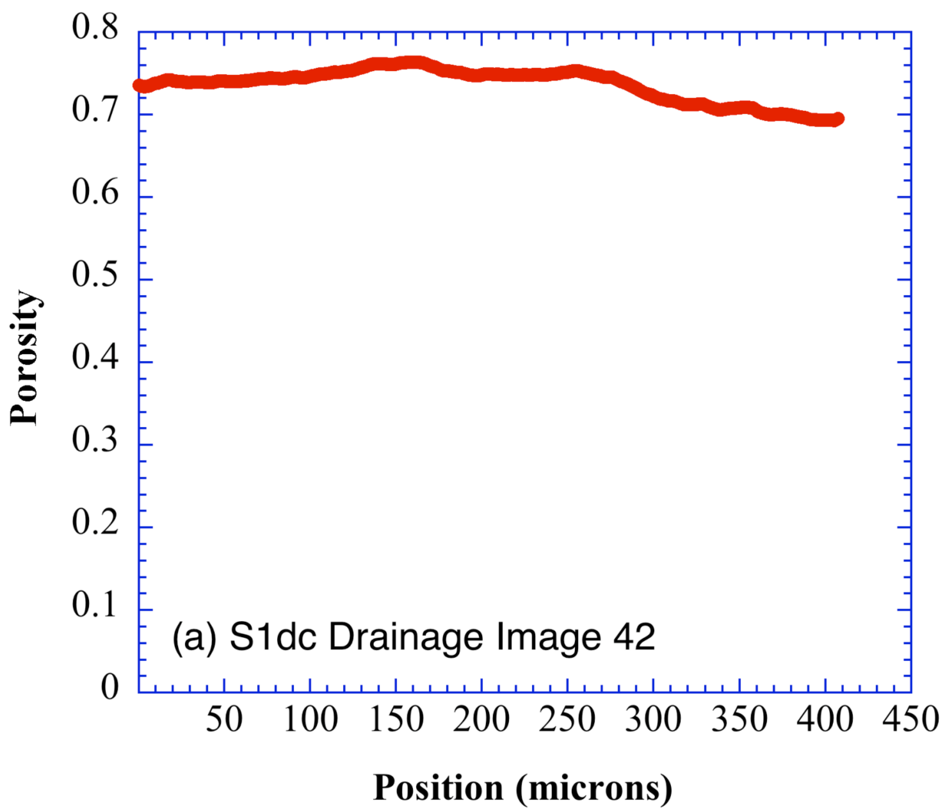
$$\gamma^{ns} = 0.022 \text{ mN/m}$$

Fit from function of state

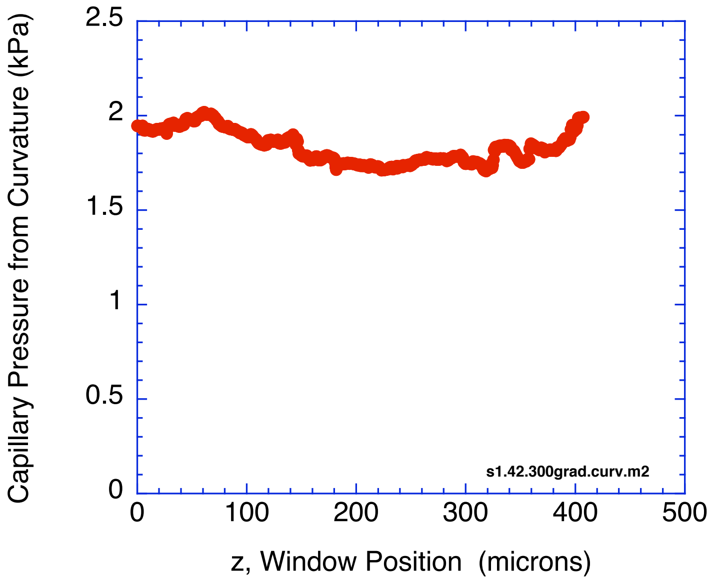
Definition: Micro-Ensemble

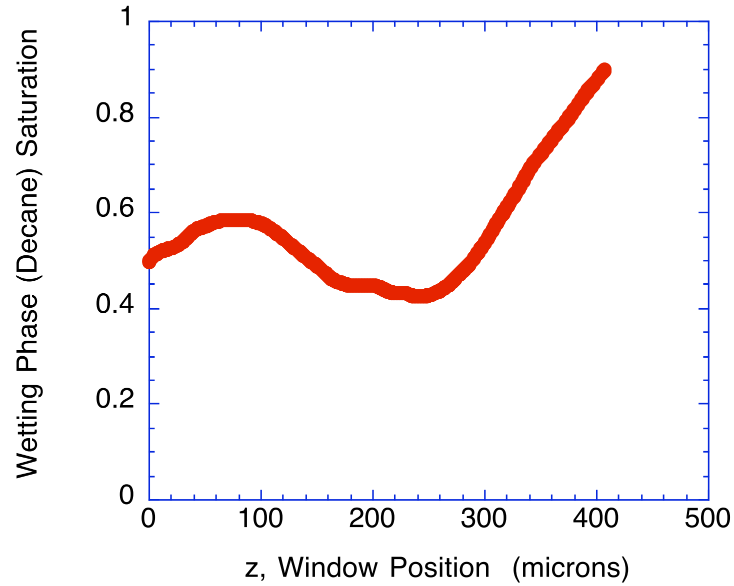
Micro-Ensemble: A moving window is used to obtain local parameters from one image, i.e. one image generates multiple data points. Globally, one image is at a constant pressure as measured externally.





p^c, s^w a^{wn} as a function of window position





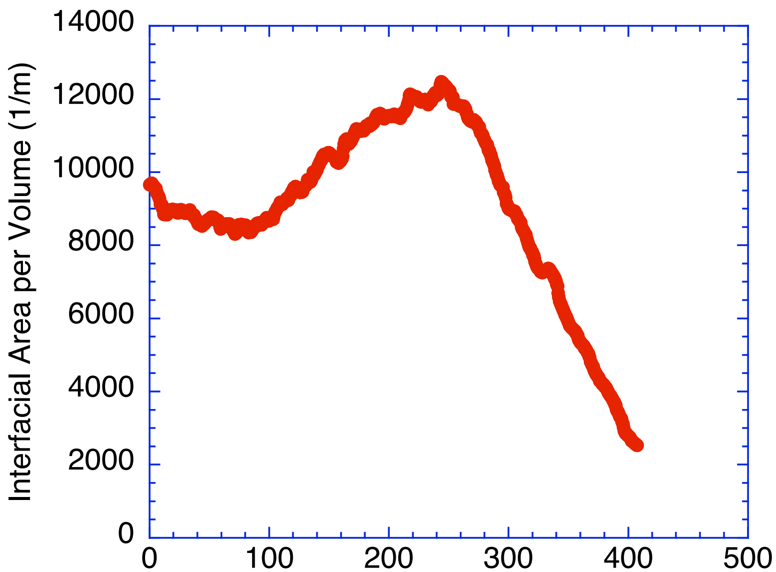
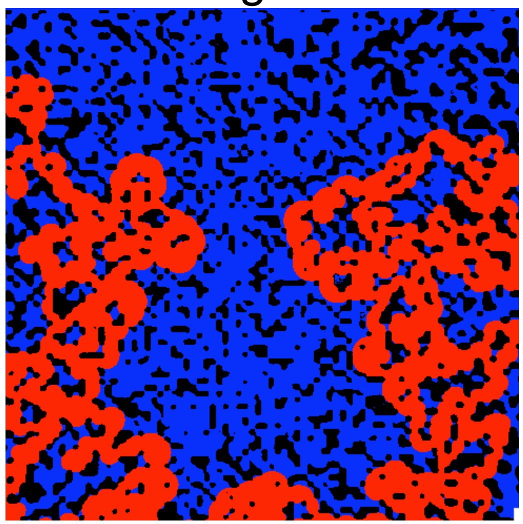


Image 42



p^c versus s^w

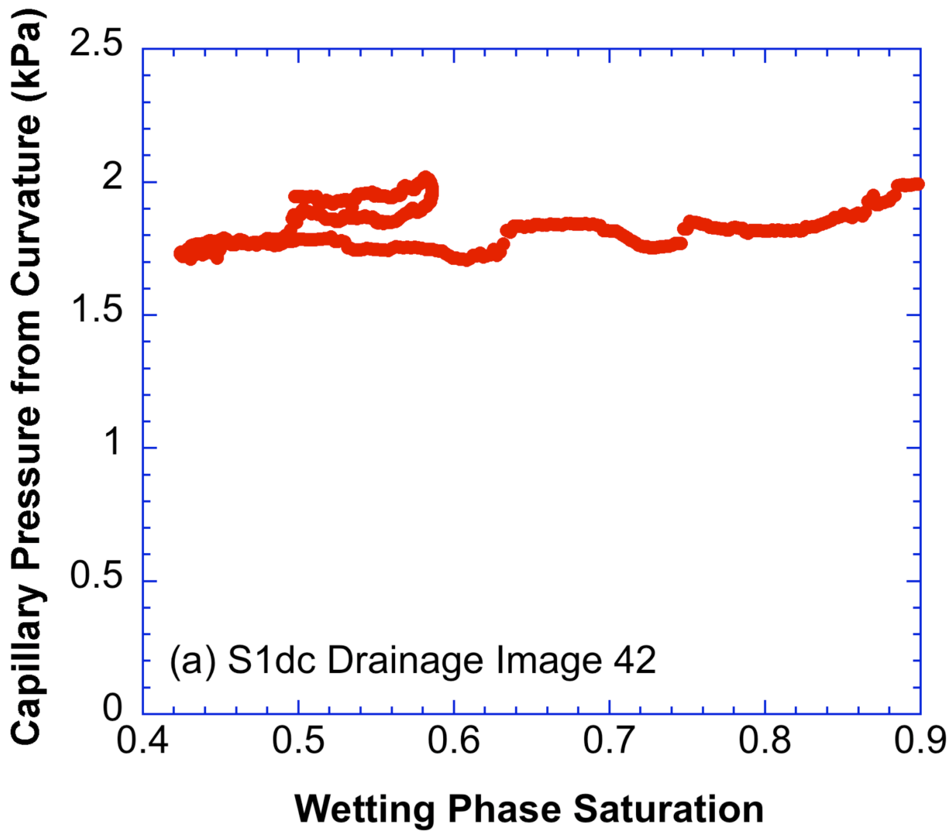
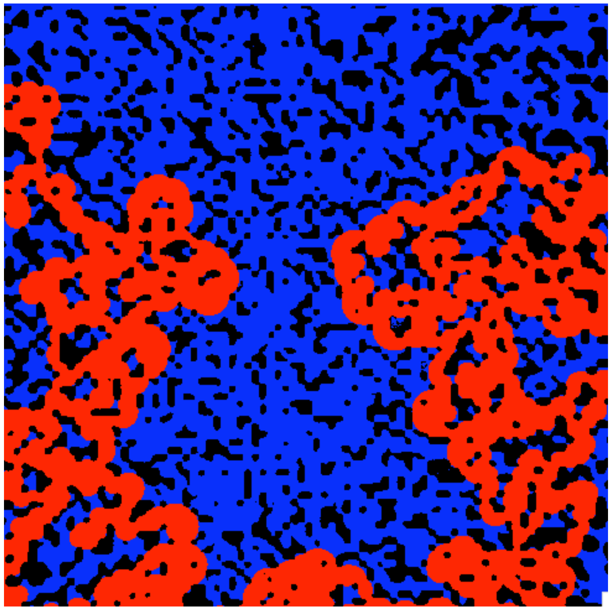
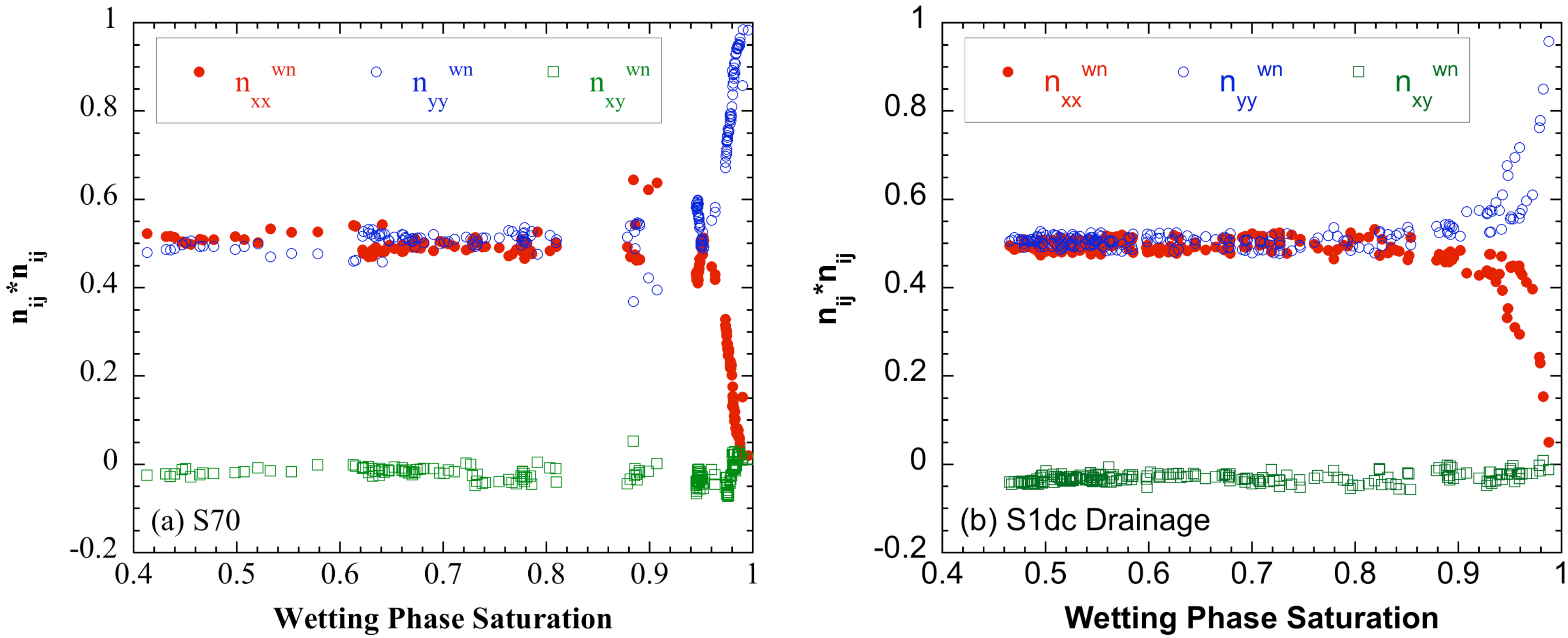


Image 42



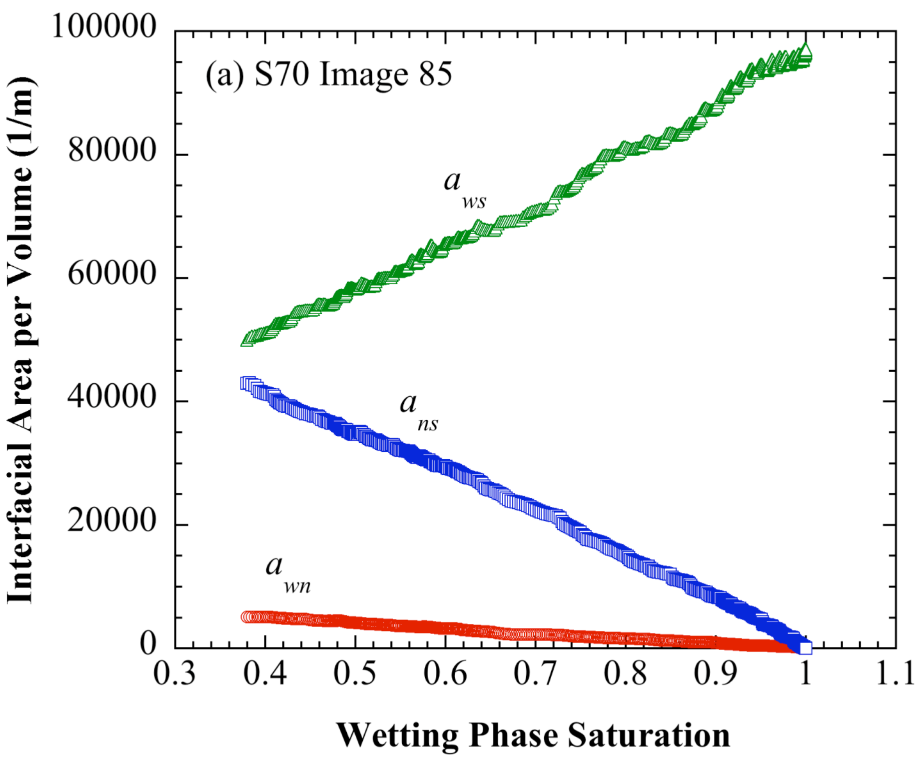
Direction of Interfaces



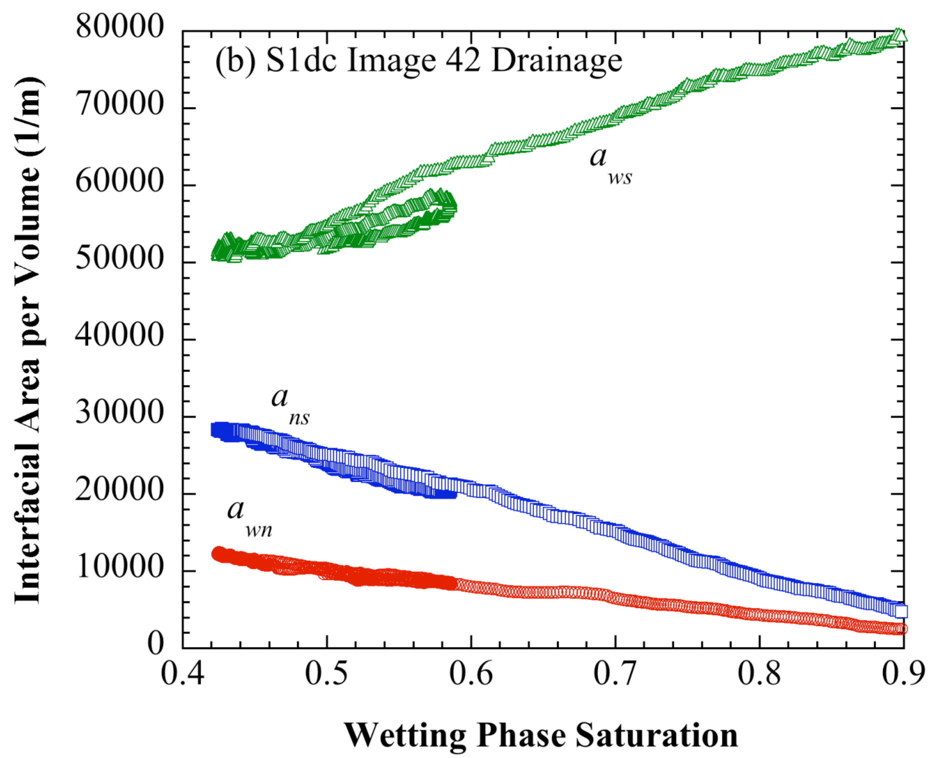
Gradients

$$P^c = -\frac{1}{\phi}\left[\gamma^{wn}\left(\frac{\partial a^{wn}}{\partial s^w}\right)\Bigg|_{energy} + \gamma^{ns}\left(\frac{\partial a^{ns}}{\partial s^w}\right)\Bigg|_{energy} + \gamma^{ws}\left(\frac{\partial a^{ws}}{\partial s^w}\right)\Bigg|_{energy} \right]$$

Sample s70



Sample s1



p^c from a & s data

$$P^c = -\frac{1}{\phi}\left[\gamma^{wn}\left(\frac{\partial a^{wn}}{\partial s^w}\right)\Bigg|_{T,\phi,\Gamma^{\alpha\beta},A^{\alpha\beta}} + \gamma^{ns}\left(\frac{\partial a^{ns}}{\partial s^w}\right)\Bigg|_{T,\phi,\Gamma^{\alpha\beta},A^{\alpha\beta}} + \gamma^{ws}\left(\frac{\partial a^{ws}}{\partial s^w}\right)\Bigg|_{T,\phi,\Gamma^{\alpha\beta},A^{\alpha\beta}} \right]$$

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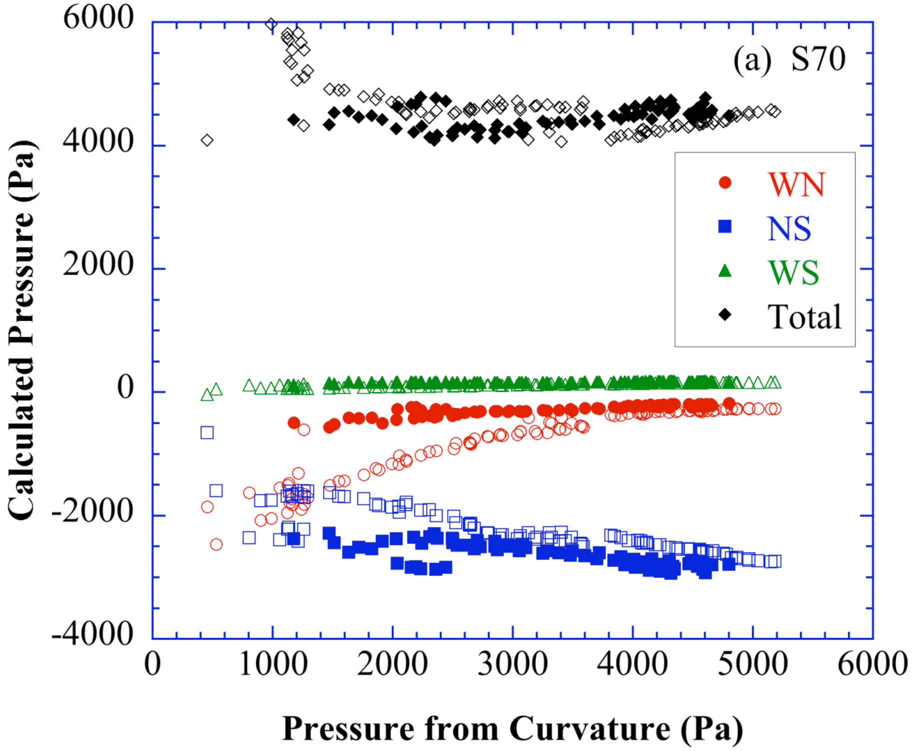

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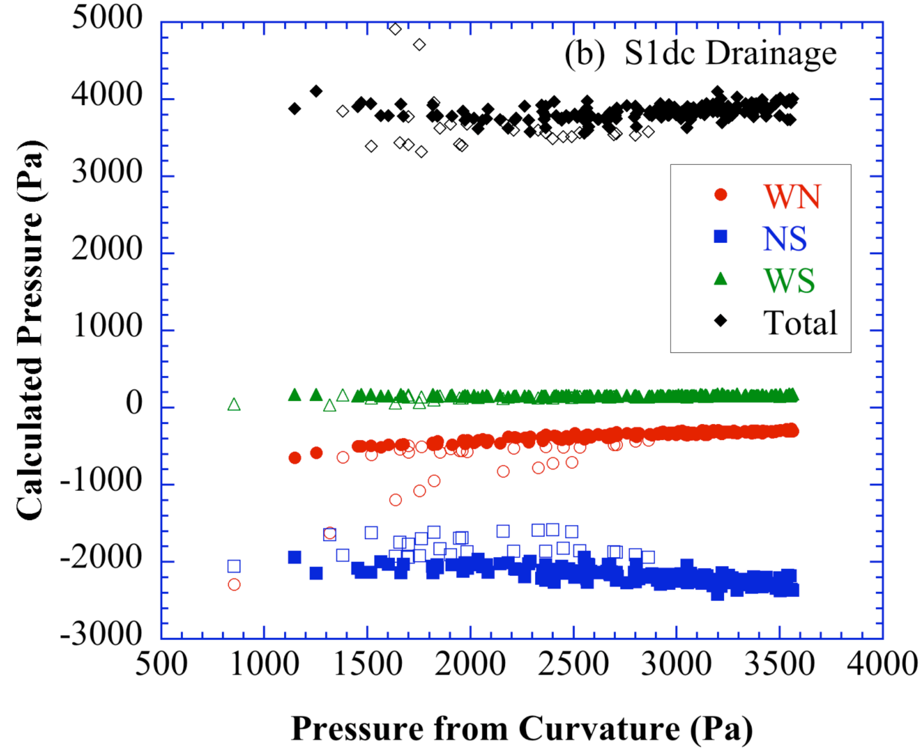
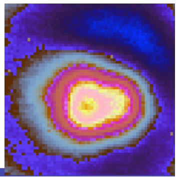


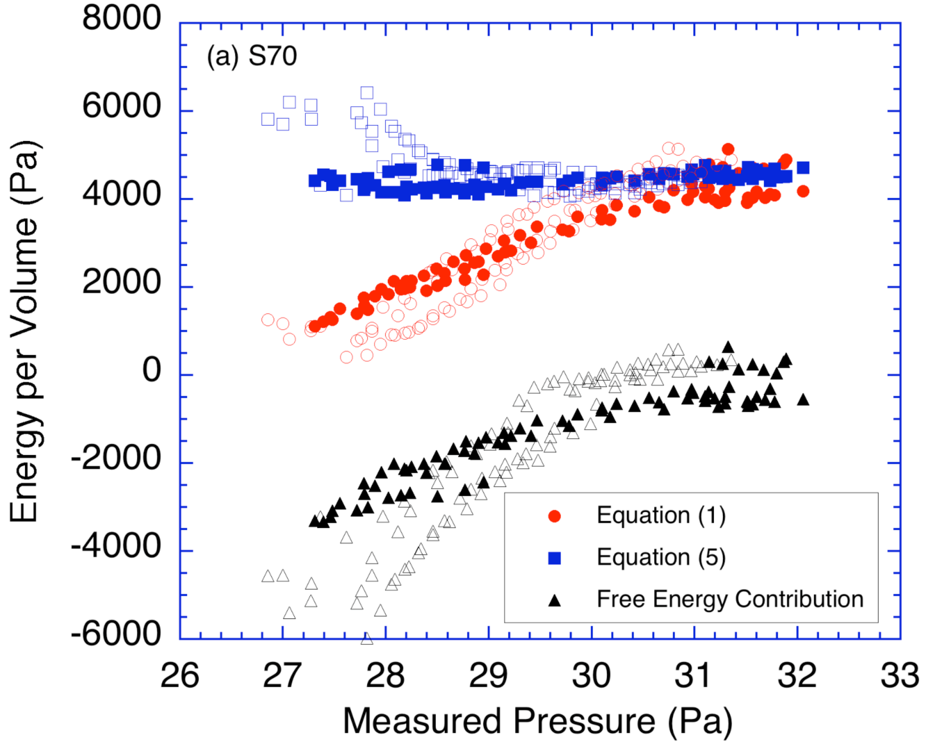
Free Energy Contribution



$$P_{cap} - \left[-\sum_{\alpha\beta} \frac{\gamma^{\alpha\beta}}{\phi} \left(\frac{\partial a^{\alpha\beta}}{\partial s^w} \right) \Bigg|_{T,\phi,\Gamma^{\alpha\beta},A^{\alpha\beta}} \right] = -s^w \rho^w \frac{\partial A^w}{\partial s^w} - s^n \rho^n \frac{\partial A^n}{\partial s^w}$$

$$P_{cap} = \gamma^{wn} \kappa$$

Sample s70



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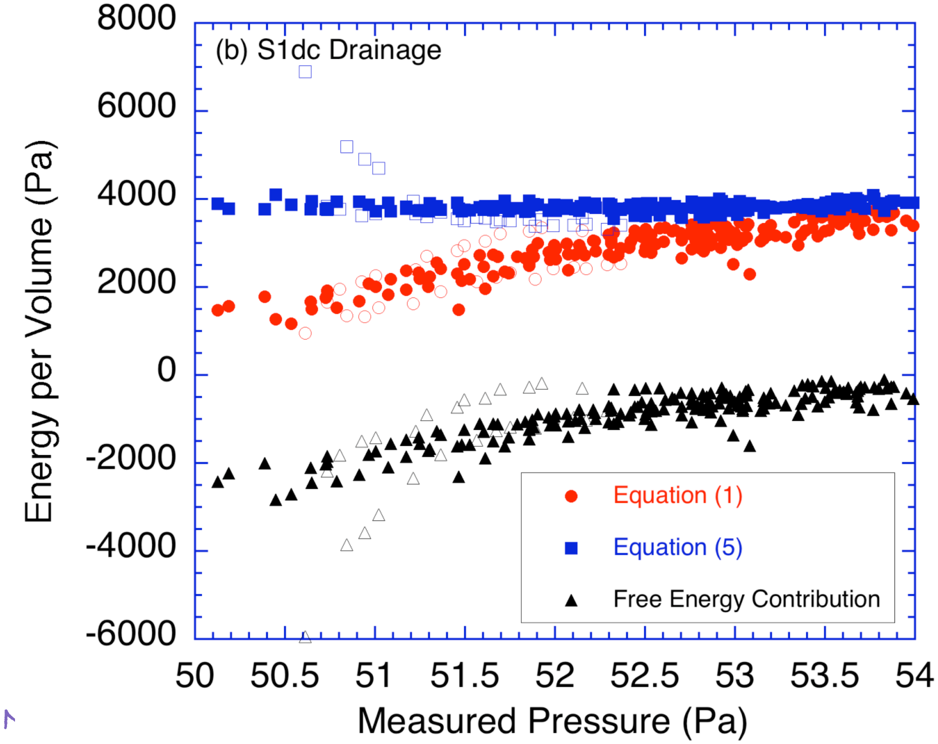
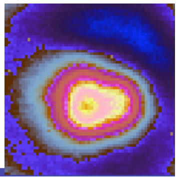


Topics:



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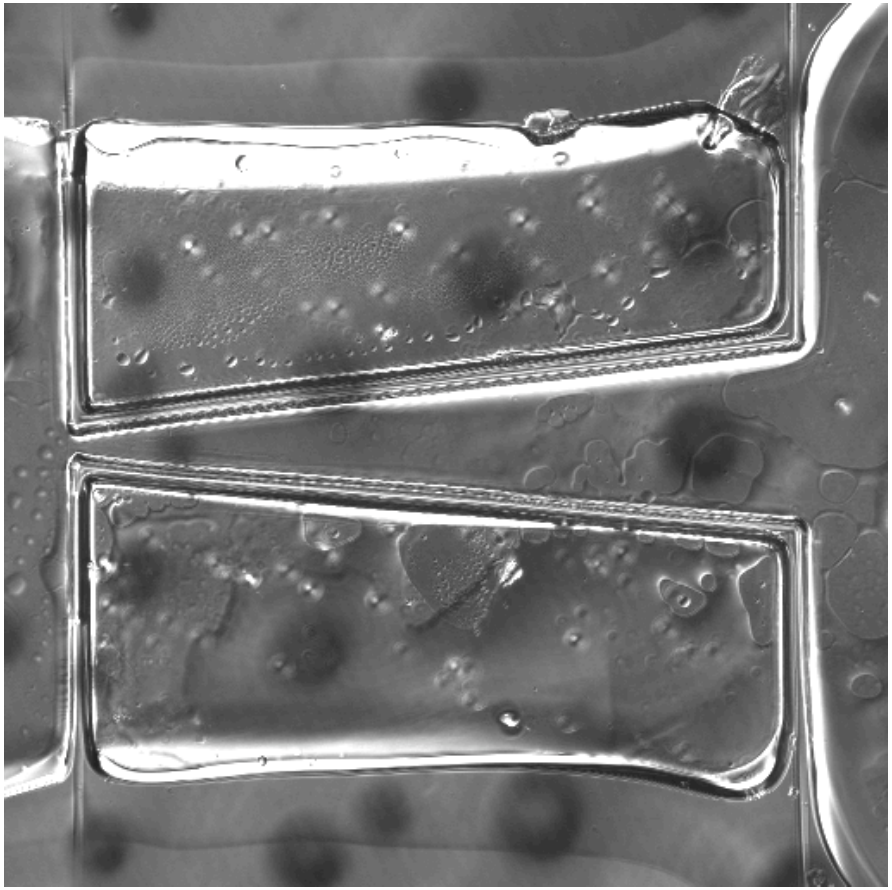
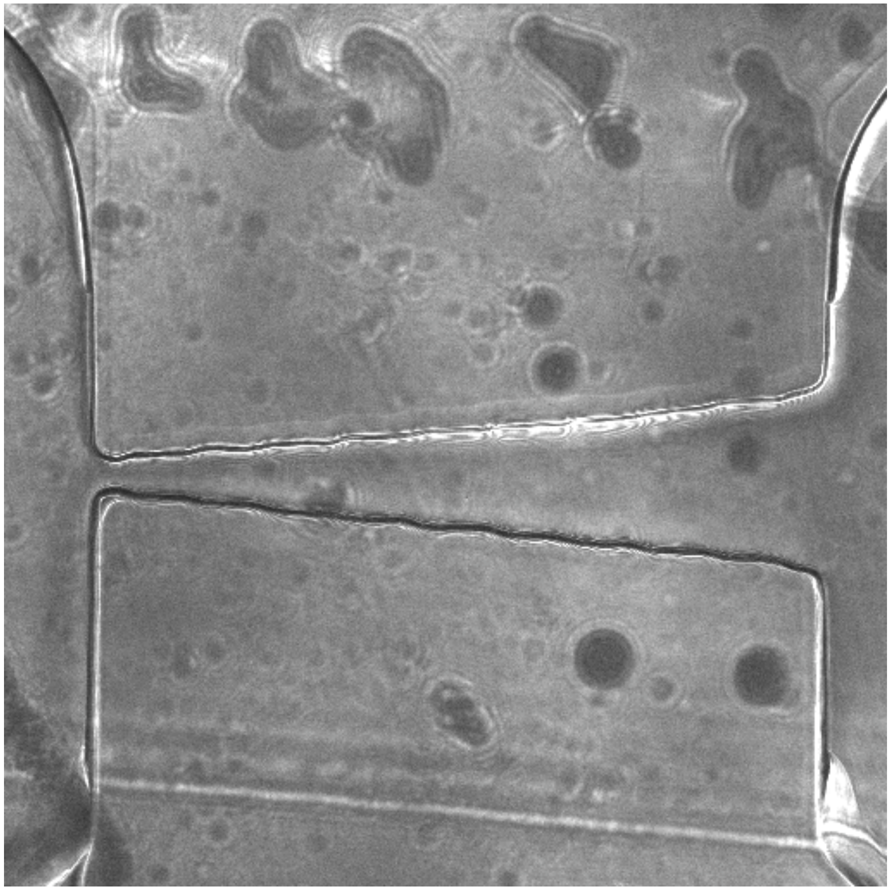
* Current Directions: Simple Wedge, Micro-capacitors, 3D

Simple Wedge

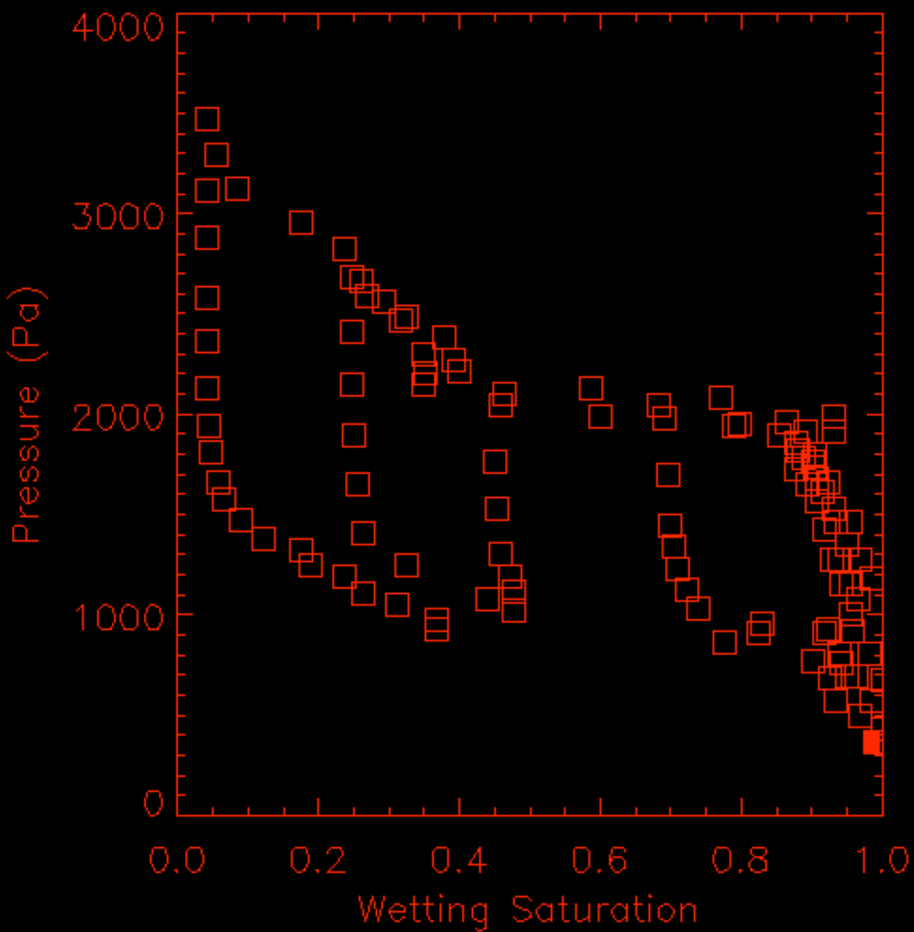
Laser Written

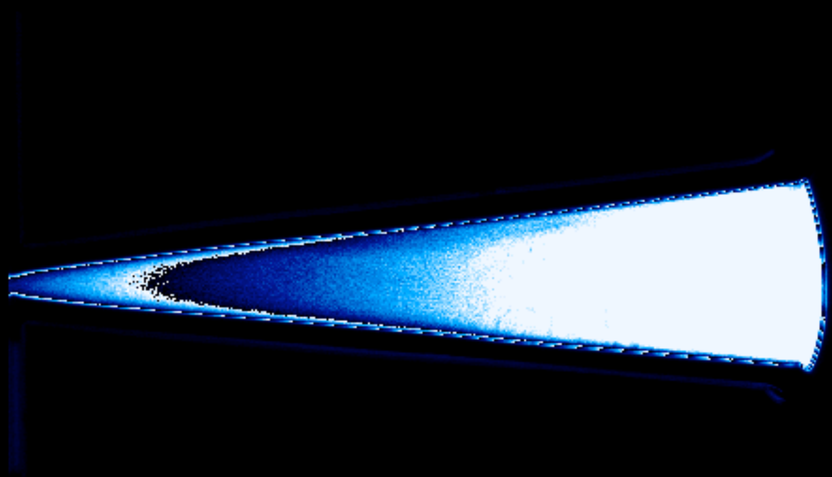


Contact Lithography



Simple Wedge: Capillary Pressure - Saturation





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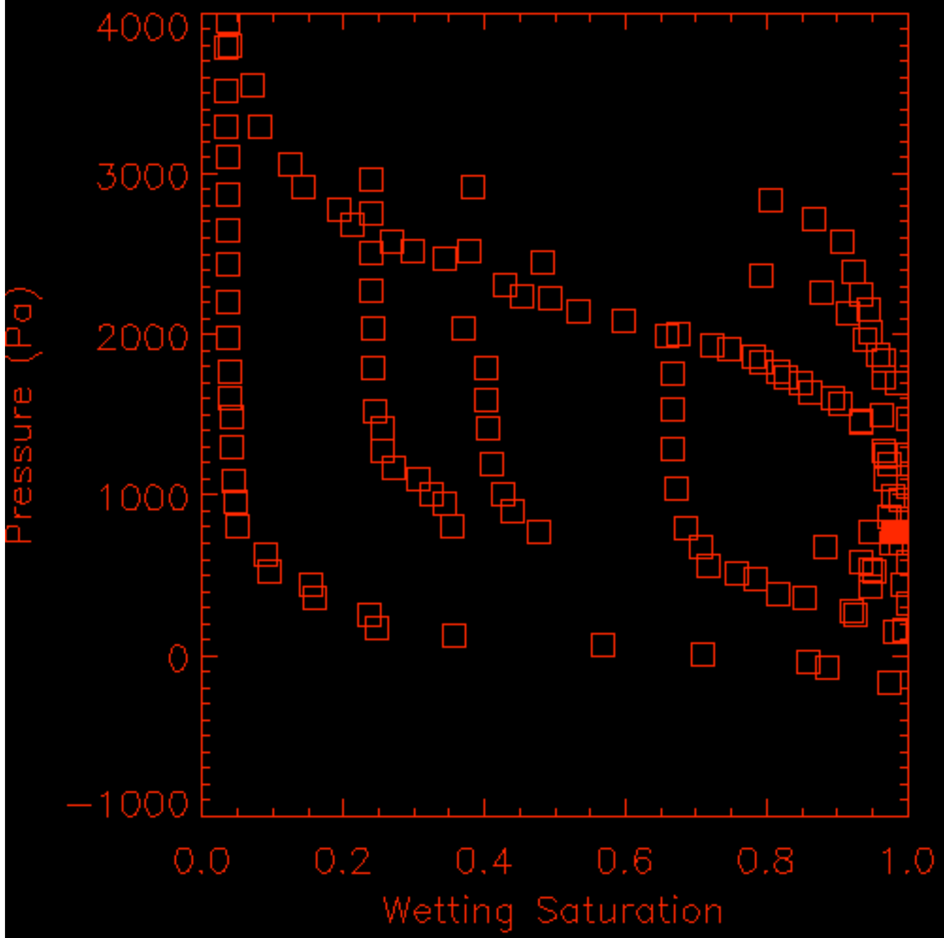
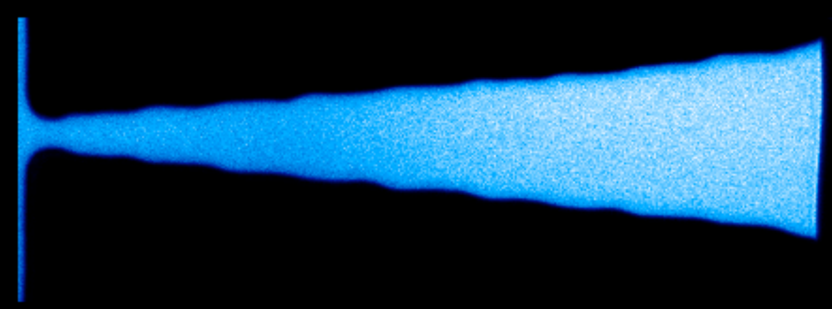
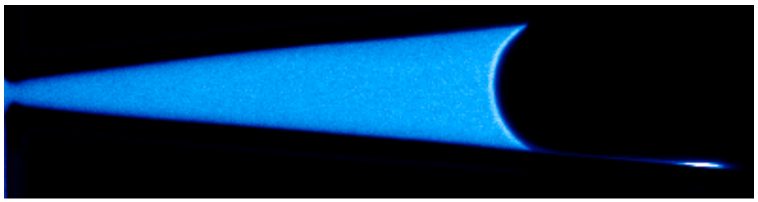
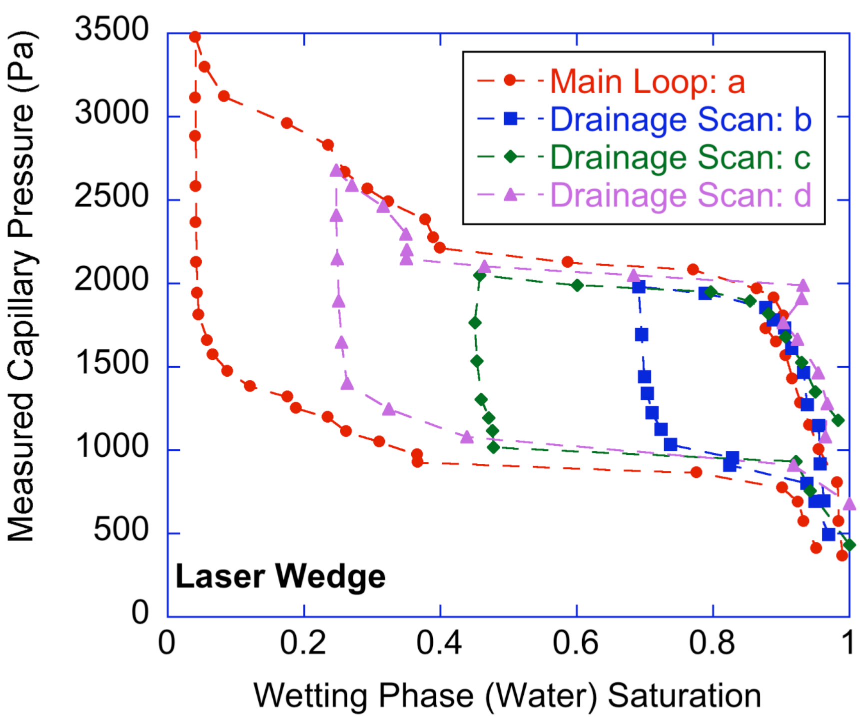




Simple Wedge: Capillary Pressure - Saturation

Laser Written





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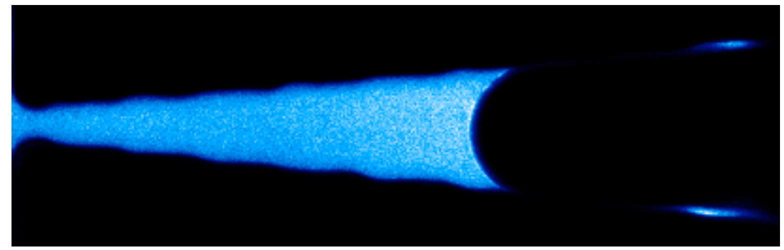
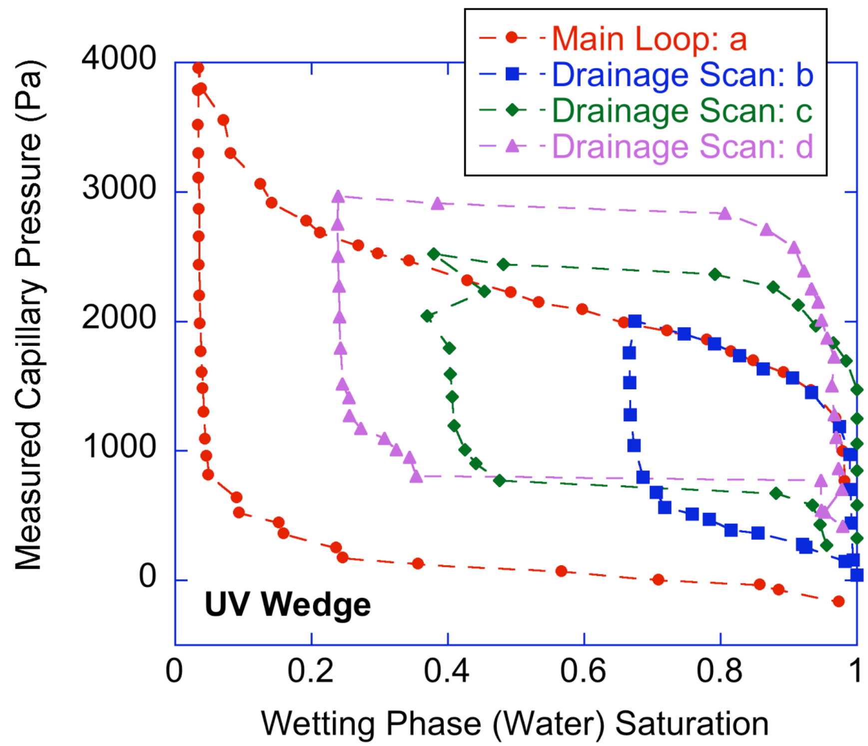
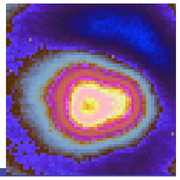
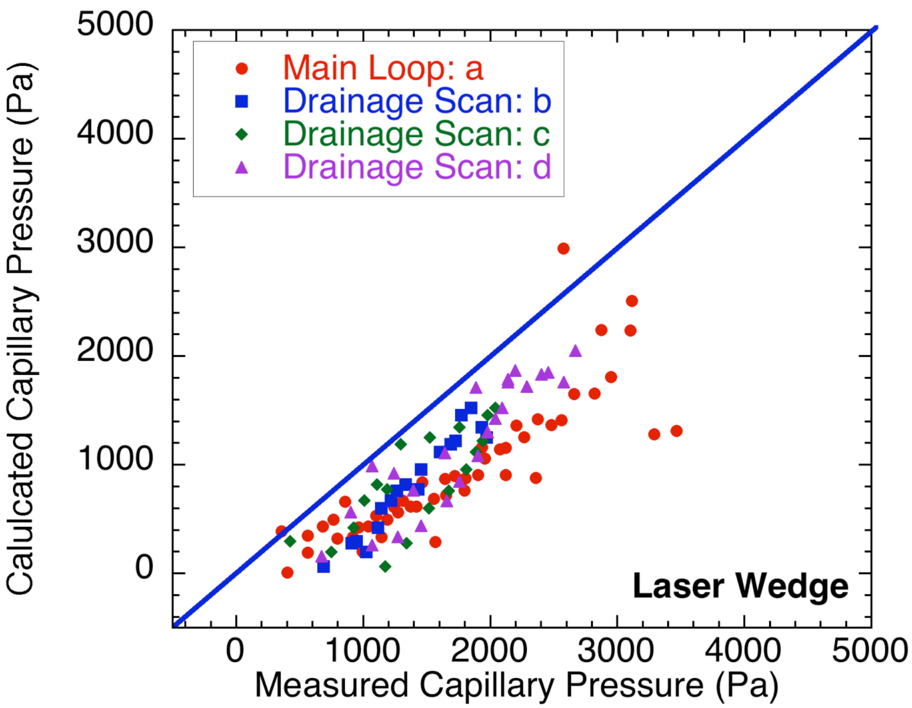




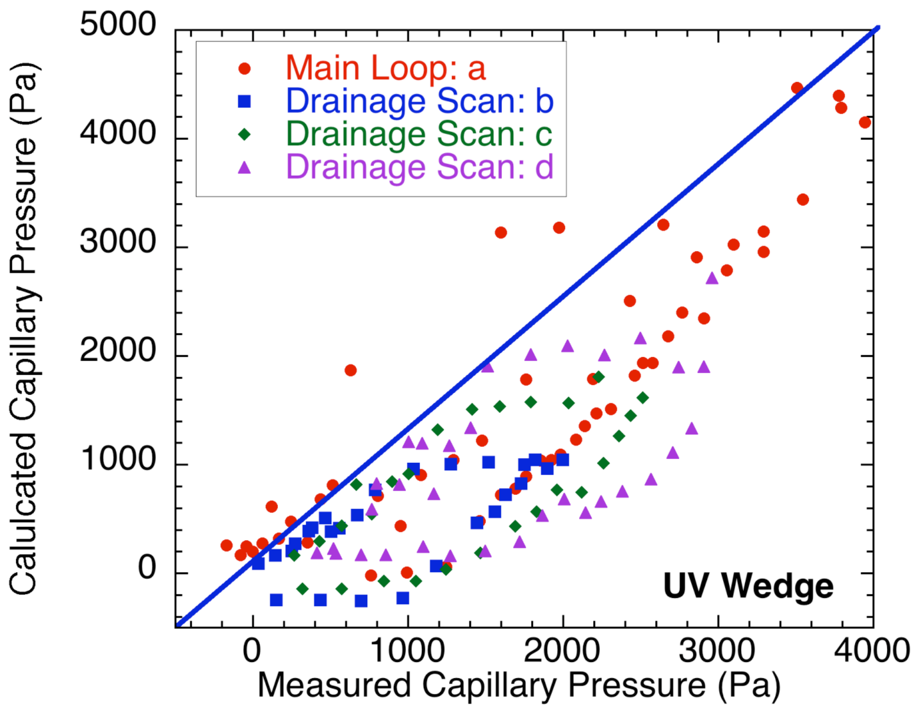
Capillary Pressure from Curvature



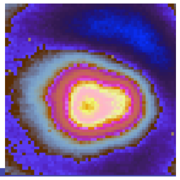
Laser Written



Contact Lithography



Topics:



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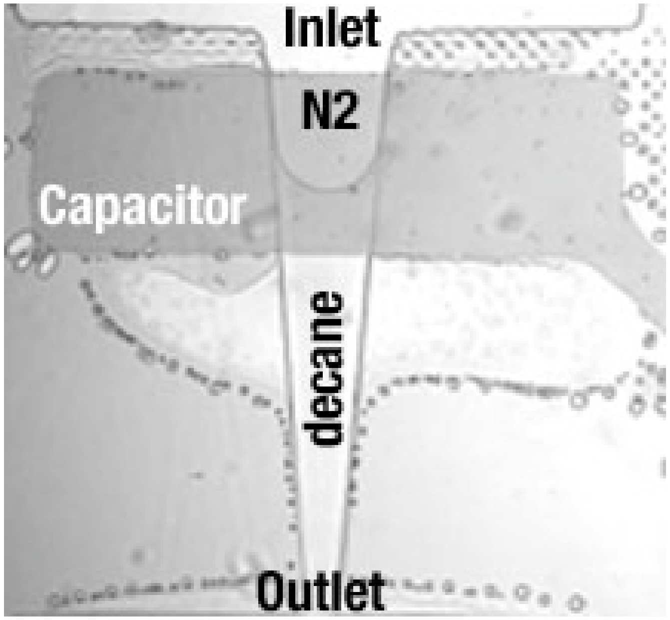
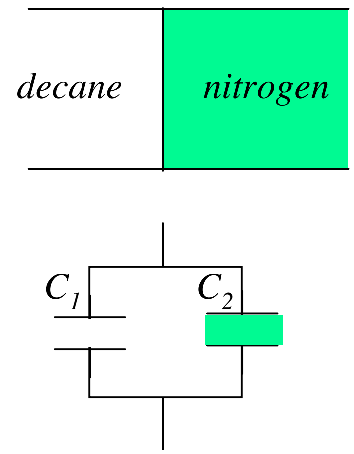
$$P^c = -\frac{1}{\phi}\left[\gamma^{wn}\left(\frac{\partial a^{wn}}{\partial s^w}\right)\Bigg|_{T,\phi,\Gamma^{\alpha\beta},A^{\alpha\beta}} + \gamma^{ns}\left(\frac{\partial a^{ns}}{\partial s^w}\right)\Bigg|_{T,\phi,\Gamma^{\alpha\beta},A^{\alpha\beta}} + \gamma^{ws}\left(\frac{\partial a^{ws}}{\partial s^w}\right)\Bigg|_{T,\phi,\Gamma^{\alpha\beta},A^{\alpha\beta}} \right]$$

* Current Directions: Simple Wedge, Micro-capacitors, 3D

Equivalent Capacitance

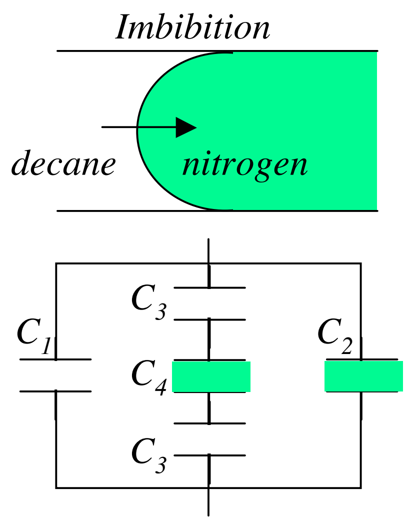


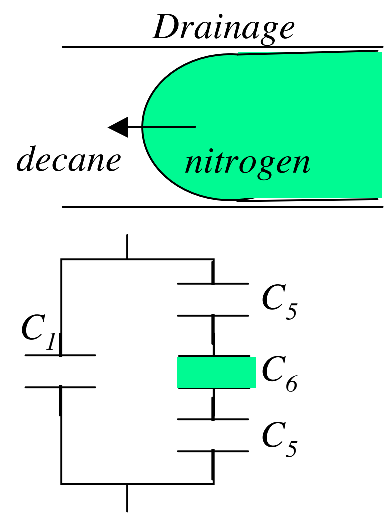


$$C_{equiv} = \frac{A}{d}\left[\left(\varepsilon_{do} + \left.\frac{\partial \varepsilon_d}{\partial \rho}\right|_{\rho_o^d}\left(\rho_d - \rho_o^d\right)\right)S_d + \varepsilon_{no}\rho_n\left(1 - S_d\right)\right]$$

$$C_{equiv} = C_1 + C_2 = \frac{\kappa_1 \varepsilon_o A_1}{d} + \frac{\kappa_2 \varepsilon_o A_2}{d} = \frac{\varepsilon_o w}{d}\kappa_2 L + (\kappa_1 - \kappa_2)x$$

Equivalent Capacitance

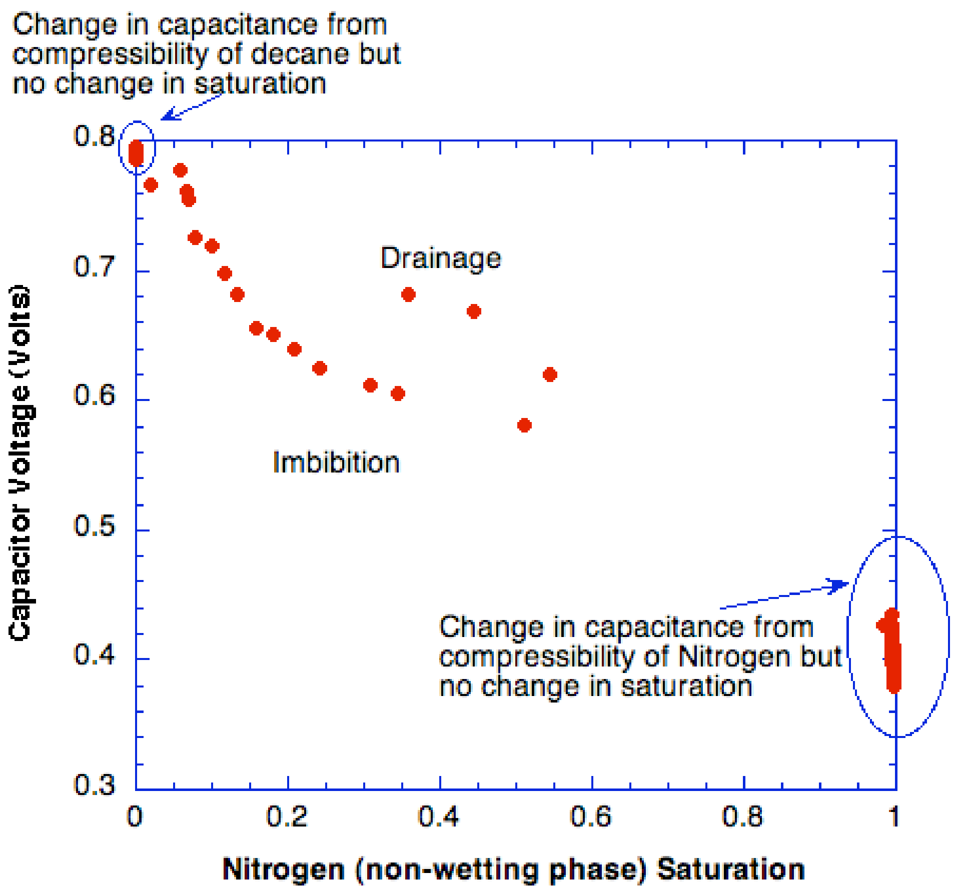


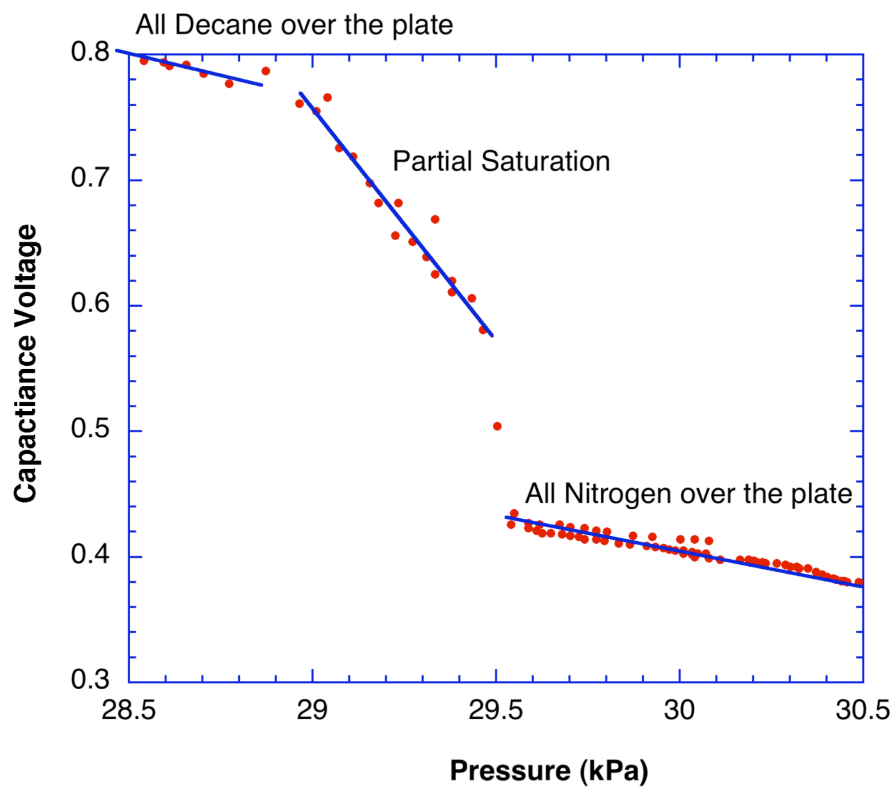


$$C_{equi} = C_1 + C_2 + \left(\frac{1}{2} \frac{C_3 C_4}{C_4 + C_3} \right)$$

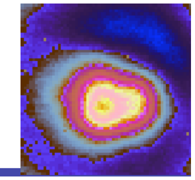
$$C_{equiv} = C_1 + \left\{ \frac{C_5 C_6}{2C_6 + C_5} \right\}$$

Current Directions: Capacitors





Topics:



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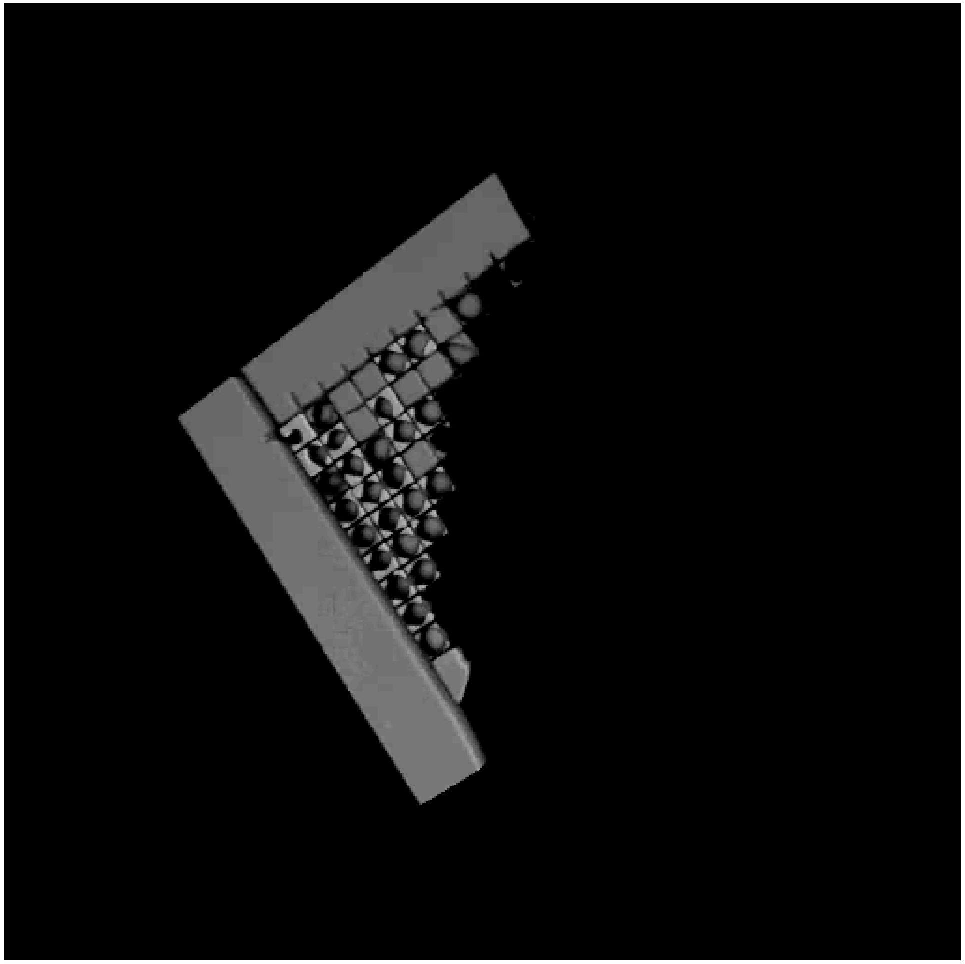
Towards Three Dimensional Micro-Models



Towards Three Dimensional Micro-Models