Project Description: MODELING FLOW IN VUGGY MEDIA

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The University of Texas at Austin

Summer School in

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Vuggy Porous Media

Pipe Creek Reef Outcrop

Pipe Creek Reef outcrop, Red Bluff River near Pipe Creek, Texas.

River basin area of the Glen Rose formation, looking upstream.

Pipe Creek Reef Outcrop

Wide view of region with both in-situ fossils and fossil debris.

Rudist Caprinids at Pipe Creek Reef

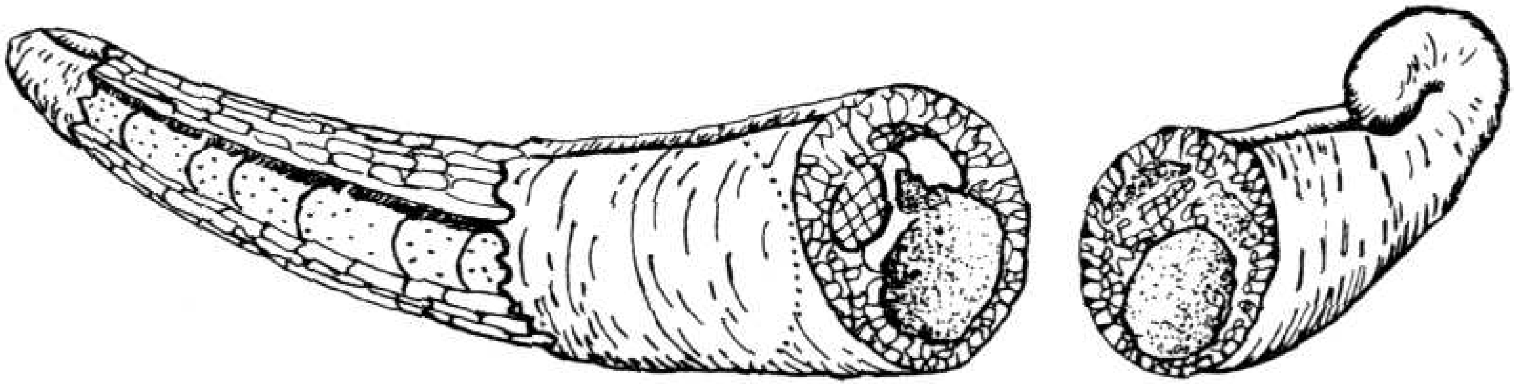


Illustration of a typical 4-6 cm diameter
Cretaceous (Albian) caprinid fossil.

Pipe Creek Reef Outcrop

Closeup of in-situ caprinids (found as in original colonies)

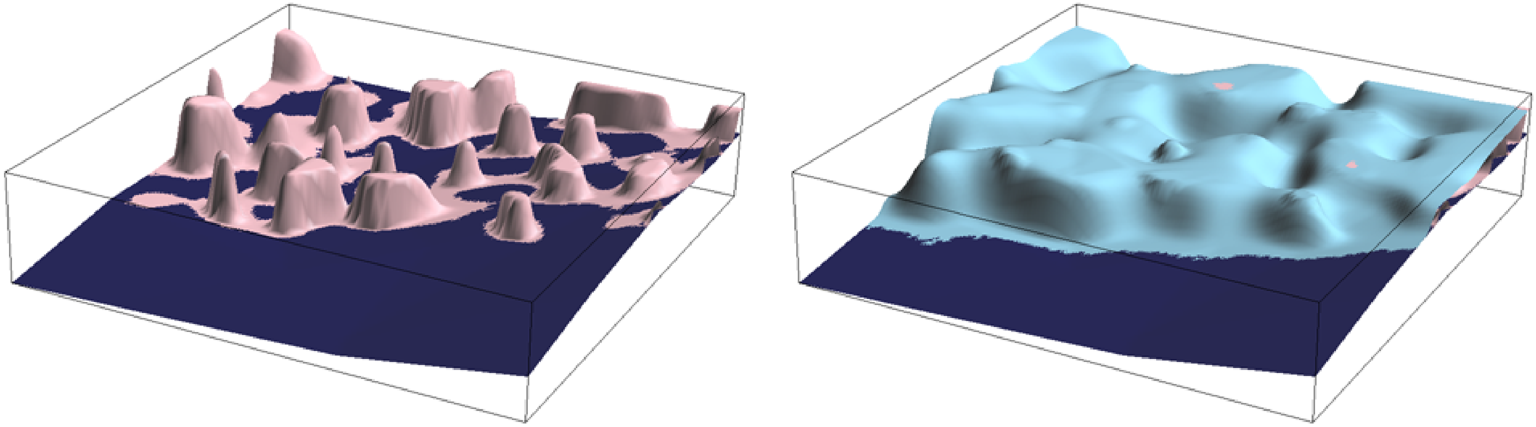
Pipe Creek Reef Outcrop

Closeup of caprinid fossils within a debris region.

Pipe Creek Reef Outcrop

Closeup of caprinid fossils within a debris region, showing the internal structure of the shells.

Pipe Creek Reef Mounds



Conceptual stochastic model of mound cores (left) and debris deposits (right) in a one square kilometer segment of a patch reef belt in the Pipe Creek area based upon outcrop observations and interpretations from 18 cored wells (factor of 10 vertical exaggeration).

Experimental Wells

Two wells were drilled approximately 5 feet apart
to a depth of about 20 feet.

Core Sample

A core sample from the formation.

Main Questions

How does fluid flow in such a system?

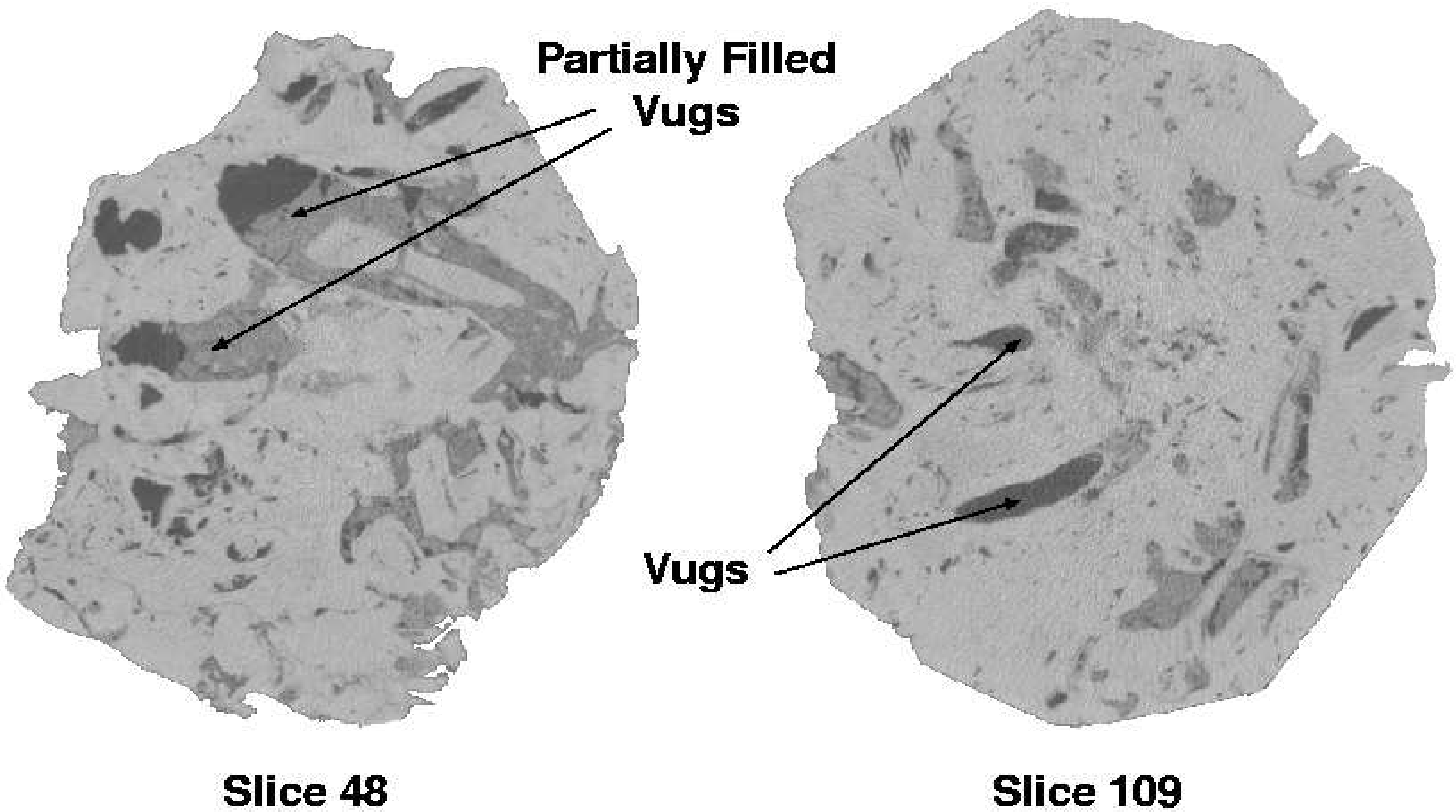
1. Are the vugs interconnected? If so, over what distances?
   - The well test suggests that they are connected for at least 5 feet.
   - Fluid flows much more rapidly in an interconnected vug system than in ordinary porous medium.
2. Is there a well defined structure to the vug network?
3. Is there an effective permeability for the system?
   - It appears so, but finding it is problematic.
4. How do tracers (i.e., chemicals) transport in such a system?
   - They exhibit exotic behavior.

Geometric Analysis of a Large Sample

Pipe Creek Sample Location

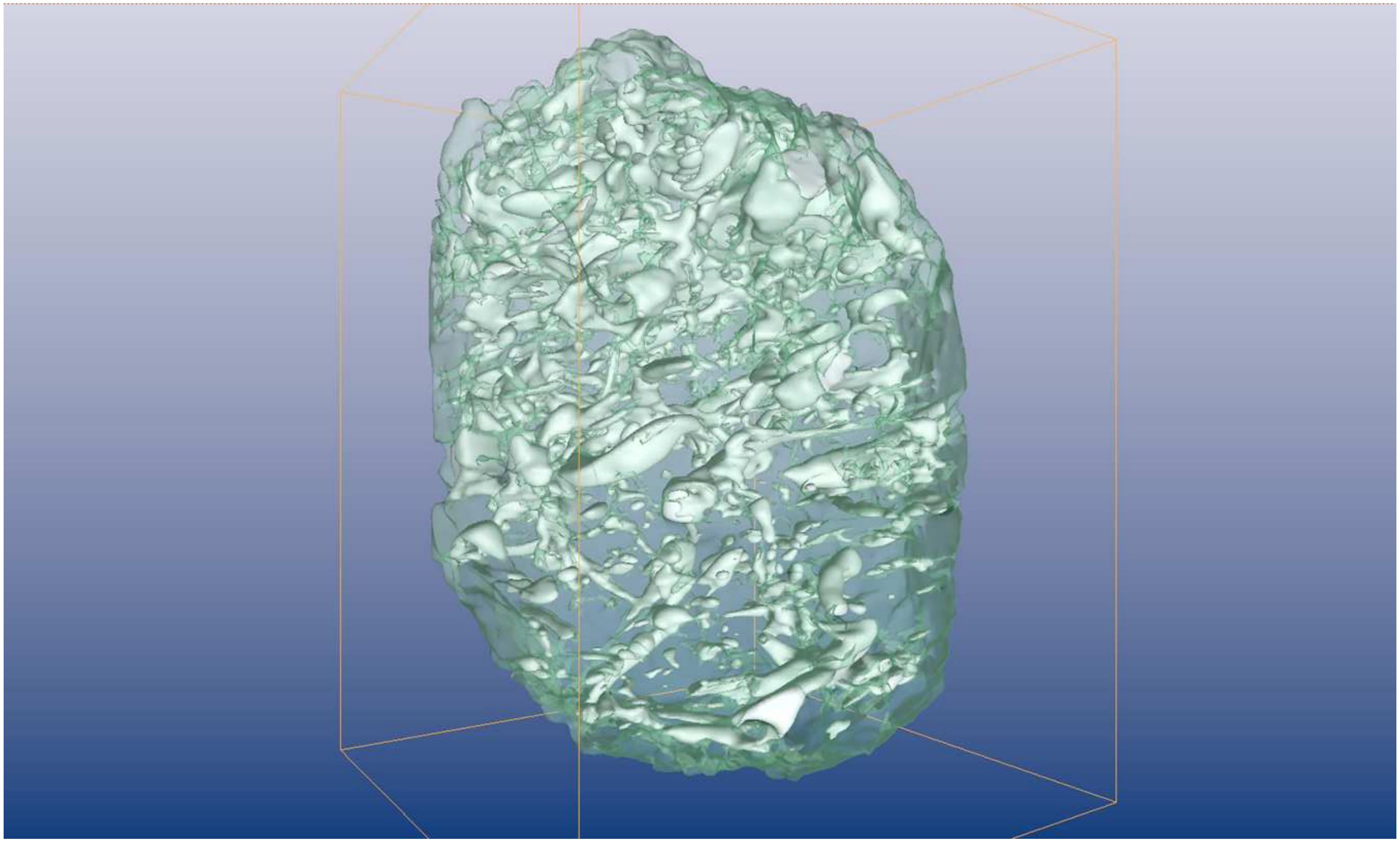
Closeup of debris region from where first large sample was taken.

Internal Structure of the Vuggy Network



CT Scans of Pipe Creek Reef sample (about $30 \times 30 \times 36\,\text{cm}^3$)
240 slices 1.5 mm thick with 512×512 pixels $0.547 \times 0.547\,\text{mm}$ square

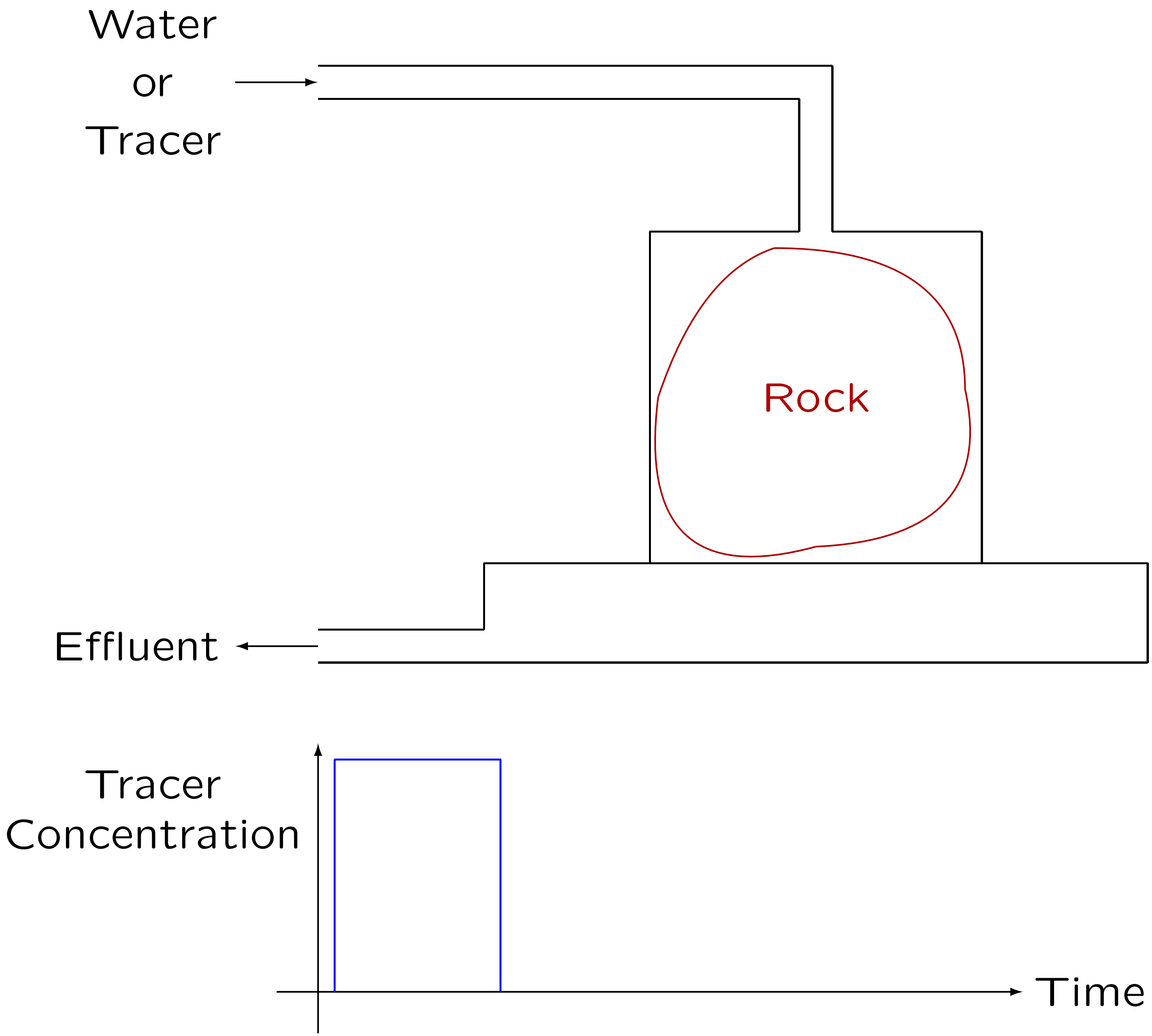
Interior Vug Network



Reconstructed vug network from CT scans.

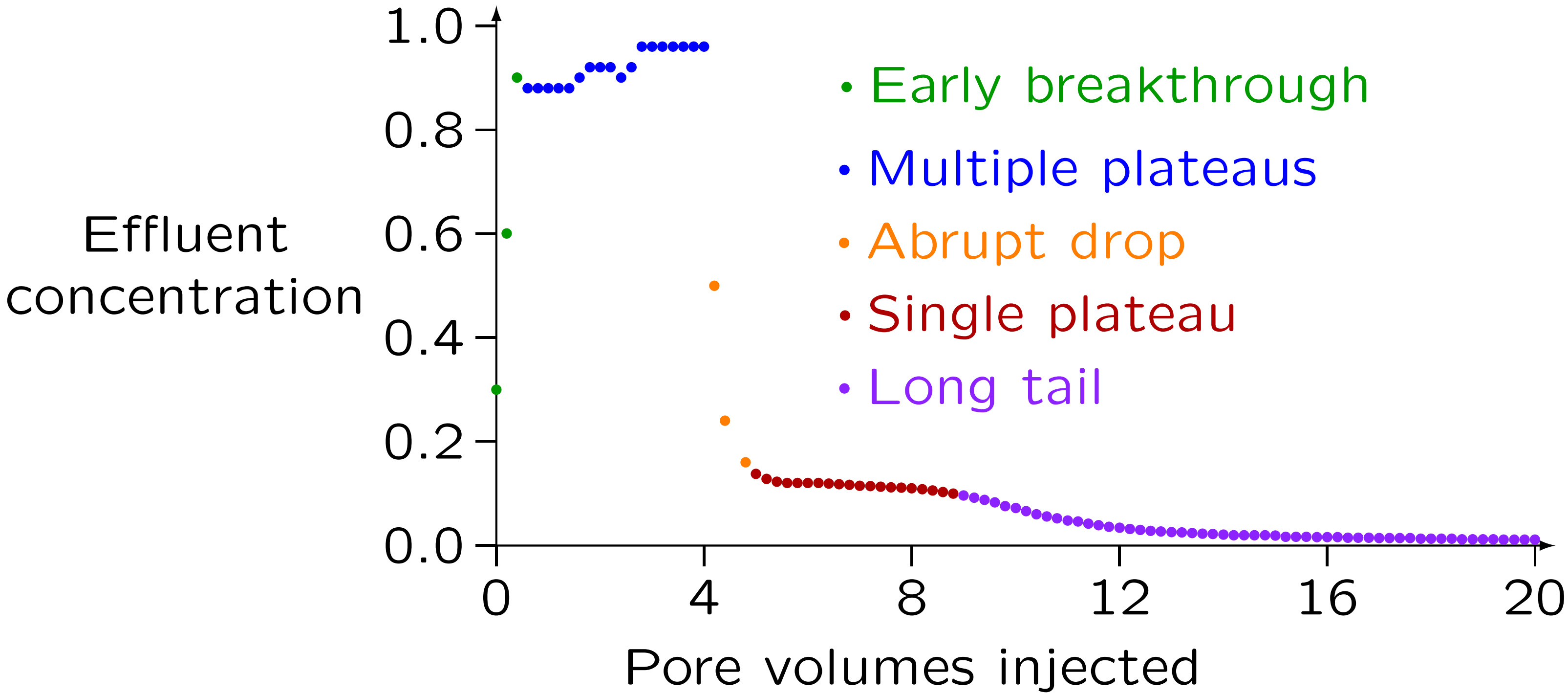
Tracer Experiments

Experimental Design



Injection history (slug injection).

Typical Experimental Results



Typical effluent concentration history.

Note that the curve has five definite features.

Theoretical Micro-Model

The Governing Equations

Notation:

- D is the symmetric gradient: $(Du)_{i,j} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$
- μ is the fluid viscosity
- K is the rock permeability
- f is a gravity term
- q is a source term

Vuggy region: Ω_s

$$\begin{aligned} -2\mu\nabla\cdot Du + \nabla p &= f \qquad \text{Stokes Equations} \\ \nabla\cdot u &= q \qquad \text{Conservation} \end{aligned}$$

Rock matrix: Ω_d

$$\begin{aligned} \mu K^{-1}u + \nabla p &= f \qquad \text{Darcy Law} \\ \nabla\cdot u &= q \qquad \text{Conservation} \end{aligned}$$

The Interface and Boundary Conditions

Notation:

- ν is the outer unit normal vector
- τ is a unit tangent vector
- α is the Beavers and Joseph slip coefficient

Darcy-Stokes Interface: $\Gamma = \partial\Omega_s \cap \partial\Omega_d$

$$u_s \cdot \nu_s = u_d \cdot \nu_s \qquad \text{Continuity of flux}$$

$$2\nu_s \cdot Du_s \cdot \tau = -\frac{\alpha}{\sqrt{K}} u_s \cdot \tau \qquad \text{Beavers-Joseph-Saffman slip condition}$$

$$2\mu\nu_s \cdot Du_s \cdot \nu_s = p_s - p_d \qquad \text{Continuity of normal stress}$$

External Boundary: $\partial\Omega$

$$u_s = 0 \quad \text{on } \partial\Omega \cap \partial\Omega_s \qquad \text{No slip}$$

$$u_d \cdot \nu = 0 \quad \text{on } \partial\Omega \cap \partial\Omega_d \qquad \text{No normal flow}$$

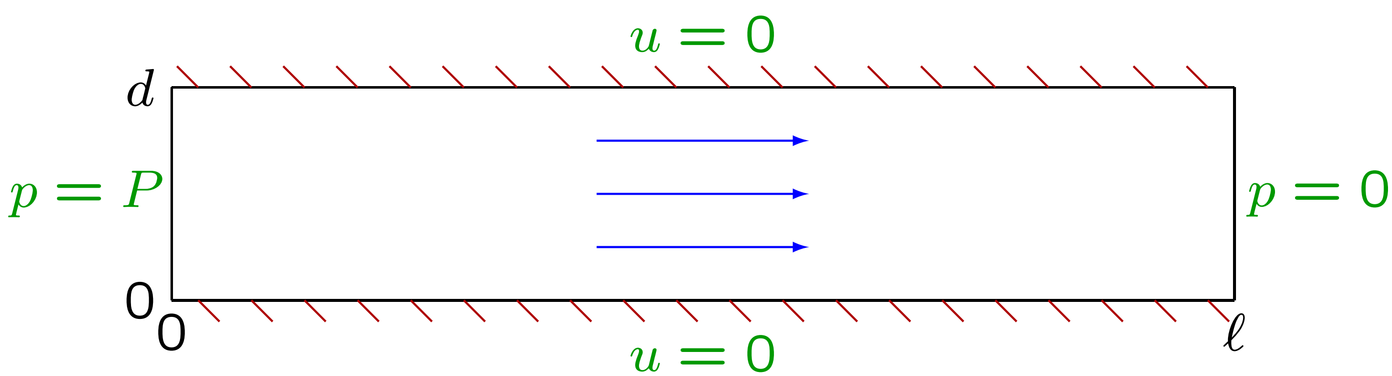
Simplification to Pipe Flow

The flow in the matrix rock is very slow, so ignore it. We are left with Stokes flow in a channel, which is like a pipe.

We can solve the Stokes equations for pipe flow,

$$-2\mu\nabla\cdot\nabla u + \nabla p = 0,$$
$$\nabla \cdot u = 0,$$

when the domain is simple, by exploiting symmetries.



This is a rectangle in 2-D but a cylinder in 3-D.

It turns out that the average velocity is proportional to the pressure gradient P/ℓ, which gives us the permeability as a function of the diameter d, for a pipe-like vug.

Caution: But this is not what we have in the rock!

Potential Project Activities

Data Available

- A data file of CT scan pixels converted into a index: vug, rock, or "surface mud."
- Overall porosity $\approx$ 20%.
- Permeability data
  - Matrix rock permeability $\approx$ 10 mD
  - Lab test of the large sample effective permeability $\approx$ 100 Darcy
  - Lab test of the well core effective permeability $\approx$ 1 Darcy.
  - Field well test effective permeability $\approx$ 1 Darcy over 5 feet.
- Tracer slug test typical behavior.

Questions and Activities—1

Progress can probably be made on the following within two weeks.

1. Determine the vug structure of the large sample, and show that it is interconnected. This could be approached by analyzing the pixel data. It would involve writing a data analysis code.
   - Two vug pixels that share a face are most likely interconnected.
   - What about vug pixels that share only an edge or a point?
   - What about dead end channels?
2. Can we treat the system as a Darcy system with a large permeability in the vugs? This could involve solving single phase flow problems for different "large" vug permeabilities.
   - Should we take the vug permeability to be infinity?
   - Is the system sensitive to the assumed vug permeability?
   - Can we determine the effective permeability of a sample this way?
   - Do ideas from pipe flow help here?
3. How does the vug network structure affect effective permeability? This could involve treating the system as a Darcy system with different patterns of vug channels.
   - Corners? Branchings? Constrictions? Pinchouts? Dead ends?

Questions and Activities—2

The following may be too ambitious for the two week project period.

4. Can we explain the features in the tracer experimental data?
   - Postulate a reason for the various features.
   - Run simulations to test the hypotheses. This involves running flow and transport codes.

Contacts

Todd Arbogast.

I am here through Monday, July 24 (I leave Tuesday morning).

Steve Bryant.

Available to answer questions by e-mail for the full two weeks:

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