Fluid flow through single fractures

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ABSTRACT: A self-similar solution is used to analyze streamline and continuum fluid flow through single fractures. The solution is obtained by using the self-similar solution for the problem of self-similar flow through a fracture. The solution is applied to a set of rectangular fractures. In each case, the solution is used to calculate the flow rate through the fracture.

INTRODUCTION

A major challenge in the analysis, design, and building of underground structures is to understand how changes in effective stress affect the hydraulic properties of a fractured rock mass. The key to understanding the hydraulic, mechanical, and seismic properties of natural fractures is in the ability to describe the way in which fractures change in size and shape and the way in which the voids in a fracture set deform. In order to understand the flow through fractures, an understanding of the mechanics of hydraulic fractures is needed. In this paper, we will focus on the mechanics of hydraulic fractures and the way in which flow through fractures is affected by the properties of the fracture.

Fluid flow through a fracture is fundamentally different from fluid flow through a porous medium. In a fracture, fluid flow occurs in a thin, two-dimensional plane and is controlled by the distribution of apertures. The complex flow paths of a fracture system is due to the presence of a fracture system with two rough surfaces in parallel contact. Between the areas of contact, there exist voids of variable porosity and aperture. If a fracture is placed under stress, the fracture voids deform, resulting in an increase of contact area, a reduction of void apertures, and a reduction in fracture void volume. The void geometry of a fracture will be influenced by the roughness of the individual fracture surfaces and the orientation of the two surfaces (Shapira, 1983; Brown et al., 1986). Several investigations have made measurements of fracture system porosity and size distribution and distribution of fracture apertures. The size distribution of fracture apertures is important in understanding the mechanical properties of fractures.
fracture. They concluded that if the rock specimen is smaller than the large scale roughness wavelength then the fluid flow measurements made on the specimen will not be representative of the large fracture behavior.

MODEL DESCRIPTION

Examples of the flow path geometry in a natural fracture at three different stress levels are shown in Figure 1. Flow paths are white while the contact areas are black. At the lowest stress (Fig. 1a) the contact area appears as isolated "islands" of contact. At high stress (Figs. 1b and 1c) these areas of contact become continuous with "tiers" of metal connected by filamentary seams "streams" of metal. These images show that the distribution of voids and contact areas are heterogeneous but correlated. Thus, a void site has a high probability of being surrounded by other void sites, and, conversely, a point of contact has a high probability of being surrounded by other points of contact.

We have chosen a stratified continuum percolation model to investigate fluid flow (saturated and unsaturated) in fractures, the change in fluid flow with stress in fractures, and the effect of sample size on saturated fluid flow through fractures. This model (Noble et al., 1989) incorporates the randomness of standard continuum percolation and the scaling nature of fractures, and reproduces the type of flow geometry observed in experiments. A continuum model is used because the distribution of void spaces is continuous and there is no underlying lattice structure in a fracture. To model the flow path geometry, a pattern is generated by placing N random sites within an area called a tier. Each one of these sites represents the center of a new tier which is smaller in size than the preceding tier by a scale factor b. In each of the new tiers, N sites define the center of yet another series of tiers, which are smaller than the preceding tier by the same scale factor b, and are again randomly distributed. This process can continue for as many tiers as desired. The final result is a correlated pattern. The pattern in Fig. 2a represents a fracture under low stress because of the small amount of contact area (white areas in Fig. 2). This pattern was generated using a five-tier model with twelve points per tier and a scale factor of 2.37 between tiers.

Fig. 1. Composite micrograph of a portion of a natural fracture at effective stresses of (a) 3 MPa; (b) 35 MPa; and (c) 85 MPa. Black represents contact area and white flow paths.
The aperture distribution of a generated pattern is related to the density of sites in the construction of the pattern. Figure 2a is a map of the aperture distribution of its pattern shown in Figure 2a. White areas in Figure 2b represent contact areas, and black areas represent sites of largest aperture. It is observed that the aperture distribution is correlated, that is, sites of large apertures have high probability of being surrounded by other sites of large apertures.

SATURATED FLUID FLOW AND MECHANICAL DEFORMATION

Previously, measurements of micromechanical displacement and fluid flow through three different natural fractures in a suite of quartz monzonite were carried out (Pyne-Nolle et al., 1987). The fluid flow data were found to deviate from a "cubic law" behavior and to show a dependence on apparent mechanical aperture much greater than cubic.

Pyne-Nolle et al. (1987) found that deviations from cubic law behavior can be explained by assuming (1) a cubic-law dependence of flow on the actual local aperture at the microporous level; (2) conservation of rock volume when defining the fracture; and (3) macroscopic flow properties are determined by the critical neck (the smallest aperture along the path of highest aperture through the fracture). These assumptions were applied to fracture flow path geometries generated from the simplified percolation model. The apparent mechanical aperture (i.e., the displacement) of the pattern was determined using a micromechanical approach assuming that rock volume is conserved during fracture deformation. If the fluid flow is reduced to one site of aperture, the result in the apertures does not simply vanish or reappear into the opposite fracture surface. Thus, the apparent aperture is not measured by fracture closure, but rather by some function of the aperture reduction, depending on the relative area covered by the aperture. By assuming conservation of volume, a non-linear relationship exists between apparent aperture and void aperture closure which depends upon the topology of the void space and areas of contact. The amount of void aperture closure is greater than the amount of mechanical displacement. This results in the critical neck being closed faster than the apparent aperture.

The non-linear relationship between the apparent aperture and void aperture closure is important to understand the results. Figure 2c shows the relationship between fluid flow through the fracture and changes in apparent fracture aperture. Deviations from cubic law behavior would not exist if fluid flow through the fracture depended on the apparent mechanical aperture, or if the apparent aperture closure and void aperture closure were equal. However, because of the non-linear relationship between apparent mechanical aperture and void aperture closure, deviations from cubic law behavior were.
SCALING BEHAVIOR OF SATURATED FLUID FLOW

A major practical problem in studying the flow of fluids through geologic formations, is how to relate laboratory measurements to behavior in the field. Laboratory measurements are performed on relatively small samples with sizes as the order of a few centimeters up to, perhaps, a meter, while flow through geologic formations may occur over kilometers. Do the laboratory measurements have anything to do with macroscopic behavior, or do different mechanisms dominate at different scales? Can hydraulic measurements performed in the laboratory on core samples be used to quantitatively predict behavior in situ? One step towards solving this dilemma involves the use of renormalization techniques. Renormalization group theory deals specifically with the question of how the physical properties of random patterns scale with the size of observation (Pelton and Touloucan, 1977). More specifically, renormalization techniques can be applied easily to percolation problems to quantify the size dependence of permeability probabilities.

We have applied these renormalization techniques to our model of stratified continuum percolation through single fractures. The two properties we study are the spanning probability, $R(0,L)$, and the size of the spanning cluster, $L_s(0,L)$. The spanning probability is the probability that a continuous flow path will span a sample of size $L$ in a specific void space area. The strength of the spanning cluster is the fraction of the void space area that belongs to a spanning flow path. The renormalization procedure to determine these two probabilities is simple: divide a flow path pattern into subsections of size $L_1$ and count how many of the subsections have a spanning flow path. Provide this number by the total number of sections to obtain $R(0,L)$. For these sections that do have a spanning path, count the fraction of the void space area that belongs to that flow path, to yield the probability $L_s(0,L)$. The results from Monte Carlo simulations for $R(0,L)$ and $L_s(0,L)$ are shown in Figure 3 and Figure 4 for stratified continuum percolation patterns with three times.

The spanning probability $R(0,L)$ has the important feature that the probability is invariant of sample size at the percolation threshold (Fig. 3). This is called the fixed point, and provides a direct method to determine percolation thresholds. Stratifed percolation has the useful property that the percolation threshold remains invariant when experimenting with fractal patterns (Pelton, 1948). Below the threshold, the percolation probability decreases with increasing sample size. The opposite is true above the threshold: the percolation probability increases with increasing sample size. These opposite trends can create confusion when experimental results from different samples or different sample sizes are presented.

The size of the spanning cluster $L_s(0,L)$ rises sharply with increasing void space coverage (Fig. 4). The

Fig. 3. Spanning probability of three-tier stratified percolation patterns calculated using renormalization group techniques for sample sizes of $L_1$, $L_2$, $L_3$, and $L_4$. The threshold occurs at $p_c \approx 0.35$. (Noble, 1949) Area normalized with respect to flow path pattern area.

Fig. 4. The size of the spanning cluster (three-tier stratified percolation patterns for grid sizes $L_1$, $L_2$, $L_3$, and $L_4$) as a function of area fraction of void space. Area normalized with respect to flow path pattern area.
correlation that are naturally invoked in saturated percolation cause the clusters to coalesce together. Therefore when a spanning cluster occurs, it is likely that most of the void space will belong to that cluster. Also, the apparent permeability threshold moves to lower fracture void space as the sample size decreases. For our smallest size in the simulations, the apparent threshold is near $A < 0.15$.

The permeability estimates described here can in principle be obtained experimentally from core samples by measuring the flow path geometry using the metal injection technique of Ryska-Mooib et al. (1974). Such a statistical analysis would establish whether a given fracture was above or below the permeation threshold. This information is of significant qualitative importance because fracture geometry in the field may not support flow (if the void geometry is below the permeation threshold) even if core samples do have connected paths across the sample.

The final difficulty is to find quantifiable values for the flow. Unfortunately, there is no direct method of measuring flow rates from percolation probabilities through obvious innate voids; increased permeation probability will yield higher flow rates. The most critical point is that fluid conductivity apparently decreases with increasing sample size, if at all remains the same. This large size scaling relationship for systems near the percolation threshold value can be used to estimate conductivities measured at different scales. The available dependence of the conductivity (Stauffert, 1985) is given by $k_d = L_t^{v_x} v_{x}$, where $v_x$ is the conductivity exponent (Hagerman et al., 1985) and $v = 4.5$ is the correlation exponent.

UNSATURATED FLUID FLOW

Unsaturated fluid flow through single natural fractures was also analyzed using unsaturated continuum percolation theory. When two or more percolation paths are present in the fracture, we investigated the relative permeabilities of two immiscible fluids in a simulated fracture geometry and considered the effect of stress on relative permeability. This investigation does not deal with long-term or dynamic solution. We allow the non-saturation to occupy the large voids and assume steady-state conditions when we calculate relative permeabilities. The results from this unsaturated fluid flow investigation are based on three simulations of fracture flow field, one for each of which used a grid model with twelve points per side and a scale factor of 3.37. A representative pattern is shown in Figure 2.

Analysis of unsaturated flow through a simulated fracture geometry, by assuming the void space with wetting phase. The non-wetting phase is introduced into the largest fractures and then allowed to occupy passively smaller fractures. The phases are assumed to have the same density and viscosity, but different surface tension. As the non-wetting phase is allowed into lesser fracture spaces, it eventually forms a connected path of highest permeability, and begins to flow (Fig. 3). Both the wetting phase and the non-wetting phase then flow through the fracture. If the non-wetting phase is allowed into even smaller fractures, it will eventually cut-off the percolating path of the wetting phase which then cannot flow (Fig. 3b).

The critical neck for the wetting phase, which is also the minimum aperture along the connected path of highest permeability, is openable until it is occupied by the non-wetting phase which then begins to flow. When the non-wetting phase begins to flow, the path is always flowing along the critical path because the non-wetting phase has been introduced into the largest aperture. The critical neck for the wetting phase is the critical connection that maintains a percolating path for the wetting phase. It is open until filled with non-wetting phase, causing the wetting phase to cease to flow.

To determine the permeability of the two phases in a fracture, relative flow of each phase was evaluated. To calculate fluid flow through the model, a serial order approach is taken that includes only the simplest dependence, which are: (1) the cubic law describes the local dependence of fluid flow on aperture; and (2) the two-dimensional critical behavior is isolated by a scaling law that leaving changing invariance. Laminate, flow between parallel plates (cubic law) is saturated for fluid flow of both phases through the critical neck. Invariance is important only for calculating the wetting phase permeability because the non-wetting phase is introduced; the wetting phase is added to its high permeability path or critical path, and its path becomes more tortuous or 3D narrower.

The expression for wetting phase flow through the pattern is:

$$Q_w = \left[ \left( \frac{4}{3} \pi \left( \frac{L_t}{2} \right)^3 \right) \right] \left( \frac{1 - \beta \rho}{\beta \rho} \right)$$

where
- $w$ = wetting phase
- $c$ = critical
- $Q$ = flow
- $a$ = area normalized with respect to whole area of the fracture
- $b$ = aperture of critical neck

$\beta = 1.2$ is final or first critical neck. Invariance is incorporated into the expression for relative permeability of the wetting phase $(Q_w / Q_s)$ through a scaling term, $\left( \frac{1 - \beta \rho}{\beta \rho} \right)$, where $\beta$ is the normalized neck area occupied by the wetting phase, $a_s$ is the normalized neck area of the wetting phase at permeation threshold, and the exponent $b$ is a critical exponent. The critical exponents, i.e., range between 1.5 and 2.7 for standard random continuum percolation. We have assumed a value of 1.9 for the critical exponent.
The non-wetting phase flow, in contrast, is always dominated by the main critical path because it occupies the largest apertures of the pattern. Therefore, the aperture of the non-wetting phase does not change with increasing non-wetting phase saturation. However, the width of the non-wetting phase flow path does change with increased non-wetting phase saturation and must be accounted for in the relative flow expression. The expression for non-wetting phase flow is

\[ Q_{nw} = \frac{Q_{nw}}{Q_{nw} + Q_{nw} - Q_{nw}} \]
Fig. 7. Effect of stress (σ) on relative permeability for a reduction in aperture of S, 20, and 50 units. (b) An enlargement of the cross-over region of (a), the arrows indicate the minimum and maximum values of wetting phase saturation for the cross-overs in relative permeabilities as a function of stress. The cross-overs in relative permeability are essentially invariant of stress.

wetting phase saturation. This leads to the important conclusion that if the percentage saturation of one of the phases is known, one can determine which phase dominates the flow at any stress.

CONCLUSIONS

A unified continuum percolation model was used to simulate experimentally-observed flow path geometries. This model, which is based on a fractal construction, with correlated contact area distributions and aperture distributions. A percolation model was chosen in order to see the "wetting body of knowledge of percolation theory in analysing fluid flow through single fractures. Using the results of percolation theory and the principle of conservation of volume, we were able to understand the deviations from cubic law behavior that have been observed for saturated fluid flow through single fractures. Also, we were able to analyse unsaturated fluid flow through single fractures using the "unified percolation model".

A major issue in experimental work concerns the relationship between behavior measured in small laboratory samples (less than a meter in dimension) and fluid flow through fractures in the field (with dimensions perhaps of kilometers). Using mean-field theory, we have begun to address this issue of the effect of sample size on the hydraulic properties of single fractures. Renormalization group theory deals specifically with the question of how the physical properties of random patterns scale with the size of observation. From this analysis, it is concluded that the probability that a fracture will support field flow is dependent on sample size.

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