feature article

The tangled tale of phase space

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Phase space has been called one of the most powerful inventions of modern science. But its historical origins are clouded in a tangle of independent discovery and misattributions that persist today.

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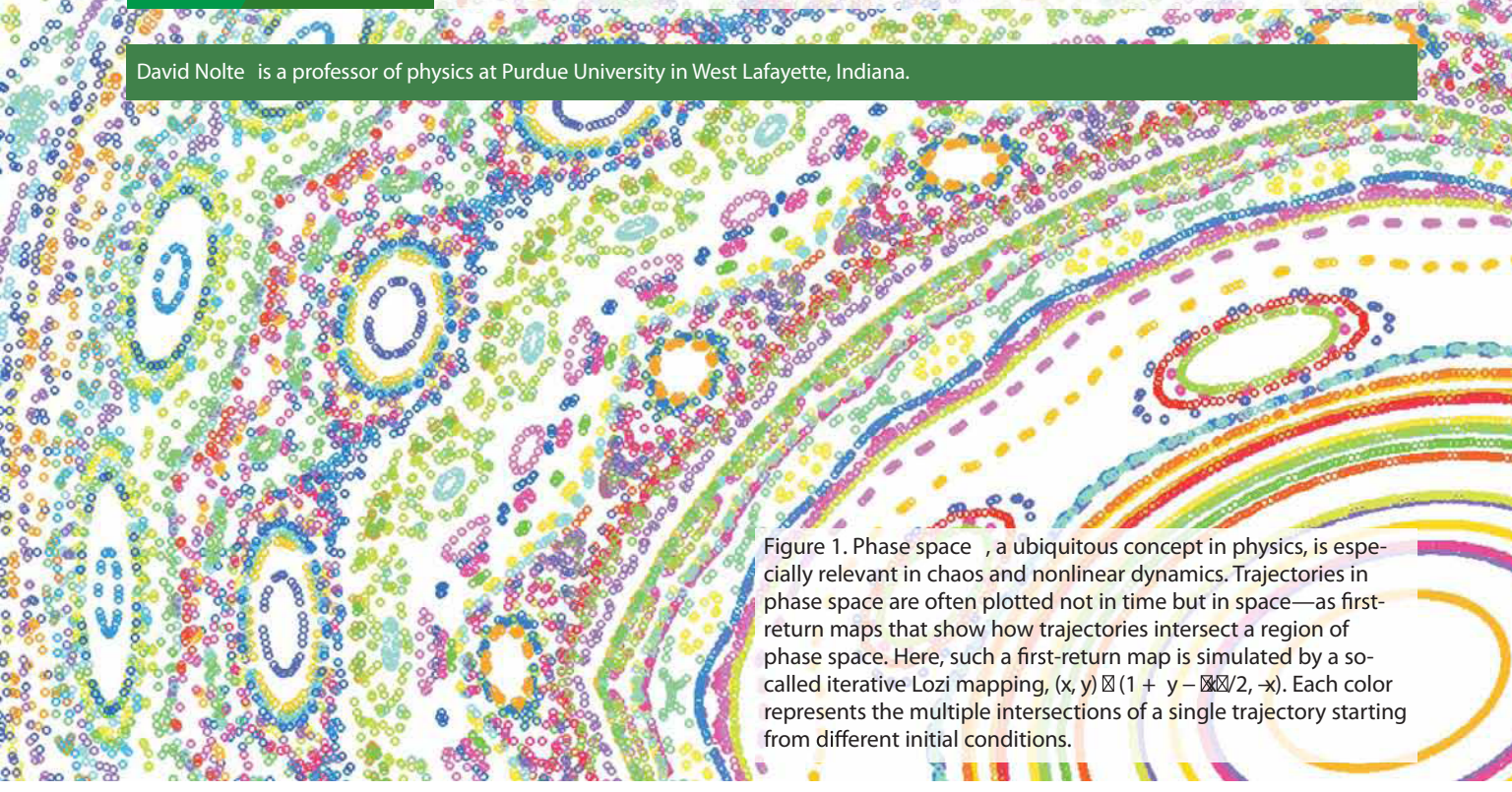

Figure 1. Phase space , a ubiquitous concept in physics, is especially relevant in chaos and nonlinear dynamics. Trajectories in phase space are often plotted not in time but in space—as first-return maps that show how trajectories intersect a region of phase space. Here, such a first-return map is simulated by a so-called iterative Lozi mapping, (x, y) (1 + y – /2, x). Each color represents the multiple intersections of a single trajectory starting from different initial conditions.

Hamiltonian Mechanics is geometry in phase space.
—Vladimir I. Arnold (1978)

Listen to a gathering of scientists in a hallway or a coffee house, and you are certain to hear someone mention phase space. Walk down the science aisle of the local bookstore, and you will surely catch a glimpse of a portrait of a strange attractor, the powerful visual icon of phase space. Though it was used originally to describe specific types of dynamical systems, today "phase space" has become synonymous with the idea of a large parameter set: Whether they are stock prices in economics, the dust motes in Saturn's rings, or high-energy particles in an accelerator, the degrees of freedom are loosely called the phase space of the respective systems. The concept and its name are embedded in our scientific fluency and cultural literacy. In his popular book Chaos on the history and science of chaos theory, James Gleick calls phase space "one of the most powerful inventions of modern science." [1] But who invented it? Who named it? And why?

The origins of both the concept of phase space and its name are historically obscure—which is surprising in view of the central role it plays in practically every aspect of modern physics (figure 1). The historical origins have been further obscured by overly generous attribution. In virtually every textbook on dynamics, classical or statistical, the first reference to phase space is placed firmly in the hands of the French mathematician Joseph Liouville, usually with a citation of the 1838 paper in which he supposedly derived the theorem on the conservation of volume in phase space. [2] (The box on page 34 gives a modern derivation.) In fact, in his paper Liouville makes no mention of phase space, let alone dynamical systems. Liouville's paper is purely mathematical, on the behavior of a class of solutions to a specific kind of differential equation. Though he lived for another 44 years, he was apparently unaware of his work's application to statistical mechanics by others [3] even within his lifetime. Therefore, Liouville's famous paper, cited routinely by all the conventional textbooks, and even by noted chroniclers of the history of mathematics, as the origin of phase space, surprisingly is not!

How did we lose track of the discovery of one of our most important modern concepts in physics? If it was not discovered by Liouville, then by whom and when and why? And where did it get its somewhat strange name of "phase" space? Where's the phase?