Rock Mechanics
Proceedings of the
33rd U.S. Symposium

Edited by
I.R. TILLERSON & W.R. WAWERSIK
Sandia National Laboratories, Santa Fe

OFFPRINT

A.A. BALKEMA / ROTTERDAM / BROOKFIELD / 1992
Immiscible fluid flow in a fracture

Lauri Pyrak-Nolte, Dan Helgesen & Guy M. Haley
Department of Earth & Atmospheric Sciences, Purdue University, West Lafayette, Ind., USA

James W. Morris
Department of Chemistry, Purdue University, West Lafayette, Ind., USA

ABSTRACT: A stratified continuum model is used to generate fracture void geometries to investigate the effect of trapping of immiscible flow through a fracture. For wetting-phase invasion a fracture initially saturated with a non-wetting phase, the non-wetting phase occurs in local maxima that are surrounded by smaller apertures. Trapping of the non-wetting phase results in high residual saturation and low values of wetting phase relative permeability. An essay cast of a natural fracture was used to visually observe trapping in a fracture.

1 INTRODUCTION

Energy sources such as coalbed methane, geothermal springs, and oil fields often occur in fractured reservoirs and involve the flow of two phases in fractures. Multiphase flow through fractures is also a concern for containment transport, and the rejection of radioactive waste. An underlying question of the movement of two fluids through a fracture is how the fracture geometry affects residual saturation and relative permeabilities. In reviewing the literature, few experimental measurements have been made of relative permeability in fractures (Barton, 1972; Merritt, 1985; Haswell & Rogers, 1985). These experiments were performed on artificial fractures, or fractures represented by parallel glass plates. Several investigators have undertaken theoretical investigations of multiphase flow in fractures. Some models involve the use of capillary theory to study multiphase flow through fractures idealized as parallel glass plates (Graves, 1965; Evans & Huang, 1985; Rasmussen et al., 1985) and wedge-shaped fractures with continuously varying apertures (Rasmussen, 1987). Pruess & Tsang (1986) numerically analyzed relative permeabilities of a rough-walled fracture for a bimodal aperture distribution and various spatial correlations. They found that relative permeabilities are sensitive to the nature and range of spatial correlation of the apertures. Pyrak-Nolte et al. (1990) examined unlagented flow in single fractures for the case of a non-wetting phase invading a wetting phase fluid (such as mercury injected into water saturated with air in rock).

Pruess & Tsang (1990) investigated numerically the effect of different aperture distributions on two-phase flow through a fracture using global accessibility. Accessibility determines which apertures will be occupied by the fluid. For wetting-phase invasion, global accessibility allows all apertures to be occupied even if they are not connected to the inlet. This paper will examine the effect of global accessibility compared with inlet accessibility with trapping for a wetting phase invasion of a fracture initially saturated with a non-wetting phase. Numerical and experimental results will be presented.
2 EXPERIMENT

A laboratory study was undertaken to observe trapping in natural fractures. To visually examine the distribution of each phase in the fracture, an epoxy cast of a natural fracture in granite was made. The natural fracture measured 52 mm in diameter. A mold of each fracture surface was made using Wood's metal, i.e., a Bi-Sn-Sb-based, low melting point metal. The molds of each surface were filled with epoxy and allowed to solidify. After the epoxy solidified, the mold with epoxy was placed in boiling water to remove the Wood's metal. The two casts of the fracture surfaces were placed together to form the fracture. The sides of the specimen were sealed except for two ports for the fluid inlet and outlet. These ports were approximately 3.18 x 10^-4 m in diameter and were diametrically opposed. The fracture was first saturated with a non-wetting phase (oil) with a surface tension = 0.07 dynes/cm, and a viscosity = 1.47 cp using an oil injection pressure of 63.9 Pa. The wetting phase was a mineral oil (surface tension = 30.3 dynes/cm, viscosity = 2.065 cp). The oil was brought into the dye-saturated fracture using an injection pressure of 1044 Pa. The velocity of invasion of the oil was approximately 1.8 x 10^-5 m/s and was kept at this rate to avoid viscous fingering. After a connected path across the cast was formed and stabilized, the oil flow rate was measured at 5.0 x 10^-5 m/s. The entire invasion of oil into a dye-saturated fracture was captured on video tape. Figure 1.a is a drawing from a video image when the distribution of the wetting phase (oil) and the non-wetting phase (dye) reached steady-state. In the image, large regions of trapped dye (non-wetting phase) are observed and the oil (wetting phase) flow path is tortuous.

3 MODEL

In this analysis of the effect of trapping on two-phase flow, fracture void topologies are generated using a stylized continuum model. A continuum model is used because the distribution of void apertures is continuous and there is no underlying lattice structure. The simulated fracture void patterns are based on a fractal construction that produces spatially correlated aperture densities with a power-law size distribution.

Figure 1. Oil (wetting phase black) invaded into a fracture saturated with dye (non-wetting phase white) under capillary pressure effects.
4 FLUID ACCESSIBILITY

The relationship between capillary pressure and saturation of a fracture will affect how two phases are introduced into a fracture. We assume that the distribution of each phase in the fracture will depend only on capillary pressure effects and neglect the effects of buoyant and viscous forces. The configuration of the flow paths of the wetting and non-wetting phases are based on the local fracture geometry and are independent of global pressure gradients. We make the assumption that the area occupied by each phase is directly dependent on the capillary pressure. The capillary pressure is taken to be inversely proportional to the local aperture. When the non-wetting phase is in a small aperture, this corresponds to a high capillary pressure. The simulations begin with the fracture saturated with a non-wetting phase. Flow is assumed to occur left to right in simulations with zero-flow boundary conditions at the top and bottom of the pattern.

Global accessibility and inlet accessibility with trapping are used to introduce each

Figure 2. Aperture contour map of a simulated fracture void geometry using the stratified continuum model. White regions represent areas of contact. Increasing shades of gray represent increasing aperture. Contour interval 20 units.
phase into the fracture. For wetting phase invasion, global accessibility shows all sites of a given aperture or less to be occupied even if they are not connected to the inlet. Later accessibility with trapping reduces to the approach when wetting phase is introduced from the inlet of the fracture simulation and occupies sites with wetting phase for all sites of aperture b or less connected to the invading front, unless the non-wetting phase occupying that site is surrounded by the wetting phase. If the non-wetting phase is surrounded by the wetting phase, the non-wetting phase is trapped and no longer participates in the flow and can never be occupied by the wetting phase.

Figure 3 shows the distribution of the wetting phase and non-wetting phase in a fracture using inlet accessibility with trapping at breakthrough (Figure 3 top) and at maximum possible saturation (Figure 3 bottom). Breakthrough occurs when the first connected path of wetting phase is formed that spans the fracture. Black in Figure 3 represents the invading wetting phase and white regions represent the non-wetting phase. Gray regions represent node-to-end contact. Trapping causes a high value of residual

Figure 3. Invasion of wetting phase (black) into the fracture saturated with non-wetting phase at breakthrough (top) and for maximum saturation (bottom). The aperture distribution is shown in Figure 2. Contact area is represented by gray regions.
saturation of non-wetting phase in the fracture. If global accessibility were used, the entire pattern in Figure 4(a) would be black. Because of the correlated and continuous nature of the void geometry of the simulated fracture, the non-wetting phase becomes trapped in regions of local maxima. Regions of local maxima are surrounded by voids of smaller apertures through which the wetting phase preferentially flows. The amount of trapping in a fracture depends on the spatial correlations of the apertures in the fracture.

5. CAPILLARY PRESSURE-SATURATION RELATION

The capillary pressure-saturation is calculated by assuming that the capillary pressure is inversely proportional to the aperture of the fracture. Throughout the simulation the volume and area of the wetting phase were recorded as a function of invaded aperture size as the wetting phase was introduced into the fracture. The wetting phase saturation is calculated by dividing the volume of wetting phase in the simulated fracture by the total volume. Figure 4 shows an average for five fracture void simulations for both global accessibility and inlet accessibility with trapping. The capillary pressure curve for a log-normal aperture distribution in a fracture is similar to the customary J function for three-dimensional porous media (Frank & Tsang, 1996). The important feature of Figure 4 is the difference between the results for inlet accessibility with trapping and global accessibility. Trapping of the non-wetting phase by the wetting phase has a dramatic effect on the capillary pressure curves for a single fracture. Because of trapping, lowering the capillary pressure will never result in complete wetting phase saturation. For the simulations, at the lowest capillary pressure, the maximum wetting phase saturation is roughly thirty-five percent.

6. FLOW & RELATIVE PERMEABILITY

To determine the relative permeability of each phase in the simulated fractures, relative flow of each phase through the fracture is evaluated. Relative fluid flow through the

![Figure 4. Capillary pressure as a function of wetting phase saturation for global accessibility and inlet accessibility with trapping.](image-url)
Fracture aperture is calculated for each increment of capillary pressure (increase of aperture) assuming steady-state equilibrium conditions. At calculating this flow through the fracture, a second-order approach is taken that includes only the capillary pressure. For determining the wetting phase flow, Qw, "cobblestone" (Williamson et al., 1999) behavior is assumed to describe the local dependence of fluid flow on aperture. The two-dimensional critical behavior is included by a scaling law that describes changes in tortuosity.

\[ Q_w = \frac{\Delta P \cdot W \cdot C}{\Delta L} \cdot \frac{(w_h - b_h)}{b_w} \]  

where \( b_h \) is the effective aperture of the wetting phase critical neck and is given by

\[ b_h = \frac{V_{inj}}{\pi \cdot (w_h - b_h)^2} \]

and the constant of proportionality, \( C \), is given by

\[ C = \frac{2}{\sqrt{\pi}} \cdot \left( \frac{\text{Area}_{w}}{\text{Area}_{nbw}} \right) \]

where

- \( Q_w \) - Flow
- \( \Delta P/\Delta L \) - Pressure Gradient
- \( b \) - Aperture
- \( a \) - Normalized Area
- \( w \) - Wetting Phase
- \( w - nbw \) - Non-Wetting Phase
- \( c \) - Critical neck of either \( w \) or \( nbw \)
- \( crw \) - Critical Neck of Pure \( w \)
- \( \mu \) - Viscosity
- \( W \) - Fracture Width
- \( l \) - critical exponent

The area \( nbw \) occupied by the non-wetting phase is normalized by the area of the entire simulated fracture. The critical area, \( nbw \), is the normalized area of the wetting phase at the throw, and \( l \) is a critical exponent taken to be 1.9 for these simulations. The critical exponent for standard random continuum percolation ranges between 2.1 < \( l < 2.7 \) (Hegger et al., 1995). Tortuosity is imposed for determining the wetting phase flow because the path of the wetting phase is constantly changing with changes in saturation. An effective critical aperture, \( b_h \), is used for the wetting phase to account for the parallel flow paths that are established as the wetting phase is allowed into larger apertures. Because the non-wetting phase dominates the critical path of the path and only flows along this path, the tortuosity of the non-wetting phase flow path does not change with a change in saturation. The non-wetting phase flow is given by

\[ Q_{nbw} = \frac{\Delta P \cdot W \cdot b_{crw}}{\Delta L \cdot \frac{\text{Area}_{w}}{\text{Area}_{nbw}}} \cdot \left( \frac{b_{crw}}{b_{w_h}} \right) \]

where \( b_{crw} \) is the critical neck of the critical path of the fracture simulation, and \( b_{w_h} \) is the largest aperture the wetting phase has entered for a given capillary pressure. The quotient \((1 - (b_h/b_{w_h}))\) in equation (2) represents the change in the width of the non-wetting phase critical neck in the wetting phase flow entered into larger apertures. Equations (1) and (2) differ slightly from the equations put forth by Pyeak-Noltz et al. (1976).
Relative permeabilities of each phase in the fracture void simulations were determined from relative flow values:

\[ k_{wi} = \frac{Q_{wi}}{Q_{wi,0}} \quad \text{and} \quad k_{sw} = \frac{Q_{sw}}{Q_{sw,0}} \]

where

\[ Q_{wi,0} = \frac{\Delta P \cdot W_i \cdot \eta_i}{A \cdot L \cdot 

\eta \text{ is the permeability of the wetting phase, and } k_{wi,0} \text{ is the single phase permeability. Flow for each phase was normalized by the flow through the simulation for complete saturation by the wetting phase. The global pressure gradient was assumed equal for both phases. Viscosities are based on the values for methane (108.7 \text{ x } 10^{-6} \text{ poise}) and water (0.01 poise). In coal, methane is the wetting phase and water is the non-wetting phase because coal is hydrophobic (Fuerstenau et al., 1990).}

Figure 5 shows the effect of trapping on the relative permeability of each phase. Trapping results in a maximum wetting phase permeability of six percent for a maximum wetting phase saturation of thirty-five percent. The high residual saturation is a direct result of the fracture topologies. Because of the correlated geometry of the simulated fracture, the non-wetting phase becomes trapped in regions of local maxima. Regions of local maxima are surrounded by voids of smaller aperture through which the wetting phase preferentially flows.

7 CONCLUSIONS

From the experiments, trapping of a non-wetting phase by a wetting phase is observed for a natural fracture. This phenomenon can be modeled based on fundamental physical

![Figure 5. Relative permeability as a function of wetting phase saturation for global accessibility and infill accessibility with trapping.](image_url)