Model of Soil Freezing and Thawing

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ABSTRACT: Consequences of soil freezing and thawing are widely known but their prediction is quite difficult. Frost heave is connected with the permanent upward displacement of masses of soil surface. In an effort to understand this phenomena in depth we introduce and study a basic model in 2D representing, e.g., a cross-section of a testing sample of soil.

Motivated by [1], the model is based on a modified heat equation for the soil temperature u (in °C) which covers phase change in a neighbourhood of the freezing point depression $u^\star$. The equation has form

$$C\frac{\partial}{\partial t}u(t,x)+L\frac{\partial}{\partial t}\theta(u)=\lambda\Delta u(t,x)$$

where C, L, λ are constants which have meaning of the volumetric heat capacity, the volumetric latent heat of fusion water and thermal conductivity, respectively. The phase change related behavior is described by the power function θ,

$$\theta(u)=\eta\varphi(u),\quad \varphi(u)=\begin{cases} 1 & : \ u\geq u^\star \\ |u^\star|^b/|u|^b & : \ u<u^\star \end{cases}$$

where η is the soil porosity of melt-state soil, φ represents the liquid pore water fraction and b is a positive constant related to material characteristic of soil. This part of the model can be mathematically analysed providing the information on the solution existence.

We generalize the model by considering changes in the soil structure caused by the water-ice phase change. We solve the model numerically and are interested in results considering periodically alternating conditions for freezing and thawing of the sample.

REFERENCE:

[1] http://permafrost.gi.alaska.edu/content/model-soil-freezing-and-thawing