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FLUID PERCOLATION THROUGH SINGLE FRACTURES

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Abstract. Large deviations from "cubic-law" dependence of laminar fluid flow through fractures on the apparent mechanical aperture of a fracture can be explained by assuming: 1) cubic-law dependence of flow on the actual local aperture at the microscopic level; 2) conservation of rock volume when deforming the fracture; and 3) macroscopic flow properties are dominated by the critical neck (the smallest aperture along the path of highest aperture through the fracture).

Introduction

The flow of fluids through fractures in rock is a process that has importance for many areas of the geo-sciences, ranging from ground-water hydrology to oil recovery to nuclear-waste isolation. These fields require a detailed understanding of how fluids flow through fractures and sets of fractures in rock masses. Before this more global problem can be addressed, it is necessary to investigate the flow behavior of fluids through the primary unit of these networks: a single fracture in rock. In particular, it is important to be able to predict how stress fields in-situ will affect the fluid transport through a fracture. The rate of fluid flow through fractures in rock depends on the complicated three-dimensional topology of the flow paths through the fracture. The flow is constrained by the areas of contact between the two fracture surfaces, and by the distribution of apertures in the void spaces adjacent to these areas.

Measurements of the topographies of natural rock surfaces, for example Brown and Scholz [1985] and Brown et al. [1986] show that apertures, and therefore contact areas, are not independent, but are spatially related to each other. This relation is a consequence of the correlations of the fracture surfaces: points of contact or small apertures are likely to be surrounded by other points of contact or small apertures, while points of large aperture are likely to be surrounded by other points of large aperture. Distributions of contact area between natural fracture surfaces in granite have been measured experimentally [Pyrak-Nolte et al.,1987] and have been modeled using the fractal dimensions of flow path patterns to quantify the degree of correlation [Nolte et al.,1986]. The present paper includes a brief description of the model which is used to generate apertures in the void space that are directly related to the distribution of contact area. (Contact area is defined as zero aperture.) Once the flow path topology has been defined, the effects of normal stress on the flow rate through the model fracture can be analyzed. Normal stress across the fracture changes the flowpath topology by diminishing the apertures and increasing the areas of contact. The flow is strongly dominated by the path of highest aperture, called the critical path, and in particular the point of smallest aperture along this path, called the critical neck. The effects of stress on flow through fractures can be understood largely by describing the mechanical deformation of the critical neck.

Analysis

The dependence of fluid flow on fracture aperture has often been approximated to lowest order by considering the fracture to be parallel plates [for example, Lomize, 1951; Witherspoon et al., 1980]. Flow between parallel plates depends on the cube of the plate separation, or aperture, leading to the "cubic-law" for flow. The validity of the cubic-law for flow through fractures has been debated extensively in the literature with considerable disagreement. It is important to start by defining the term "apparent fracture aperture" precisely. In experimental work, displacements are measured across a gauge length containing the fracture, d_t, as a function of the normal stress across the fracture. The displacement induced by this stress across the fracture itself is then determined by subtracting from this measurement the displacement produced across an equal length of intact rock, d_r, by the same stress. This difference, $(d_t - d_r) = d_{mech}$, is frequently referred to as the fracture deformation [Witherspoon et al., 1980; Raven and Gale, 1985]. Curves of fracture deformation against stress are typically non-linear and asymptote to some constant maximum value, d_{max}, at the highest values of stress. The apparent aperture of the fracture, $\underline{b}$, at any stress, may be defined as the difference between this maximum value and the fracture deformation; $\underline{b} = d_{max} - d_{mech}$.

Some experimental evidence supports the cubic-law [Witherspoon et al.,1980] dependence on apparent aperture. However several investigators using natural fractures [Raven and Gale,1985; Pyrak-Nolte et al.,1987] have found that flow depends on the apparent aperture of a fracture raised to exponents considerably larger than cubic. This behavior is described by

$$Q - Q_\infty \propto (\underline{b})^n$$

$$\propto (d_{max} - d_{mech})^n \tag{1}$$

where d_{mech} and d_{max} have been defined above and Q_∞ is the residual or irreducible flow. The role of increasing contact area with increasing stress is implicitly included in Equation (1) through the dependence of tortuosity on apparent aperture. Several investigators have considered the influence of changing tortuosity on fluid flow. Walsh [1981] concludes that because flow depends on the cube of the aperture, but depends only linearly on tortuosity, the effects of tortuosity can be neglected. In contrast, a theoretical investigation of Tsang [1984] found that tortuosity and surface roughness can greatly affect flow. We found that the empirical dependence of flow on mechanical aperture can be explained partially, but

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not entirely, by explicitly including the effect of increasing tortuosity on flow through fractures, derived from the principles of percolation theory [Nolte et al.,1986]. The dependence of flow on mechanical aperture, after tortuosity has been included, continues to have exponents larger than cubic, which indicates that mechanical and hydraulic aperture are related in a non-trivial way. This problem has been approached numerically by Brown [1987] using a model with correlated (fractal) surfaces, but neglecting mechanical deformation of asperities and the adjacent half-spaces of the rock. The goal of the present paper is to explain the large values observed for the exponent n in Equation (1) in a simple way that includes mechanical deformation and essential results from percolation theory without requiring detailed fluid transport calculations. We make three basic assumptions: (1) cubic-law dependences are explicitly assumed at the microscopic level; (2) the details of the deformation mechanics of the fracture surface are included to lowest order by requiring conservation of rock volume; and (3) the macroscopic flow properties are dominated by the deformation of the critical neck. This last point will be justified based on results of percolation theory. From these assumptions we find macroscopic dependences of flow on apparent aperture that have exponents larger than cubic, but with cubic dependence as a lower bound.

To model fracture contact area and local apertures, including correlations, a stratified percolation model has been developed[Nolte et al.,1986]. This model is a hybrid between the standard random continuum percolation construction (in which small void areas are randomly positioned within a given area, see Pike and Seager [1974]), and the recursive structures called fractals (see Mandelbrot [1983]). In the stratified percolation model, the procedure begins by randomly distributing N sites within a given square region, called a tier. Each site defines the center of a new tier which is smaller, by a scale factor, than the parent tier. Within each of the new, smaller tiers, N sites are randomly distributed which define the centers of yet new tiers that are again reduced in size by the same scale factor from the immediately preceding tier. The procedure continues for as many iterations as is necessary, or possible, within the resolution of the graphics. At the last iteration, squares that are reduced in size from the original square size by the scale factor raised to the number of tiers are finally plotted. The results for five tiers are shown in Figure (1a) with a scale factor of 2.37 between tiers. There are nine areas per tier which culminate on the smallest level with the black squares that represent the void area. The total number of black squares plotted in the figure is equal to $(9)^5$ = 59,049. The void area fraction for this pattern is 83%, and has an approximate fractal dimension of D = 1.94.

As the final squares are randomly plotted, many squares will overlap. The aperture at a given site in Figure(1a) is taken to be directly proportional to the number of times a black square was positioned over that site during the construction of the pattern. The aperture distribution that results is displayed in the contour plot of Figure (1b). This distribution occurs as a result of the correlations introduced by the iterative construction; we have assumed no specific functional form for the aperture distribution. The critical path and the critical neck for flow through this aperture distribution can be recognized from the figure.

The distribution of apertures, P(x), is defined through the relation

$$P(x)\,dx = \text{fraction of fracture area with apertures between } x \text{ and } x + dx$$

To simulate the effects of normal stress on this distribution of local apertures, we treat the rock as incompressible yet deformable. The incremental displacement of the rock mass

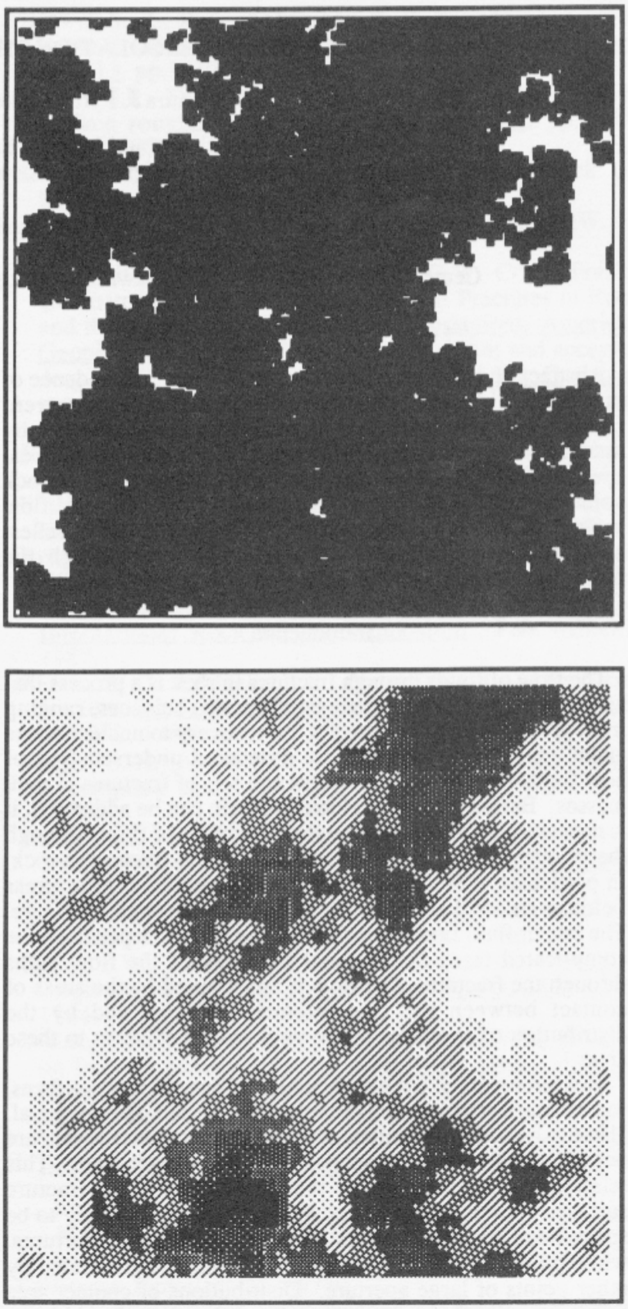

Fig. 1 Stratified percolation model of the void space in a fracture in rock. The black and white plot has 9 points per tier for 5 tiers with a scale factor of 2.37 between tiers. The contact area fraction for this pattern is 17%. In the corresponding aperture contour plot, white denotes contact while black denotes the largest apertures. The critical neck is visible in the center of the figure.

far from the fracture is therefore determined entirely by the closure of the void volume in the fracture, conserving the volume of the rock mass. As the apertures are closed by compressive stress, the average far-field displacement of the rock mass, d_{mech}, is not equal to the actual displacement of the void aperture, d_{void}. Expressions for the void and contact areas are derived from the aperture distribution P(x) as functions of the void space closure, d_{void}. These are, respectively

$$A_{contact}(d_{void}) = A\left\{P(0) + \int_0^{d_{void}} P(x)\,dx\right\}$$

$$A_{void}(d_{void}) = A\int_{d_{void}}^{\infty} P(x)\,dx \qquad (2)$$

where A is the cross-sectional area of the fracture plane, and P(0) is the zero-stress fraction of contact area. Changes in the far-field displacement, $\delta(d_{mech})$, will be less than changes in the void aperture displacement $\delta(d_{void})$, reduced by the ratio of the area of the void space to the cross-sectional area of the fracture plane:

$$A\,\delta(d_{mech}) = A_{void}(d_{void})\,\delta(d_{void}) \qquad (3)$$

Equation (3) is the statement of conservation of volume. Integration yields the relationship between far-field displacement and void aperture closure:

$$d_{mech}(d_{void}) = \frac{1}{A}\int_0^{d_{void}} A_{void}(y)\,dy \qquad (4)$$

In the limit of high stress, the d_{mech} approaches the asymptote defined by $d_{max} = d_{mech}(\infty) = V_{void} / A$, where d_{max} is the maximum mechanical displacement of the fracture (measured from the far field) and V_{void} is the zero-stress void volume of the fracture. The relationship between apparent fracture aperture and the closure of the void space apertures is shown in Figure (2) for the pattern of Figure (1). To generate this curve, for every unit closure of aperture, the mechanical displacement at the far field was calculated based on conservation of volume from Equation (3). The straight line asymptote denotes the case where $d_{mech} = d_{void}$. There is clearly a non-linear relationship between mechanical and void space aperture displacement, which will be reflected in the dependence of flow on mechanical displacement. The aperture of the critical neck is also shown. This critical aperture is determined from the contours in Figure (1b) as the smallest aperture along the critical path.

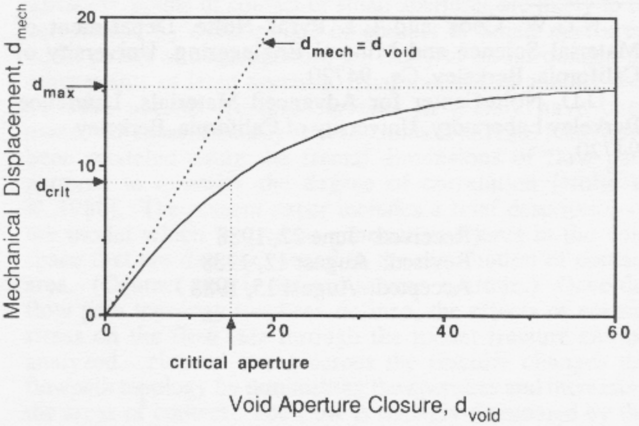

Fig. 2 Mechanical displacement, d_{mech}, vs. reduction in the void space aperture, d_{void}. The straight line denotes equality between the two quantities. The mechanical displacement approaches an asymptote defined by the original void volume of the fracture. The critical aperture is also shown with the corresponding critical mechanical displacement required to close the critical neck.

The use of far-field displacement in Equation (1), though convenient, would not be expected to yield a cubic-law dependence of flow on mechanical fracture displacement. The relation for flow through the critical neck is

$$Q - Q_{\infty} \propto (b_{crit})^3 \qquad (5)$$

where b_{crit} is the aperture of the critical neck as a function of stress, and cubic dependence on the critical aperture has been explicitly assumed. As stress is increased, b_{crit} decreases until it is closed completely at the critical mechanical displacement d_{crit}. We continue to include the irreducible flow Q_{∞} because our assumption of conservation of volume is only an approximate way to include the deformation properties of the fracture. In practice, the void space will not close completely, even at high stresses, which will allow some residual fluid flow.

The deformation of the critical neck dominates the macroscopic flow properties of the fracture. Therefore the macroscopic flow through the fracture should also behave as Equation (5). Justification for this approximation can be found from percolation studies of networks of nodes connected with strongly inhomogeneous conductances. Several researchers [Ambegaoker et al.,1971; Shklovskii and Efros,1971; Pollak,1972] found that the macroscopic conductivity is determined by the conductance of the elements that first create percolation across the distribution, i. e. the critical neck. To satisfy the criterion of strong inhomogeneity, the conductances must vary by more than 4 orders of magnitude[Seager and Pike,1974]. This criterion is satisfied in our model because the fluid conductance depends on the square of the aperture, and the apertures are distributed over 2 orders of magnitude. Furthermore, the correlations inherent in our model should relax this requirement of strong inhomogeneity because of the tendency to "channel" the flow along a single path, reducing the possibility of parallel flow channels. It is important to compare fluid transport measurements with electrical transport through saturated fractures. Electrical measurements have been one method used to probe void space geometry[Stesky, 1986]. Yet the percolation analysis based on the critical neck for fluid flow cannot be applied directly to electrical conductivity. Electrical conductance depends linearly on local apertures and therefore the local conductances may not satisfy the criterion for strong inhomogeneity for the critical neck to be applicable.

We are now able to plot the dependence of flow from Equation (5) on the apparent aperture ($d_{max} - d_{mech}$) from Figure (2). These quantities are displayed in the log-log plot in Figure (3). The straight line shows the dependence expected from Equation (1) for an exponent of 3. At the lowest stresses, the slope of the simulated curve approaches the slope of the cubic relationship. However, at even moderate stresses the exponent increases significantly from cubic and can be arbitrarily large at the highest stresses. The influence of each of our three assumptions on the results of Figure (3) can now be discussed separately. Assumption 1): The assumption of cubic-law flow on the local microscopic aperture greatly simplifies the analysis and makes the connection with laminar fluid transport. The bound on the slope in Figure (3) for low stresses is a direct consequence of this assumption. Assumption 2): Conservation of rock volume during deformation is a simple yet important way to include the deformation mechanics of the asperities and surrounding rock mass. Deviations from cubic-law dependence at low and moderate stresses result from the non-linear relationship between far-field displacement and the closure of the void space, expressed in Equation (4). Assumption 3): The dramatic deviation from cubic-law dependence at high stress (neglecting irreducible flow) is a consequence of the dominance of the critical neck on flow through the fracture. Conservation of volume cannot alone explain this high-stress behavior.

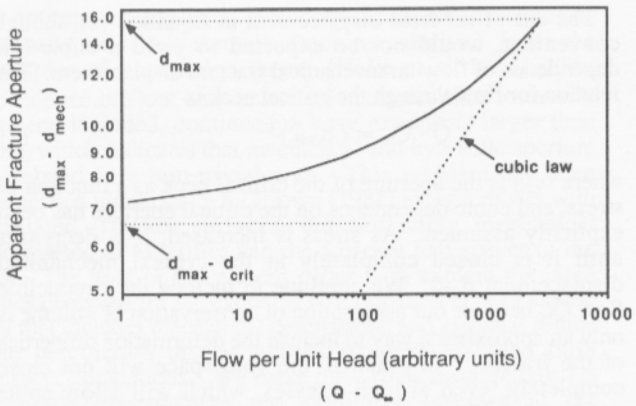

Fig. 3 Relationship between fluid flow, $(Q - Q_\infty)$, and the apparent fracture aperture, $(d_{max} - d_{mech})$. The straight line describes cubic-law dependence on mechanical aperture. The slope of the log-log plot approaches 3 at low stress, but increases significantly with increasing stress.

Conclusions

In conclusion, we are able to explain the large exponents observed for Equation (1) based on three basic approximations: 1) cubic-law flow at the microscopic scale; 2) conservation of rock volume during deformation of the fracture; and 3) dominance of the critical neck on the macroscopic flow properties. Although the concept of a critical neck is somewhat intuitive, we provide a firm, quantitative basis for this assumption based on percolation theory. It should be emphasized that our explanation is not sensitive to the specific model used to construct the aperture distribution, as long as the model includes the correlations observed experimentally. We have used the stratified percolation model which has these correlations automatically built-in. While the exact flow properties of a fracture will depend on the details of the fracture topology, our analysis presented here can explain the gross features of the flow properties of a fracture very quickly and simply.

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