**Pore-Scale Modeling of Non-Darcy Flow Regimes in Porous Media**

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Non-Darcy flow is porous media often observed near wells and in hydraulic fractures where relatively high velocities occur. In these regions an empirical model, Forchheimer’s equation, is used in place of Darcy’s law. It includes a quadratic correction to the linear model and has been adequately fit to many experimental data sets, while found to be insufficient in others. Furthermore, a number of numerical and theoretical approaches have shown limitations of the Forchheimer model in the laminar flow regime. It is important to understand the applicability of Forchheimer’s equation and to be able to obtain good predictions of macroscopic properties so that nonlinear flow may be properly modeled in reservoir simulators.

In this study, we use the method of homogenization to develop a filtration law in porous media that includes the effects of inertia at finite Reynolds numbers. The result is much different than the empirically observed quadratic Forchheimer equation. First, the correction to Darcy’s law is initially cubic (not quadratic) for isotropic media. This is consistent with several other authors who have solved the Navier–Stokes equations analytically and numerically. Second, the resulting filtration model is an infinite series polynomial in velocity, instead of a single corrective term to Darcy’s law. Although the model is only valid up to the local Reynolds number, at the most, of order 1, the findings are important from a fundamental perspective because it shows that the often-used quadratic Forchheimer equation is not a universal law for laminar flow, but rather an empirical one that is useful in a limited range of velocities. A major contribution of this study is that the coefficients of the polynomial law can be derived a priori, by solving sequential Stokes problems. In each case, the solution to the Stokes problem is used to calculate a coefficient in the polynomial.

Non-Darcy flow is also modeled at higher Reynolds numbers using a physically-representative pore-scale network model. Quantitative and predictive results are obtained using both computer-generated porous media and real sandstones digitized through x-ray computed microtomography (XMT). The permeability and non-Darcy coefficient, **, are determined for these isotropic and anisotropic media where Forchheimer’s equation is applicable. The numerical model is compared to existing experimental data and good agreement is found. Furthermore, limitations to Forchheimer’s equation at both low and high velocities are determined and discussed. In addition to the cubic law observed at low *Re*, a plateau is observed at high *Re*.