

## Computing the Greens functions for fractional Laplacians

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**ABSTRACT:** The heat equation with the Laplacian is used as the basis to model standard diffusion. The Green's function for the standard model is given by the Gaussian/normal density, which has an explicit form in arbitrary dimensions. For anomalous diffusions, the fractional Laplacian is used as the basic equation. In this case, the Green's functions are the Levy stable densities. Except for a few special cases, these stable densities do not have an explicit formula. We describe numerical techniques to compute general stable densities in one dimension. For multivariate problems, the problem is much more challenging. We describe methods of computing multivariate elliptical stable densities. Our approach reduces the problem to a one dimensional problem by using the amplitude  $R=|X|$  of an isotropic stable random variable  $X$ . Using the one dimensional stable densities, we describe methods of computing the cumulative distribution function and the density function of  $R$ . We also describe how to simulate multivariate Levy flights.