Invasion Percolation in an Etched Network: Measurement of a Fractal Dimension

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A wetting fluid is displaced at very low flow rate by a nonwetting fluid in a 250 000-duct, transparent, etched network. The structure formed by the injected fluid is ramified and the scale invariance is described by a measured fractal dimension $1.80 < D < 1.83$. This value agrees with theoretical results of invasion percolation with trapping.

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Recently, percolation theory has been used to describe the displacement of one fluid by another in porous media when capillary forces are very strong compared to viscous forces.[1–7] In this paper, we present experimental support for this theory by measuring the fractal dimension of a nonwetting injected cluster in a two-dimensional etched network.

All the displacement mechanisms are linked to capillary forces and randomness due to the different sizes of pores in a porous medium. Generally speaking, when one fluid (say oil) is slowly displacing another nonmiscible fluid (say water) in a capillary tube, the fluid for which the contact angle θ (between the tube and the meniscus) is smaller than $\pi/2$ is called the "wetting fluid"; the other one is the "nonwetting fluid."

Capillary forces prevent the nonwetting fluid from spontaneously entering a porous medium. It can only enter a throat (diameter D_0) when the pressure exceeds the pressure in the wetting fluid by a value P, called capillary pressure, linked to the surface tension γ by the Laplace law $P = 4\gamma\cos(\theta)/D_0$. Assuming that the porous medium can be described by a network of pores (nodes or intersections of the lattice) connected by ducts (bonds), from a statistical point of view a duct with $D > D_0$ is an "active" or "conductive" bond and a duct with $D < D_0$ is an inactive bond. The fraction p of active bonds can easily be deduced from the throat size distribution.[2]

At a given pressure P, the injected fluid invades all the percolation clusters connected to the injection face; this mechanism has been called invasion percolation.[4,8] During the displacement, the wetting phase is trapped in the network when the invading nonwetting fluid breaks the continuous path toward the exit.[2,5] Computer simulations of this invasion percolation with trapping[4,6] using two-dimensional networks (size up to 100×100) show a fractal[9] behavior at the percolation threshold (breakthrough) and also at the end of displacement when all the bonds are active (the capillary pressure is sufficient to allow the displacement in the smallest ducts). For instance, the fraction of invaded ducts (saturation S) in a $L \times L$ network decreases with the size L as

$$S \propto L^{(D-2)}. \quad (1)$$

The fractal dimension D is 1.82 in a two-dimensional network[4] and the difference with ordinary percolation ($D = 1.89$) seems significant.

We have developed a molding technique[10] using a transparent resin and a photographically etched mold to study two-phase flow in porous media and directly measure the structure of the injected cluster. The cross section of each duct of the etched network is rectangular with a constant depth $x = 1$ mm and a width d which varies from throat to throat (generally $d > 0.1$ mm). For this study we used a very large network (300×300 mm) containing 250 000 ducts with seven classes of width d at random locations. In previous work,[2] we have shown that the structure of the injected cluster is independent of the pore-size distribution, so the size distribution is broadened around 50% (Table I) to get better accuracy near the bond percolation threshold (0.5 for a square network).

The wetting fluid is paraffin oil (viscosity $\mu = 20$ cP or 0.020 S.I.), the nonwetting fluid is air ($\mu = 0.02$ cP), the contact angle is zero, and the surface tension $\gamma = 20$ dyne/cm (0.020 S.I.). The nonwetting fluid is injected by slowly decreasing the pressure in the wetting fluid (constant level container) and different experiments are run from 1 to 96 h. This time scale is characterized by the capillary number (calculated for the wetting fluid):

$$N_{ca} = q\mu/\Sigma\gamma, \quad (2)$$

where Σ is the cross-section area of the network and q the mean flow rate of the injected fluid.

For a given capillary number, the experiments are

TABLE I. Proportion and width of the different classes of ducts in the network.

Class	1	2	3	4	5	6	7
Width (mm)	0.27	0.31	0.35	0.39	0.43	0.47	0.51
Fraction (%)	45	2	2	2	2	2	45

reproducible and the structure of the injected cluster (Fig. 1) qualitatively agrees with computer simulations.[2,6] During the displacement, the nonwetting fluid presents very thin and dendritic fingers, and, at the end of the experiment, the cluster size of the trapped phase varies from the pore scale (Fig. 2) to the network scale (large clusters in black, Fig. 1).

So far there have been very few experimental studies of real fractal systems. Some measurements are based upon a relationship between a physical property and fractal dimension: surface roughness by gas adsorption with different sizes of molecules[11] or electrical transfer[12]; structure of aggregates by x-ray or light diffraction.[13,14] More often, the fractal dimension of clusters is deduced from photographs by plotting the number of particles versus the size of a large number of clusters,[15,16] or using a density-density correlation function when it is possible to digitize the photographs.[15,17,18]

In our experiments, we get only one cluster for each run and digitization is quite impossible because of the black meniscus which surrounds the nonwetting phase in each pore (Fig. 2). Consequently, we have to use a simple but laborious technique: From an origin O roughly at the center of the network, we count the number N of invaded ducts in a $L \times L$ square centered in O. A duct is counted only when the nonwetting fluid has invaded the duct and also the pore (intersection) next to this duct (ducts where the meniscus remains at one or both ends are not counted).

When the capillary number decreases, the final saturation S increases (Fig. 3). This phenomenon is due to the possibility for a fraction of the wetting fluid to "escape" by flowing along the roughness of the pores when trapping occurs.[19,20] This mechanism seems not to be relevant at large scale and does not change the fractal dimension of the cluster.

By replacing the saturation S by $N/(L \times L)$ in Eq. (1),

$$N \propto L^D. \quad (3)$$

At the end of the displacement, the variation of N as a function of L (measured in units of the mesh size) for

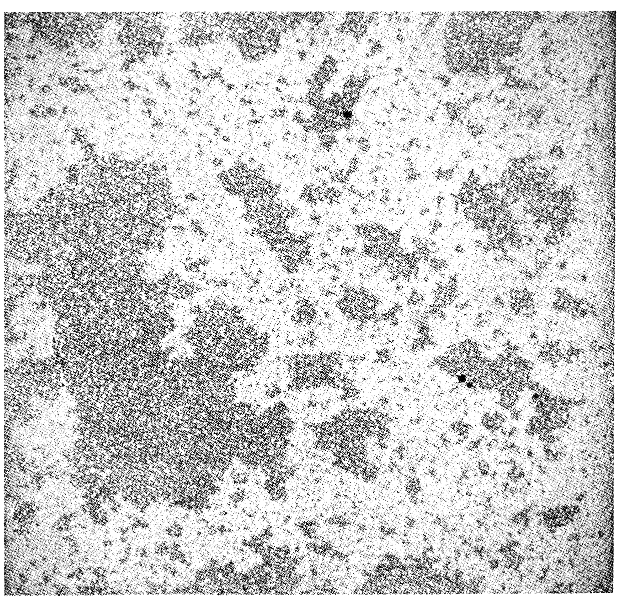

FIG. 1. Displacement of the wetting fluid (black) by the nonwetting fluid injected on the left-hand side of the network. On the right-hand side, a semipermeable membrane prevents the nonwetting fluid from flowing outside.

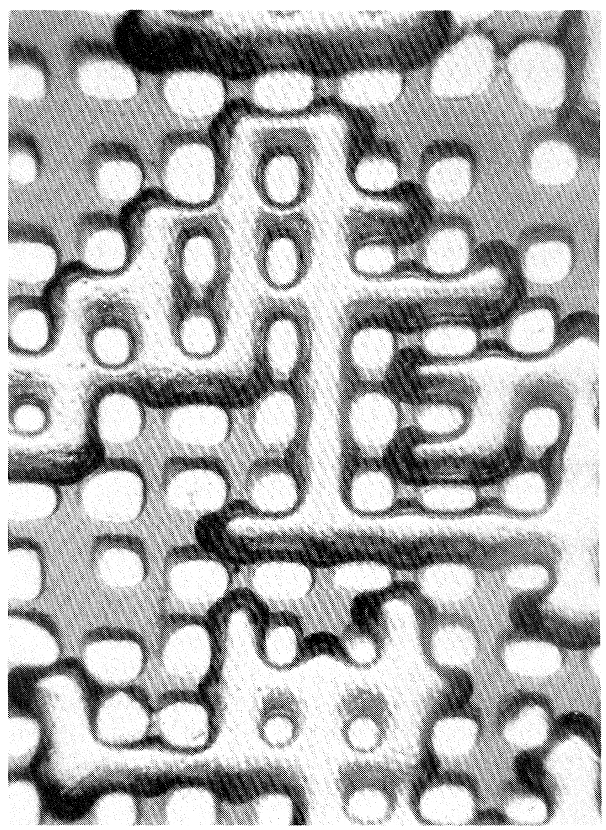

FIG. 2. Close-up of the situation of the wetting fluid (black) and nonwetting fluid (white) in the ducts of the etched network. The distance between two nodes is about 0.8 mm.

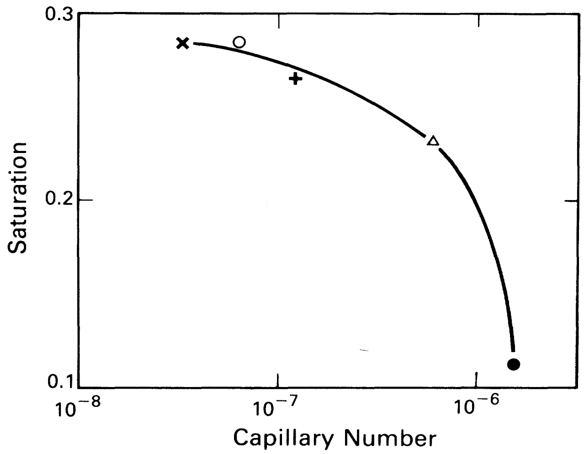

FIG. 3. The mean saturation of the injected fluid in the central zone of the network (125×125) as a function of the capillary number (calculated for the wetting fluid).

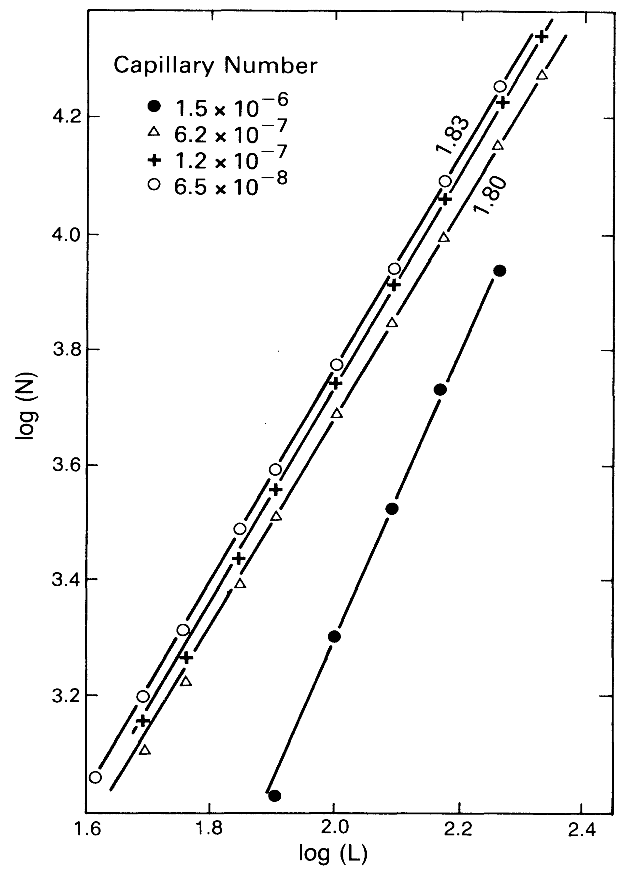

FIG. 4. Number of filled ducts vs size of the $L\times L$ square for different capillary numbers. The slopes 1.83 and 1.80 are least-squares fit to the data. (The data for $N_{ca}=3.3\times 10^{-8}$ are exactly the same as for $N_{ca}=6.5\times 10^{-8}$ and are not shown in this figure.)

different capillary numbers is plotted on a log-log scale in Fig. 4. The curves are linear when the size L is greater than about 70 meshes and a least-squares fit for the slope leads to $D=1.83\pm 0.01$ for the three slowest displacements ($N_{ca}=3.3\times 10^{-8}$, 6.5×10^{-8}, 1.2×10^{-7}) and $D=1.80$ for $N_{ca}=6.2\times 10^{-7}$. These measurements are in good agreement with the theoretical value $D=1.82$.

The highest capillary number ($N_{ca}=1.5\times 10^{-6}$) leads to a different value ($D>2$) and this experiment reveals the weakness of the method used to calculate the fractal dimension. Generally, an accurate method consists in plotting the density-density correlation function $C(r)$ versus the distance r separating the bonds,[21]

$$C(r)=\sum_{r'}\rho(r')\rho(r'+r). \tag{4}$$

The density $\rho(r)$ is defined to be 1 for the occupied bonds and 0 for the others. But instead of averaging on all the values of r', our method uses only one origin O ($r'=0$), which leads to a kind of radius of gyration R (defined in a square):

$$N(R)=\int_0^R \langle\rho(0)\rho(r)\rangle\, d^2r. \tag{5}$$

This equation shows that the origin must be in the cluster [$\rho(0)=1$]. This condition is satisfied for all the experiments except that at the highest capillary number where a large cluster of wetting fluid remains in the central part of the network. This explains the behavior of this experiment. Thus the main problem is not the finite size of the network (for instance, the number of filled ducts is of the same order as the number of particles used in computer studies of diffusion-limited aggregation[21]) but the difficulty of measuring the fractal dimension of a cluster obtained by injection through a side of the network. Consequently, it seems possible to improve the accuracy of our experiments by injecting the nonwetting fluid through one point in the central part of the network.

We conclude that experimental displacements of a wetting fluid by a nonwetting fluid in a two-dimensional random network are consistent with computer simulations of invasion percolation. In particular, the measured fractal dimension of the injected cluster is closed to the simulation value of 1.82. While it is true that the range of L values used is less than one decade, this is also the case for the computer experiments, the latter being limited by the time required at each step to check if part of the displaced fluid has been trapped. Thus, we cannot strictly exclude the possibility that, in both the experiment and simulation, the fractal dimension should really be the same as the value 1.89 for classical percolation. Nevertheless, these experiments do show that the saturation is not "homogeneous" at the scale of the network and that this result is strongly linked to capillary effects.

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[1]P. G. de Gennes and E. Guyon, J. Mec. **17**, 403 (1978).

[2]R. Lenormand, These d'Etat, Institut National Polytechnique, Toulouse, 1981 (unpublished).

[3]I. Chatzis and F. A. L. Dullien, Petroleum Society of Canadian Institute of Mining and Metallurgy Paper No. 7607, 1976 (unpublished).

[4]D. Wilkinson and J. F. Willemsen, J. Phys. A **16**, 3365 (1983).

[5]R. Lenormand and S. Bories, C. R. Acad. Sci. Ser. B **291**, 279 (1980).

[6]R. Chandler, J. Koplik, K. Lerman, and J. F. Willemsen, J. Fluid Mech. **119**, 249 (1982).

[7]P. G. de Gennes, J. Physicochim. Hydrodyam. **4**, 175 (1983).

[8]A displacement at constant flow rate will lead to the same mechanism. See, e.g., E. Guyon, J. P. Hulin, and R. Lenormand, Ann. Mines **17**, Nos. 5&6 (1984).

[9]B. B. Mandelbrot, *Fractals, Form, Chance, and Dimension* (Freeman, San Francisco, 1977).

[10]J. Bonnet and R. Lenormand, Rev. Inst. Fr. Pet. **42**, 477 (1977).

[11]P. Pfeifer, D. Avnir, and D. Farin, Surf. Sci. **126**, 569 (1983).

[12]A. Le Mehaute, A. de Guibert, M. Delaye, and C. Filippi, C. R. Acad. Sci. Ser. II **294**, 835 (1982).

[13]D. W. Schaffer, J. E. Martin, P. Wiltzius, and D. S. Cannel, Phys. Rev. Lett. **52**, 2371 (1984).

[14]J. Feder, T. Jossang, and E. Rosenqvist, Phys. Rev. Lett. **53**, 1403 (1984).

[15]A. Weitz and M. Oliveria, Phys. Rev. Lett. **52**, 1433 (1984).

[16]C. Allain and B. Jouhier, J. Phys. (Paris), Lett. **44**, L421 (1983).

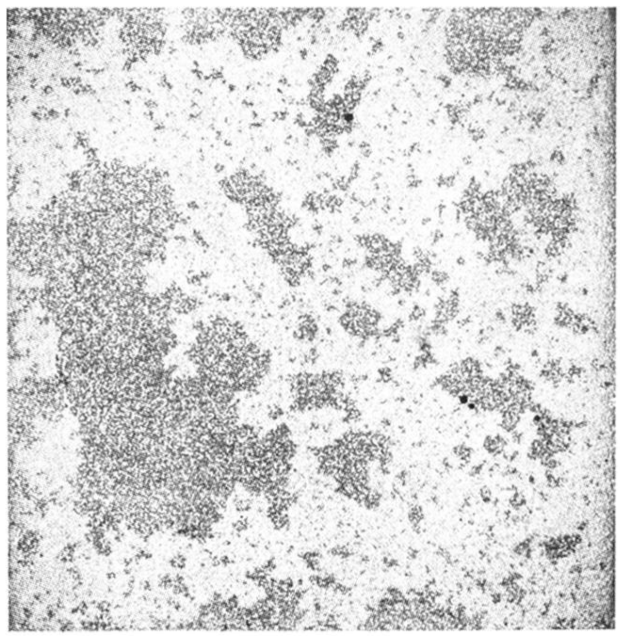
[17]L. Niemeyer, L. Pietronero, and H. J. Wiesmann, Phys. Rev. Lett. **52**, 1033 (1984).

[18]M. Matsushita, M. Sano, Y. Hayakawa, H. Honjo, and Y. Sawada, Phys. Rev. Lett. **53**, 286 (1984).

[19]R. Lenormand, C. Zarcone, and A. Sarr, J. Fluid Mech. **135**, 337 (1983).

[20]R. Lenormand and C. Zarcone, Society of Petroleum Engineers Paper No. 13264, 1984 (unpublished).

[21]T. A. Witten and L. M. Sander, Phys. Rev. Lett. **47**, 1400 (1981), and Phys. Rev. B **27**, 5686 (1983).



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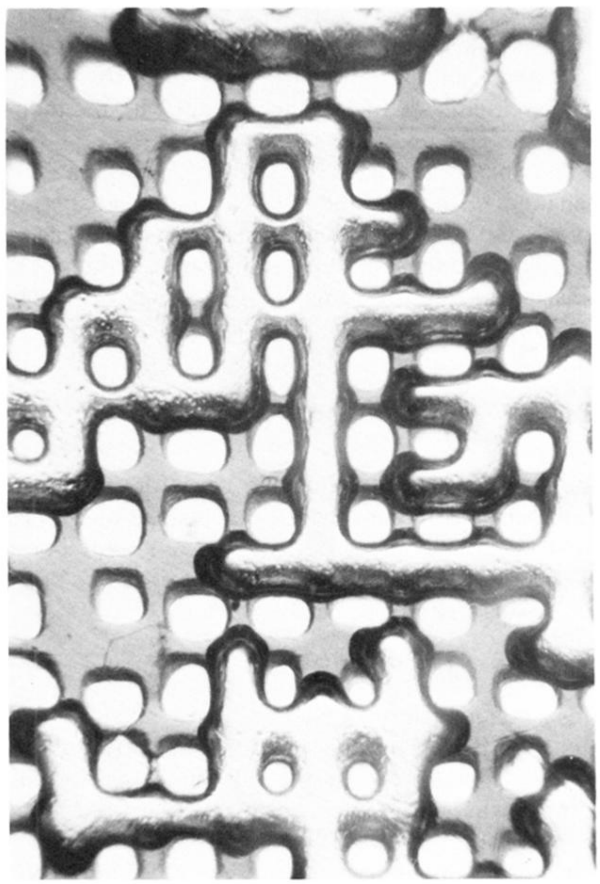

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