

# Microwave spectroscopy of electron solid and stripe phases in higher Landau levels

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## Abstract

Our recent broadband microwave and RF spectroscopic measurements of two-dimensional electron systems at Landau level filling  $\nu \gtrsim 1$  are reviewed. Resonances in the spectrum of diagonal conductivity in many discrete regions of  $\nu$  allow us to identify and study the pinned electron solids.

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## 1. Introduction

For Landau level filling,  $\nu$ , close to integers, the possibility of Wigner solidification of particles or holes from the top Landau level with occupation was considered [1] from the early days of the quantum Hall effects. More recently, “bubble” phases with clusters of electron guiding centers on a lattice, and also anisotropic striped phases were predicted [2] and studied experimentally [3,4] for higher Landau levels.

This paper reviews our microwave spectroscopic studies [5–10] of electron solids, with  $\nu$  near 1 or higher, including Wigner solids near integers but also the more complex bubble solid and striped phases. A striking feature of the isotropic solids like the analogous ones [11–13] found in the lowest Landau level, at the high magnetic field ( $B$ ) termination of the fractional quantum Hall series, is a resonance in the microwave spectrum. The electrons that form the solids all have vanishing diagonal conductivity in low temperature limit, due to pinning of the solid by disorder. The resonance is a pinning mode, or oscillation of

the solids about their pinned positions. The reviewed experiments demonstrate the value of studying the solids with broadband spectroscopy, which can quantitatively characterize the resonance.

## 2. Technique

The studies of RF and microwave spectra of the 2DES are made possible by an extremely broadband technique, in which we send a signal through a transmission line that is in close proximity to the 2DES; then calculate conductivity from the effect of the 2DES on the signal. A schematic of the microwave measurement setup appears in Fig. 1. The line is of a conventional type called coplanar waveguide (CPW) and is comprised of three metal film conductors. We pattern the film directly onto the top surface of the sample, so it is  $\lesssim 1 \mu\text{m}$  above the 2DES. The two broad side conductors are grounded, and are shorted together at the ends of the sample by the housing in which the sample is mounted. The microwave signal is applied to one end of the narrow central conductor, and detected at the other end, so that the CPW is operated like a coaxial cable, with outer, grounded shields and an inner, driven conductor.

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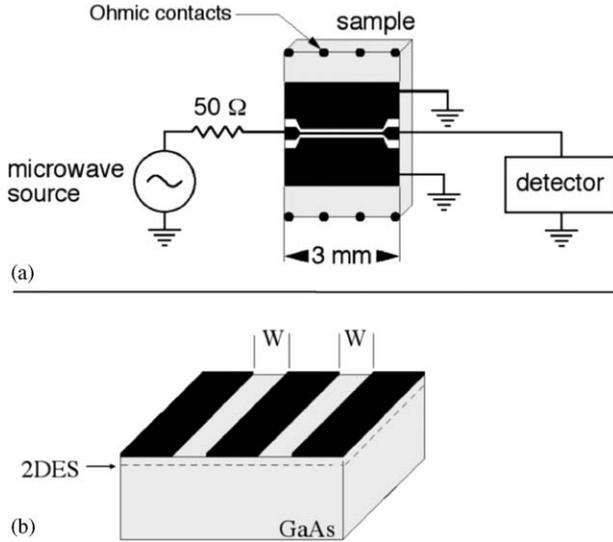


Fig. 1. (a) Schematic of measurement of a transmission line. The coplanar waveguide (CPW) transmission line is drawn to scale, as fabricated on the top of a sample. The highly conductive metal film that forms the CPW is shown as black. The metal side planes are grounded. (b) Magnified cutaway view of a quantum Hall sample near the CPW (not to scale). Typically, slotwidth  $W = 30 \mu\text{m}$ .

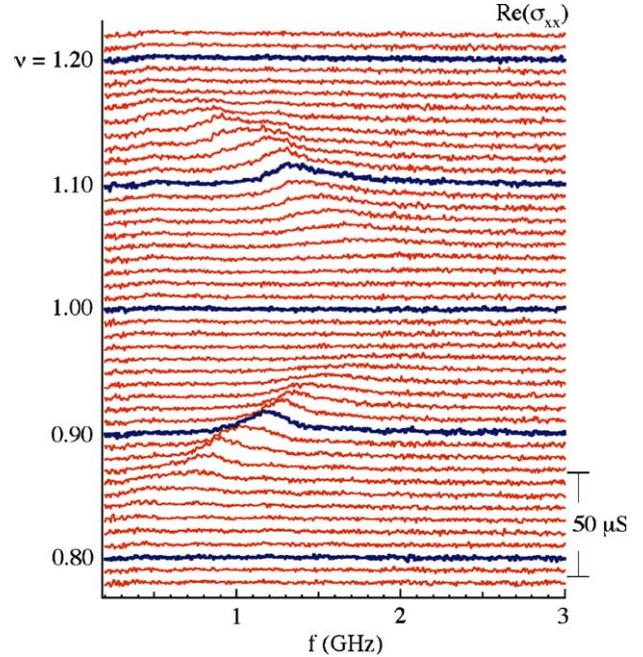


Fig. 2. Spectra  $\text{Re}(\sigma_{xx})$  vs frequency  $f$ , near  $\nu = 1$ , offset vertically proportional to  $\nu$ . The resonances are interpreted as due to the pinning mode of the IQHWC.

The width  $W$  of the slots between the center conductor and the ground planes is  $30 \mu\text{m}$  for the data shown here. The measurement is sensitive to the 2DES immediately under these slots; the in-plane microwave electric field is largely confined to these regions. We measure the loss and dispersion of the transmission lines with a room temperature source and receiver. From  $P$ , the transmitted power normalized to unity for vanishing diagonal conductivity  $\sigma_{xx}$ , we calculate the real part of diagonal conductivity of the 2DES as  $\text{Re}(\sigma_{xx}) = -W |\ln(P)| / 2Z_0 L$ , where  $L$  is the total length of the transmission line and  $Z_0 = 50 \Omega$  is the characteristic impedance of the CPW for  $\sigma_{xx} = 0$ . This formula is a good approximation for 2DES under our experimental conditions; it is valid in the high  $f$ , low loss limit, in the absence of reflections.

### 3. Resonance within integer quantum Hall effect

Likely the simplest of the electron solids of higher Landau levels are those found near integer  $\nu$ . These solids can be thought of as a Wigner crystal of top Landau level particles or holes. We refer to these phases as integer quantum Hall Wigner crystals (IQHWC), since the resonance giving evidence for them appears in parts of the filling factor range in which an integer quantum Hall effect plateau appears in DC transport. Fig. 2 shows many spectra taken near  $\nu = 1$ ; the spectra are offset vertically proportional to  $\nu$  and were taken from a  $30 \text{ nm}$  quantum well with density  $n \approx 3.1 \times 10^{11} \text{ cm}^{-2}$  and mobility  $\mu \approx 2.4 \times 10^7 \text{ cm}^2/\text{Vs}$ . There are clear ranges in which a resonance is present above and below  $\nu = 1$ , which we

interpret as due to the pinning mode of the electron and hole IQHWCs. Just at  $\nu = 1$ , where particles and holes are nominally absent, the spectrum is flat. We have seen similar resonances in this sample around  $\nu = 2, 3$  and  $4$ , and, in another high mobility wafer, a  $50 \text{ nm}$  well with  $n \approx 1 \times 10^{11} \text{ cm}^{-2}$  around  $\nu = 1, 2$  and  $3$ .

Features of the spectra are consistent with the interpretation or the resonances as IQHWC pinning modes, and can be understood in terms of the change in  $n^*$ , the density of top Landau level carriers, as  $\nu$  is varied near an integer. Taking  $\nu^*$  as the difference between the true filling and the nearby integer filling, we obtain top level carrier density  $n^* = \nu^* n / \nu$ . The upward shift in the resonance frequency  $f_{pk}$  as integer  $\nu$  is approached is analogous with observations [14] of the low  $\nu$  insulator terminating the FQHE series at high  $B$ , which at fixed  $B$  show an increase in  $f_{pk}$  as  $n$  is reduced with a backgate. In both cases the decrease in the density of the solid is explained within weak pinning [15–18] as resulting in closer association of the carriers with the impurity potential, and hence in larger pinning energy. More directly,  $n^*$  of the IQHWC resonance can be related to the integrated intensity of the resonance, using  $S/f_{pk} = n^* e \pi / 2B$  [15], where  $S$  is  $\text{Re}(\sigma_{xx})$  integrated over frequency  $f$ . The measured  $S/f_{pk}$  vs  $n^*$ , though smaller (by at most a factor of two) than the value calculated from the formula, is suggestive of the expected linear dependence.

The IQHWC resonance is evidence that localization producing the width of the IQHE plateau can be collective, involving pinning of Wigner solids. Consistent with this, the resonance, particularly when farthest from integer  $\nu$ , survives to temperatures several times larger than  $hf_{pk}/k_B$ .

At such temperatures individually localized carriers would escape causing the resonance to vanish.

The filling factor range in which we observe the IQHWC is of particular interest. At the large  $|\nu^*|$  limit, at least for  $\nu < 4$ , it is likely that the disappearance of the resonance is due to a transition to a fractional quantum Hall liquid state. The  $|\nu^*|$  at which this occurs is around 0.2. This value is in rough agreement with the  $\nu$  of 0.219 [19], below which insulating behavior and the resonance [13] first emerge in the highest quality samples, in the lowest Landau level, at the high  $B$  termination of the fractional quantum Hall effect (FQHE) series. In the lowest Landau level, the  $\frac{1}{5}$  FQHE is present with insulator both at higher and lower fillings, resulting in a “re-entrant” insulator. No such re-entrance of an insulator on either side of a well-developed FQHE has been observed for higher Landau levels, however. The disappearance of the IQHWC resonance at low  $|\nu^*|$  occurs for  $|\nu^*|$  between 0.04 and 0.09 [5,7,9]. It may signal a transition from a solid to single particle localization or an inability of a solid to support an observable resonance. More detailed study of the disappearance of the resonance as integer  $\nu$  is approached, likely with some improvement in experimental technique, will be required to answer this question.

#### 4. Bubble and stripe phases of higher Landau levels

More complicated electron solids and anisotropic “stripe” phases were predicted from Hartree–Fock calculations [2] for 2DES with at least two orbital Landau levels filled. As  $\nu^*$  is increased, the IQHWC gives way to “bubble” phases, lattices, with clusters of  $M > 1$  electron guiding centers per site. On increase of  $\nu^*$  further, to a range near  $\frac{1}{2}$ , anisotropic “stripe” phases appear, which orient themselves along a particular crystal axis of the GaAs host. DC transport experiments [3,4] gave evidence of distinct regions of top-Landau level localization (due to the pinned bubble phases or IQHWC) or anisotropy (due to the stripes) at roughly the same  $\nu^*$ s predicted by the theories.

Fig. 3 shows the IQHWC resonances around  $\nu = 4$ . The hole branch of IQHWC resonances gives way to a flat spectrum as  $\nu$  decreases below about 3.8. On the electron side, for  $\nu$  from 4.16 to 4.28, the IQHWC resonance diminishes as a lower  $f_{pk}$  resonance grows. This lower  $f_{pk}$  resonance is identified with the bubble phase. We have observed  $\nu$  ranges of resonances, interpreted as bubble phase pinning modes, around  $\frac{1}{4}$  and  $\frac{3}{4}$  filled Landau levels for both the  $N = 2$  [6] and  $N = 3$  Landau levels. The presence [9] of two resonances in a range of  $\nu$  implies coexistence of the IQHWC and bubble phases, as expected for a first order transition. The ratio of the  $f_{pk}$ 's of the two resonances is consistent with a theoretical prediction [20] for the IQHWC and  $M = 2$  bubble phase.

As with the IQHWC, the bubble phase resonance  $f_{pk}$  decreases as  $\nu^*$  and  $n^*$  increase. The bubble phase resonance also survives to temperatures much larger than

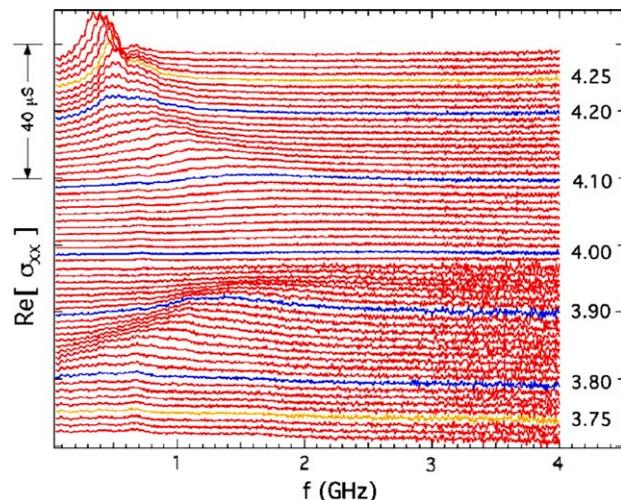


Fig. 3. Spectra,  $\text{Re}(\sigma_{xx})$  vs frequency,  $f$ , near  $\nu = 4$ , offset vertically proportional to  $\nu$ . For  $\nu > 4$  the bubble phase resonance develops out of the low  $f$  shoulder of the IQHWC resonance as  $\nu$  increases.

$hf_{pk}/k_B$ , and so provides evidence of correlated electron solids beyond the top Landau level insulating behavior seen in DC transport.

#### 5. Conclusion

Broadband, transmission line-based measurements have revealed a number of distinct regions of  $\nu$  in which striking resonances appear in the microwave or RF spectrum signalling the presence of pinned electron solids. The resonances occur not only in the lowest Landau level, but also at higher  $\nu$ , so far nearly to  $\nu = 8$ . Microwave spectroscopy is now a proven tool for study of the various electron solid phases of 2DES.

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