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PMT EFFECTIVE RADIUS AND UNIFORMITY TESTING

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January 14, 2002

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ABSTRACT

As a part of the VERITAS project we have tested several photomultiplier tubes from different manufacturers for surface uniformity of their sensitivity and effective diameter. This report presents the results of our tests and the methods implied in data analysis. The Hamamatsu 41 tube is used to illustrate the methods used in data processing. The data acquisition program was written C++ and the analysis was done using Mathematica 4.

1. NOTATION

For this report we have tested 3 photomultiplier tubes (PMT) from Hamamatsu (model: R7056ASSY), 3 from Photonis (model: XP2900/02), and 2 from Electron Tubes (model: P25VN-02) (hereafter ET). In order to distinguish tubes from the same company we append the serial number of the tube to the name of the corresponding company, for example: Hamamatsu 41 (S/N: UA0041), Photonis 25216, Electron Tubes (ET) 122 and so on. Sometimes, we will further abbreviate these names, for example, to HUA41 (Hamamatsu R7056ASSY tube with S/N UA0041), Ph25216 (Photonis XP2900/02 tube with S/N 25216) or ET122 (Electron Tubes P25VN-02 tube with S/N 122).

2. RAW DATA FORMAT

The 2D data (anode current) was taken by scanning the surface of a PMT along both the X and Y axes with a step of about 0.5mm (exact value: 0.4953mm). During this scan the photomultiplier's surface was illuminated by an optical fiber of diameter 0.6 mm that was coupled to a broadband light source. The wavelength used in

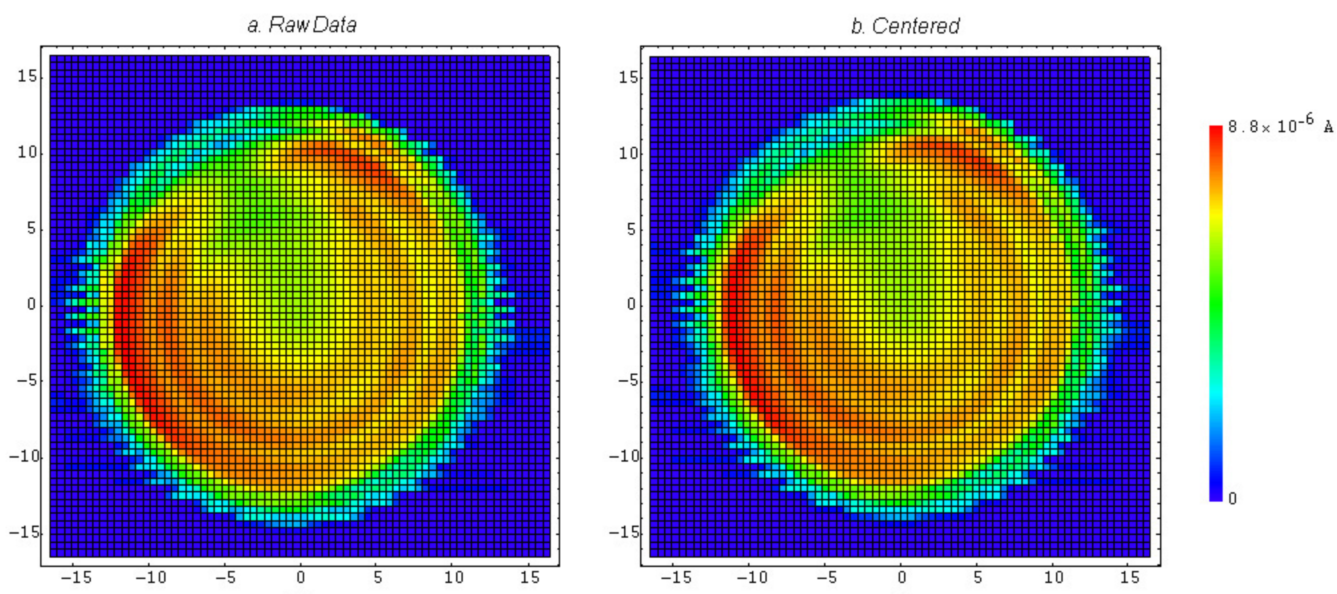


Fig. 1 Density profile for Hamamatsu 41

these measurements was λ = 420nm. The scanned area was a box of side 34mm resulting in a 2D pixel structure of 70 by 70. The tubes were placed in the center of the scanned area. In Fig. 1 a density plot of the raw data for Hamamatsu 41 (an interpolation of 1st order was used) is displayed.

This setup allowed us to obtain a quantitative measure of the "effective radius" and non-uniformity of the tubes' sensitivity. In addition, orientation of dynodes is irrelevant in our measurements (see section 4). A new method of data analysis has been developed which, compared to the methods reported at the summer VERITAS meeting in Leeds, does not require that the PMT be placed exactly in the center of the scanned region. The new method (presented in the next section) automatically finds the center of the PMT. The density plot adjusted to the new center position is presented in Fig. 1b (coordinates of the center of the PMT determined from Fig. 1b are: $x_c = -0.606504\text{mm}$ and $y_c = -0.421581\text{mm}$).

3. METHODOLOGY AND DATA ANALYSIS

Introduction

The most important characteristics studied were "effective radius" of the PMT, radial profile and deviation from uniformity of the surface sensitivity. We have considered several procedures that can be used to define this "effective radius".

The first procedure is based on the analysis of the radial profile of the sensitivity surface of the PMT. To build the radial profile of a tube we integrated the signal over circular rings of width 0.25mm for radii varying from 1mm to about 15mm. The integrals were normalized over the integration domain. From the outset this method was found to have the following defects: if the integration is done without interpolating the experimental data (in practice we average pixels which lie within a specific ring) then we introduce errors owing to the discrete nature of the data. On the other hand, the interpolation of data containing experimental errors has its own problems. The effective radius was then defined as the radius for which the intensity of the signal is ½ of the average signal inside a circle of some convenient radius (but smaller than the radius corresponding to the roll- off in the sensitivity). However, this introduces an ambiguity as to the correct choice of radius. In Fig. 2 radial profiles for Hamamatsu 41 are presented: red dots correspond to discrete averaging while green dots to the case when data are interpolated before integration. As one can observe the turn over obtained by the two methods, which is of major importance in determining the "effective radius", differ enough to produce quite different values for the effective radius.

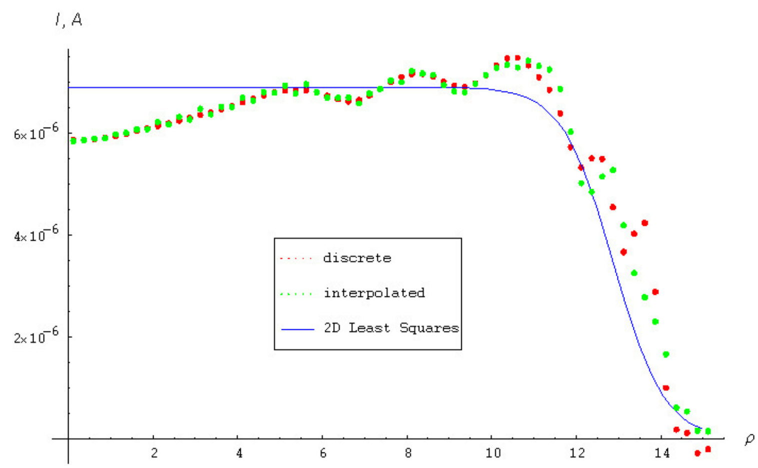


Fig. 2 Radial profile for Hamamatsu 41

Another approach was to define the effective radius as the radius of a cylinder having the same volume as that enclosed by the data with a height equal to the aforementioned average over some conveniently chosen circle. The motivation for this method comes from the fact that we expect a perfect PMT to have an intensity surface in the shape of a cylinder. This method of determining the effective radius, however, has problems similar to the previous approach.

Effective Radius

The last approach discussed in the previous section, instead, suggested to us the use of another surface as "ideal" for the PMT sensitivity surface – the surface obtained by rotating the function $z(x) := \frac{a}{2}\{1 - \tanh[b(x - r)]\}$ around an axis parallel to the z-axis and passing through a point with coordinates (x_c, y_c):

$$z(\rho) := \frac{a}{2}\{1 - \tanh[b(\rho - r)]\}, \tag{1}$$

where

$$\rho(x, y) := \sqrt{(x - x_c)^2 + (y - y_c)^2}, \tag{2}$$

or

$$z(x, y) := \frac{a}{2}\left\{1 - \tanh[b(\sqrt{(x - x_c)^2 + (y - y_c)^2} - r)]\right\}. \tag{3}$$

The two additional parameters x_c, y_c were introduced in order to take into account the possibility that a PMT is not placed exactly in the center of the scanned region. They allow us to find the position of the center of a PMT in an automated way.

In Fig. 3 we plotted the function (1) using the following parameters: $a = 1, b = 2, r = 12.5, x_c = y_c = 0$. Fig. 4 shows a 3D view of the function (3) with the same parameters. This function has the following useful properties: it is a continuous function; it is almost flat for a large range of values of ρ (for example, for the function in Fig. 3, $z(10) = 0.999955$ – a decrease of 0.0045% from 1 and $z(11) = 0.997527$ – a decrease of about 0.25% from its magnitude in the center and this decrease strongly depends on the steepness of the function given by parameter b); and z goes to zero as $\rho \to \infty$. The parameter a is equal to the "amplitude" of the function in the center of the coordinate system while parameter b describes the slope of the function. The parameter of interest, r, is the value of ρ for which $z(\rho = r) = \frac{1}{2}a$. One can observe a connection between this property of the parameter r and the previous definition of the effective radius (see the beginning of this section). We exploit the above described properties of function (1) and of the parameter r to define the "effective radius" of the sensitivity of a PMT:

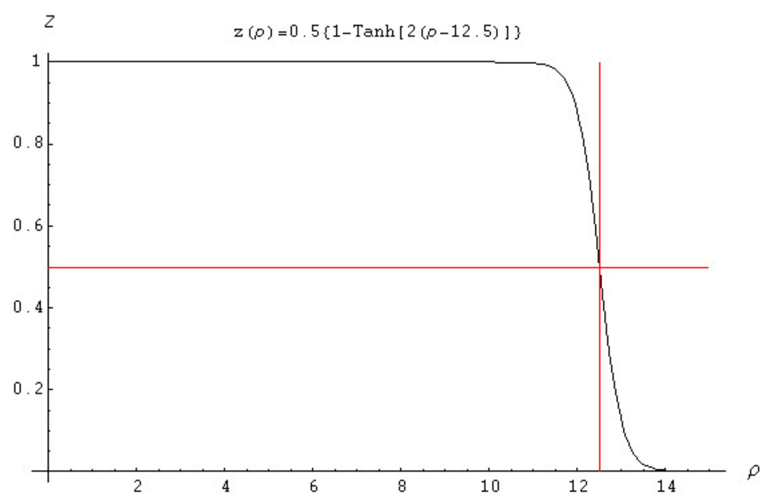


Fig. 3 Ideal radial profile

The ***effective radius*** *of a PMT is defined as the value of the radius ρ for which the function $z(\rho)$ decreases by ½ from its value in the origin (center). For function (1) or (3) it is given by the value of the parameter r.*

The two additional parameters x_c, y_c were introduced in order to take into account the possibility when a PMT is not placed exactly in the center of the scanned region. They allow us to find the position of the center of a PMT in an automated way. We used these two parameters to center the density plot in Fig. 1b.

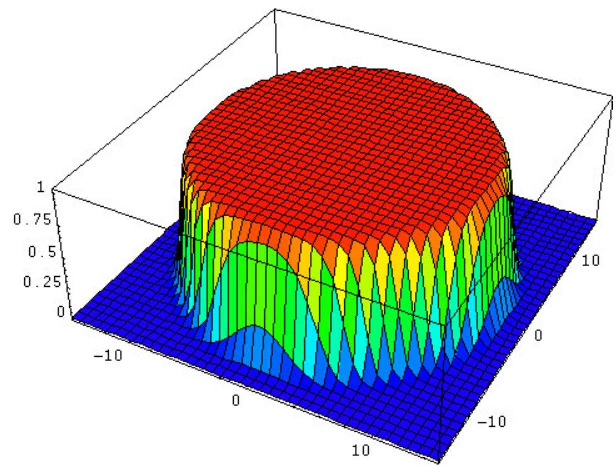


Fig. 4 3D view of the function z(x,y)

Computation of the Effective Radius

So far, we introduced the "ideal" surface of sensitivity for a PMT and defined the "effective radius" of a PMT. Now, we have to specify a method for computing the parameters of the "ideal" surface of sensitivity. Then, the parameter *r* will give us the desired effective radius of the PMT. We found that we can accomplish this by using the least squares method to fit the "ideal" surface to the experimental data. In practice, the least squares method will try to "average" the experimental data and represent them with the ideal surface.

NOTE 1: Our purpose here is to *replace* the experimental surface of sensitivity with an *equivalent ideal surface* – a simple enough surface (see, for example, Fig. 4) that allows us to give a clear and meaningful definition of the effective radius. We **do not** want to find the best approximation to experimental data; for this purpose the use of some large set of basis functions would be more advisable than the use of function (3).

NOTE 2: The background should be eliminated from the signal before applying the least squares procedure. The procedure we used for this is described in the next section.

NOTE 3: *Mathematica*'s "NonlinearFit" package contains two functions which can be used to fit function (3) to the experimental data: NonlinearFit and NonlinearRegress. Both of them accept as an option the weights of the experimental data points. For this report we did not use this option.

The continuous blue line in Fig. 2 is the radial profile of the ideal surface of sensitivity for the Hamamatsu 41 tube computed using the least squares method. We found the following value for the parameter *r*: 13.32mm.

It is interesting to compare this value with the values obtained using three methods from the preliminary report presented at Leeds (two of these methods were outlined in the introduction to this section; the third method is a variation of the second method): 13.35mm (method 1 – ½ of the average over a circle of radius of 10mm); 13.39mm (method 2 – radius of an equivalent cylinder with the volume equal to the volume enclosed by the experimental intensity surface and height computed as in method 1) and 12.75mm (method 3 – a variation of method 2; it uses a different (adaptive) way to compute the height of the cylinder; we found that this method tends to grossly underestimate the effective radius. We can see that the new value of the effective radius is close to the values obtained using methods 1 and 2 from the preliminary report.

For some PMTs we have found that the turn over of the ideal profile did not match very well the turn over of the discrete radial profile. We explain this by the way these two profiles are built: the discrete one, unlike the ideal radial profile, does not take into account global properties of the experimental data – it is built by averaging data over narrow rings. Actually, the slope of the discrete profile depends on the width of the "integration" rings. In order to have a more intuitive representation of how "well" the ideal surface fits the experimental data we built several plots in which we overlapped ideal and experimental surfaces.

Fig. 5 shows the experimental sensitivity surface of Hamamatsu 41 (plotted without mesh), the ideal sensitivity surface computed using the least squares method (plotted with mesh) and their overlap from different view points. It is interesting to see how this looks in cross-section. For this purpose, in Fig. 6 we present cuts through the *X-Z* plane of the overlapping surfaces. From the frontal view one can observe how the ideal surface "averages" the experimental data.

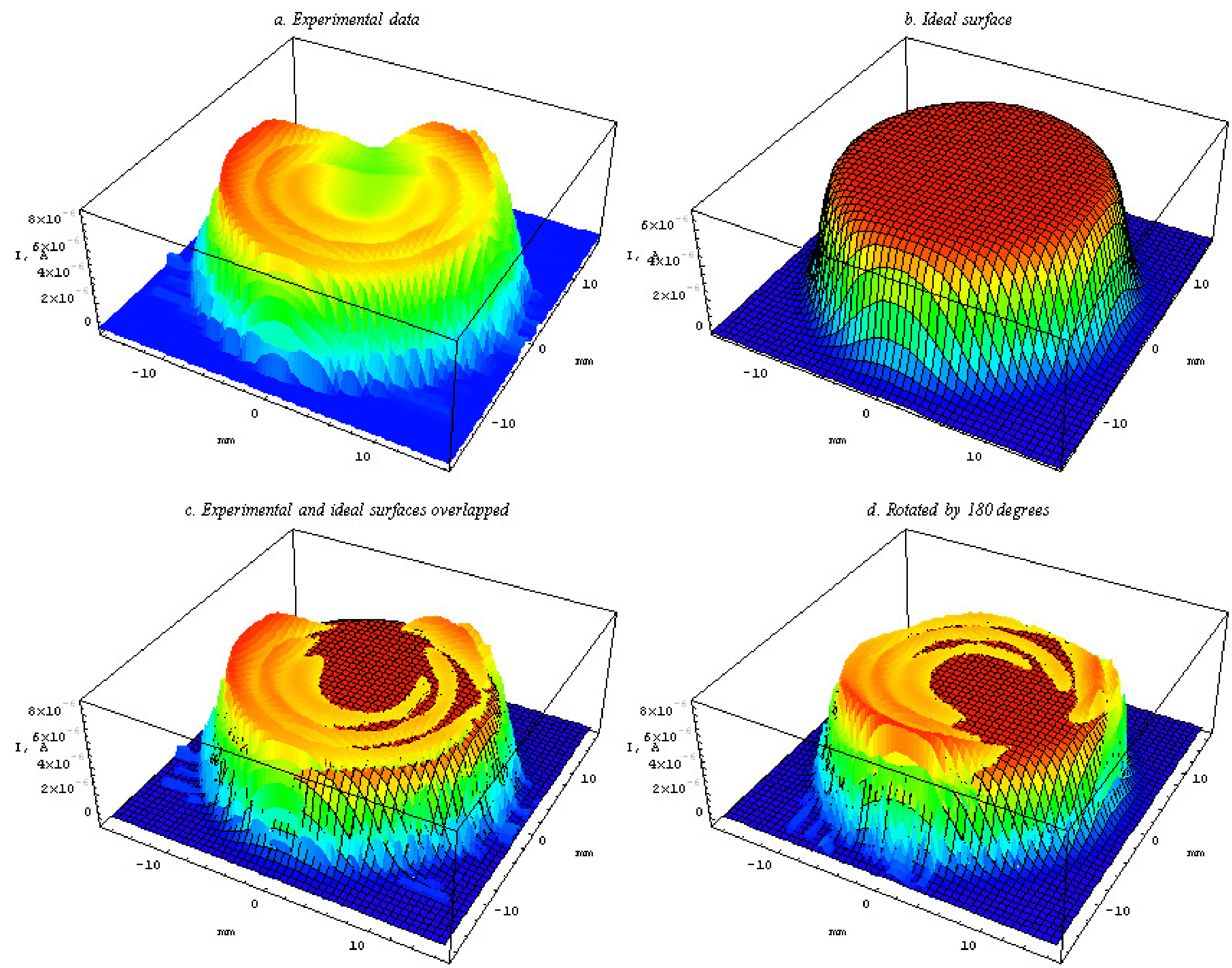


Fig. 5 Approximation of the experimental data by an ideal surface for Hamamatsu 41

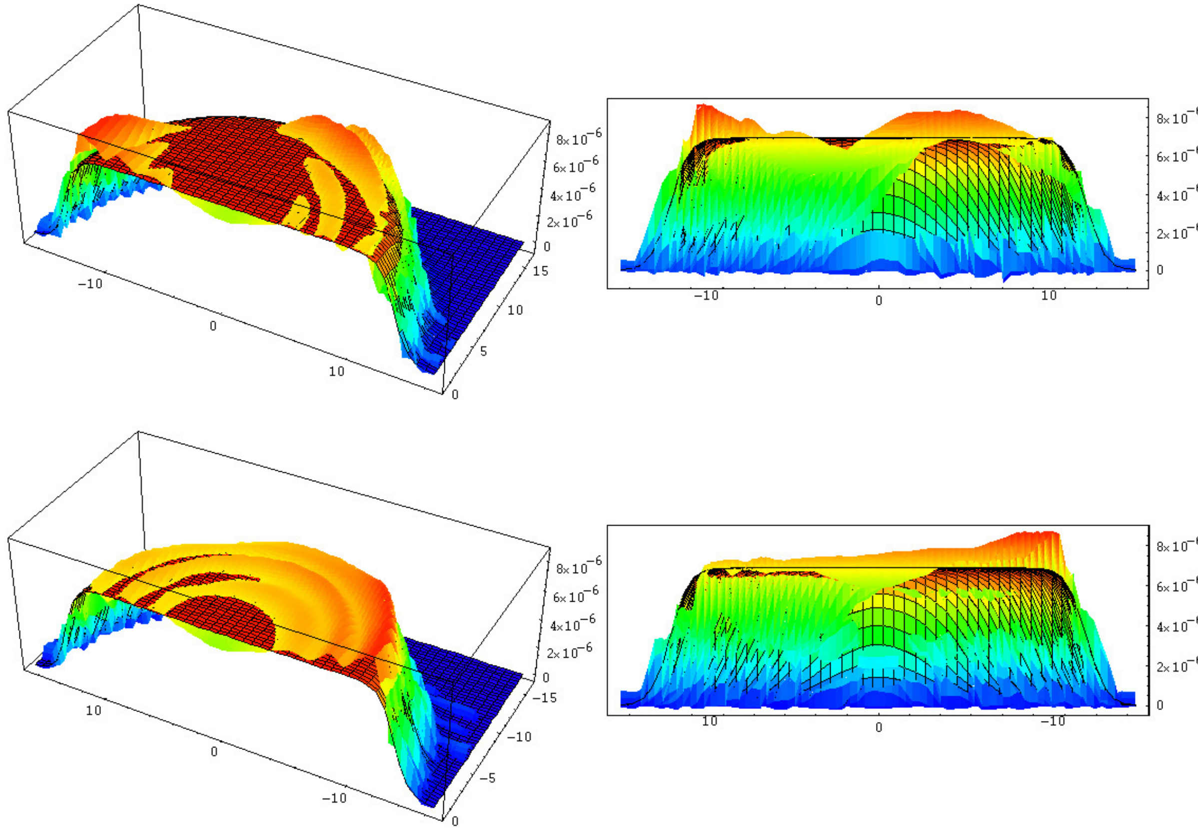


Fig. 6 Cuts of the overlapped surfaces through X-Y plane for Hamamatsu 41

Background Subtraction

The background signal in our measurements mainly comes from multiple reflections of the light emerging from the optical fiber from the walls of the dark box as well as electronics noise. This background signal is small in comparison with the useful signal (as one can see from Fig. 1, Fig. 5, or Fig. 6). Nevertheless, it is very advisable to eliminate this background.

The most simple and elegant solution is to introduce an additional parameter into the equation for the ideal sensitivity surface (3) to count for the background rather than subtract it. We considered the following function:

$$z(x,y) := \frac{a}{2}\left\{1-\tanh[b(\sqrt{(x-x_c)^2+(y-y_c)^2}-r)]\right\}+z_0 . \tag{4}$$

Unfortunately, the introduction of the parameter z_0 made the numerical algorithm unstable.

Instead, from Fig. 1 we observe that the peripheral pixels of the scanned region bear only the background signal plus some statistical noise. In order to eliminate the statistical noise we compute the average value of the signal corresponding to the peripheral pixels (for a 70×70 grid we have 276 peripheral pixels):

$$background := \frac{1}{4(n-1)}\left\{\sum_{i=1}^{n}\left(z_{1,i}+z_{n,i}\right)+\sum_{i=2}^{n-1}\left(z_{i,1}+z_{i,n}\right)\right\}. \tag{5}$$

We subtract this value from the experimental data before trying to fit the ideal sensitivity surface to the experimental data.

Stability of the Method

It is important that the method of computing the effective radius is stable to variations of the initial conditions. To check that the method described in this report is stable to statistical fluctuations we performed several numerical experiments. The two most important simulations performed consist of:

- Adding pseudo-noise to the real signal (we used HUA41 data). We observed that an addition of statistical noise at a level of 10% of the useful signal led to variations of about 0.04% in the computed value of the effective radius from its original value (13.3197): $r \in [13.3201, 13.3255]$. A noise level of 20% led to variations in r of 0.16%: $r \in [13.3133, 13.3349]$.

- Varying the computed value of the background level. Assuming that the background computed using equation (5) is susceptible to fluctuations we varied the computed value of the background to observe the effects of such variations on the resulting value of the effective radius. We observed that if the background is not eliminated or its value is changed by a factor of two the resulting error in r was less than 0.1%; for an error of 400% in computing the background we got a variation of less than 0.2% of r, 0.5% for an error of 1000% and 2.4% for an error of 5000% (factor of 50) in computing the background.

These simulations tell us that both the methodology of computing the effective radius and the numerical algorithms used are very stable to statistical fluctuations in the initial data.

Error Estimation

Mathematica's NonlinearRegress function returns the confidence intervals of the computed parameters. We will use this interval **as a measure of the error** (for nonlinear functions computation of confidence regions is a difficult problem; *Mathematica* gives only an estimation of these intervals) of the effective radius (and of other parameters). In this report we will use a **confidence level of 99.7%** to compute confidence intervals.

For example, for HUA41 and a confidence level of 99.7% we get the following confidence interval: $r \in [13.2828, 13.3567]$ with the average $r = 13.3197\,\text{mm}$. In this case we will write the result as $r = (13.32 \pm 0.04)\,\text{mm}$.

Uniformity Testing

Unlike the effective radius, it is more difficult to give an "absolute" definition of the uniformity of the sensitivity of a PMT. Here we will try to find a measure of this uniformity which can be used in comparing the uniformity of different PMTs.

For a grid of size $n \times m$ it seems natural to define the non-uniformity of a PMT as the average of the "relative errors" of data points:

$$\varepsilon = \frac{1}{n \cdot m}\left(\sum_{i,j}\left|\frac{z_{i,j} - z(x_i, y_i)}{z(x_i, y_i)}\right|\right) \times 100\%, \tag{6}$$

or, alternatively, for a model with 5 degrees of freedom:

$$\varepsilon := \sqrt{\frac{1}{n \cdot m - 5} \sum_{i,j} \left(\frac{z_{i,j} - z(x_i, y_i)}{z(x_i, y_i)} \right)^2} \times 100\%. \tag{7}$$

There is a problem with the above definitions: small values of $z(x_i, y_i)$ at the edge of the intensity surface will make the corresponding experimental points (many consisting only of statistical noise) to contribute heavily to the value of the non-uniformity computed with equations (6) and (7). We can correct this situation by replacing $z(x_i, y_i)$ at the denominator in equation (7) (or (6)) with $a \equiv z(x_c, y_c)$ (see equation (3)).

Thus, we **define the non-uniformity of the sensitivity surface of a PMT as**

$$\varepsilon := \frac{1}{a} \sqrt{\frac{1}{n \cdot m - 5} \sum_{i,j} \left(z_{i,j} - z(x_i, y_i) \right)^2} \times 100\% = \frac{1}{a} \sqrt{EstimatedVariance} \times 100\%. \tag{8}$$

4. RESULTS

Here (see Table 1) we will present effective radii and corresponding non-uniformities we have computed for different PMTs. For the HUA41 tube we repeated the measurement with the tube rotated by 90°. One can see that this didn't have (within experimental errors) any influence on the results. Also, for the HUA44 tube we measured the effective radius and its non-uniformity for several wavelengths: 420nm, 550nm (green) and 650nm (red). For all these wavelengths its effective radius is above 12.5 though for red light non-uniformity is somewhat greater.

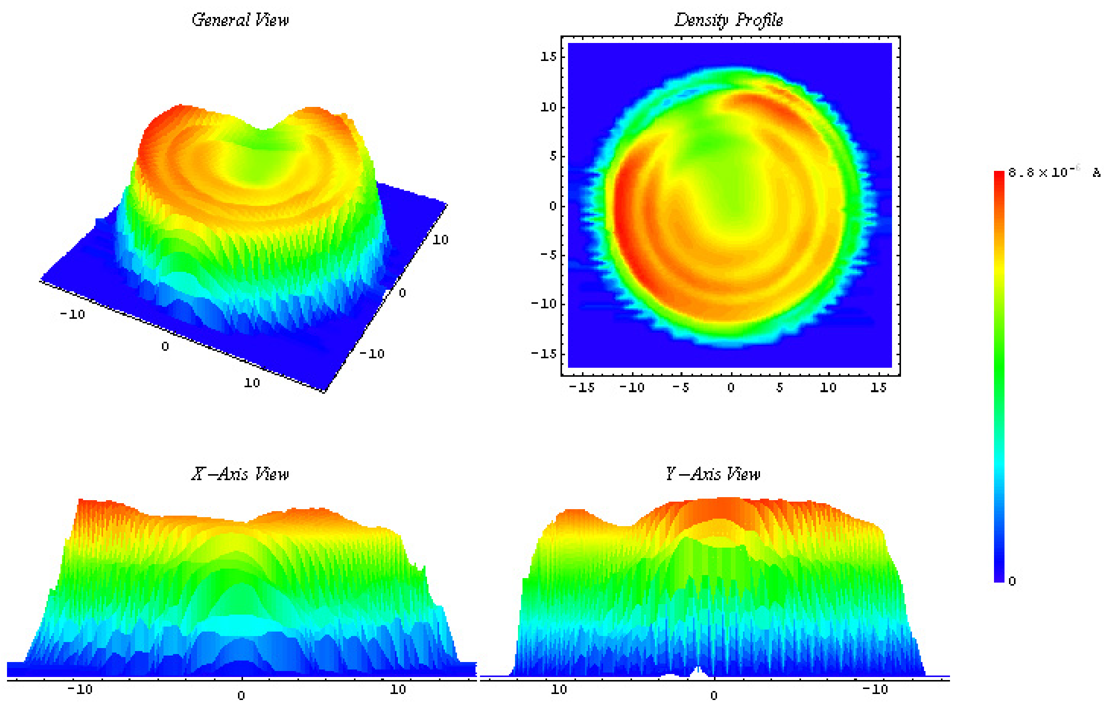
Table 1: Effective radii and non-uniformities of the studied PMTs

Manufacturer:	Serial Number:	Parameters of the ideal intensity surface			ε, %	Passed
		a, μA	*b*, mm^{-1}	**r, mm (effective radius)**		
Hamamatsu (R7056ASSY)	UA0041	6.90±0.07	0.84±0.04	**13.32±0.04**	**11.1**	Yes
	UA0041 (90º)	6.90±0.06	0.85±0.04	**13.33±0.04**	**10.9**	Yes
	UA0042	7.34±0.07	1.12±0.08	**13.47±0.04**	**13.3**	Yes
	UA0044 (420nm)	5.81±0.05	1.09±0.06	**13.51±0.03**	**11.3**	Yes
	UA0044 (550nm)	7.69±0.07	1.12±0.07	**13.60±0.03**	**12.4**	Yes
	UA0044 (650nm)	9.6±0.1	1.4±0.2	**13.69±0.05**	**19.9**	Yes
Photonis (XP2900/02)	25216	6.57±0.05	1.17±0.07	**12.54±0.03**	**10.1**	Yes
	25832	9.45±0.07	1.17±0.06	**12.32±0.03**	**9.2**	No
	25892	6.22±0.04	1.22±0.06	**12.62±0.02**	**8.0**	Yes
Electron Tubes (P25VN-02)	122	6.45±0.05	1.8±0.1	**12.50±0.02**	**10.6**	No
	125	5.26±0.04	1.4±0.1	**12.71±0.03**	**11.0**	Yes

5. INDIVIDUAL TUBES DATA

Hamamatsu UA0041

HUA41



$\text{BestFitParameters} \to \{a \to 6.90336\times10^{-6},\ b \to 0.838418,\ r \to 13.3197,\ xc \to -0.606494,\ yc \to -0.421556\}$

$\text{EstimatedVariance} \to 5.9202\times10^{-13}$

ParameterCITable →

	Estimate	Asymptotic SE	CI
a	6.90336×10^{-6}	1.91786×10^{-8}	$\{6.84642\times10^{-6}, 6.96031\times10^{-6}\}$
b	0.838418	0.014984	{0.793927, 0.882909}
r	13.3197	0.0124556	{13.2828, 13.3567}
xc	−0.606494	0.016143	{−0.654426, −0.558562}
yc	−0.421556	0.016143	{−0.469488, −0.373624}

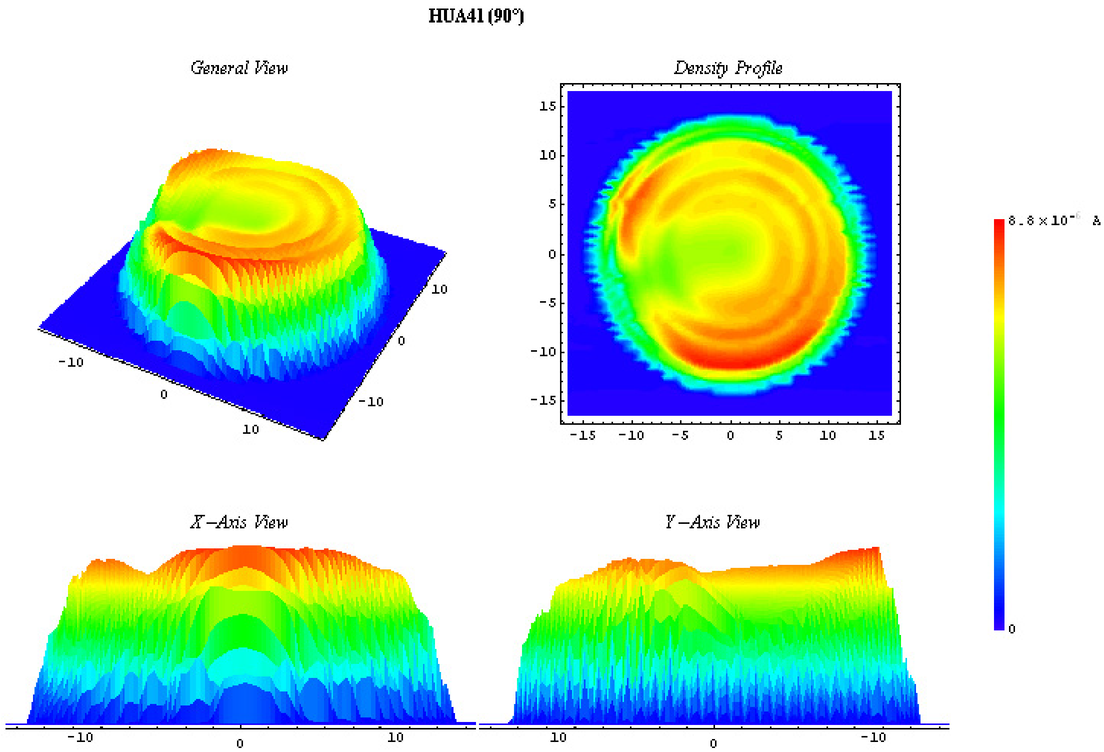
ParameterTable →

	Estimate	Asymp. SE	TStat	PValue
a	6.90336×10^{-6}	1.91786×10^{-8}	359.951	$3.1967413975\times10^{-3524}$
b	0.838418	0.014984	55.9542	$4.8638878317\times10^{-528}$
r	13.3197	0.0124556	1069.38	$3.8630083525\times10^{-5804}$
xc	−0.606494	0.016143	−37.5701	0.
yc	−0.421556	0.016143	−26.1139	0.

$$\text{AsymptoticCovarianceMatrix} \to \begin{pmatrix} 3.67819\times10^{-16} & -9.45799\times10^{-11} & -9.49405\times10^{-11} & -1.4378\times10^{-15} & -7.83891\times10^{-16} \\ -9.45799\times10^{-11} & 0.000224521 & 0.0000326335 & 3.89773\times10^{-9} & 2.10376\times10^{-9} \\ -9.49405\times10^{-11} & 0.0000326335 & 0.000155141 & 1.23395\times10^{-11} & 2.43353\times10^{-11} \\ -1.4378\times10^{-15} & 3.89773\times10^{-9} & 1.23395\times10^{-11} & 0.000260596 & 3.68425\times10^{-14} \\ -7.83891\times10^{-16} & 2.10376\times10^{-9} & 2.43353\times10^{-11} & 3.68425\times10^{-14} & 0.000260596 \end{pmatrix}$$

$$\text{AsymptoticCorrelationMatrix} \to \begin{pmatrix} 1. & -0.32912 & -0.397439 & -4.64405\times10^{-6} & -2.53195\times10^{-6} \\ -0.32912 & 1. & 0.174853 & 0.0000161138 & 8.6973\times10^{-6} \\ -0.397439 & 0.174853 & 1. & 6.13693\times10^{-8} & 1.21029\times10^{-7} \\ -4.64405\times10^{-6} & 0.0000161138 & 6.13693\times10^{-8} & 1. & 1.41378\times10^{-10} \\ -2.53195\times10^{-6} & 8.6973\times10^{-6} & 1.21029\times10^{-7} & 1.41378\times10^{-10} & 1. \end{pmatrix}$$

Hamamatsu UA0041 (rotated by 90°)



BestFitParameters → {a → 6.89694×10^{-6}, b → 0.851894, r → 13.3264, xc → −0.50865, yc → −0.0449484}

EstimatedVariance → 5.66505×10^{-13}

ParameterCITable →

	Estimate	Asymptotic SE	CI
a	6.89694×10^{-6}	1.86962×10^{-8}	{6.84143×10^{-6}, 6.95246×10^{-6}}
b	0.851894	0.0150063	{0.807337, 0.896451}
r	13.3264	0.0120744	{13.2905, 13.3622}
xc	−0.50865	0.0156766	{−0.555197, −0.462103}
yc	−0.0449484	0.0156765	{−0.0914956, 0.00159884}

ParameterTable →

	Estimate	Asymp. SE	TStat	PValue
a	6.89694×10^{-6}	1.86962×10^{-8}	368.894	$1.53363531251 \times 10^{-3574}$
b	0.851894	0.0150063	56.7692	$3.8388489788 \times 10^{-540}$
r	13.3264	0.0120744	1103.69	$5.3476702557 \times 10^{-5871}$
xc	−0.50865	0.0156766	−32.4465	0.
yc	−0.0449484	0.0156765	−2.86724	0.00415843

AsymptoticCovarianceMatrix →

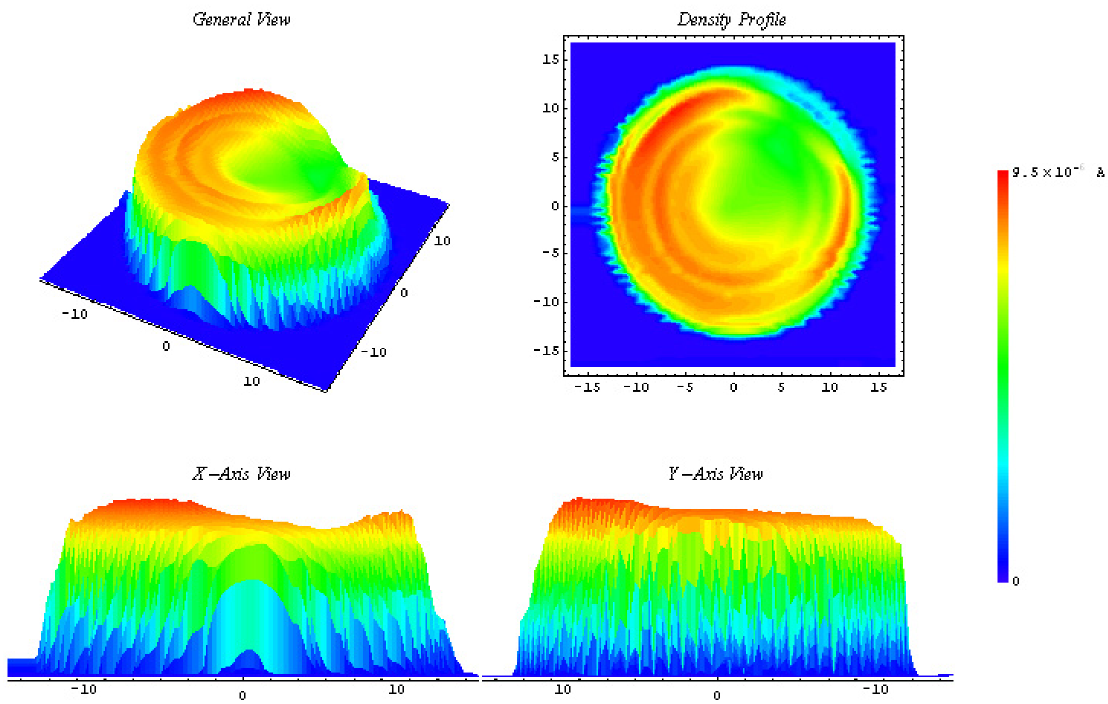
$$\begin{pmatrix} 3.4955 \times 10^{-16} & -9.15561 \times 10^{-11} & -8.88922 \times 10^{-11} & -8.38128 \times 10^{-16} & -4.79471 \times 10^{-17} \\ -9.15561 \times 10^{-11} & 0.000225188 & 0.0000311567 & 2.34027 \times 10^{-9} & 1.37106 \times 10^{-10} \\ -8.88922 \times 10^{-11} & 0.0000311567 & 0.000145791 & 1.38117 \times 10^{-11} & -3.22709 \times 10^{-12} \\ -8.38128 \times 10^{-16} & 2.34027 \times 10^{-9} & 1.38117 \times 10^{-11} & 0.000245755 & 1.61047 \times 10^{-15} \\ -4.79471 \times 10^{-17} & 1.37106 \times 10^{-10} & -3.22709 \times 10^{-12} & 1.61047 \times 10^{-15} & 0.000245754 \end{pmatrix}$$

AsymptoticCorrelationMatrix →

$$\begin{pmatrix} 1. & -0.326333 & -0.393771 & -2.8596 \times 10^{-6} & -1.6359 \times 10^{-7} \\ -0.326333 & 1. & 0.171954 & 9.94817 \times 10^{-6} & 5.82817 \times 10^{-7} \\ -0.393771 & 0.171954 & 1. & 7.29675 \times 10^{-8} & -1.70489 \times 10^{-8} \\ -2.8596 \times 10^{-6} & 9.94817 \times 10^{-6} & 7.29675 \times 10^{-8} & 1. & 6.55318 \times 10^{-12} \\ -1.6359 \times 10^{-7} & 5.82817 \times 10^{-7} & -1.70489 \times 10^{-8} & 6.55318 \times 10^{-12} & 1. \end{pmatrix}$$

Hamamatsu UA0042

HUA42



$$\text{BestFitParameters} \to \{a \to 7.3443\times10^{-6},\ b \to 1.1224,\ r \to 13.4677,\ xc \to -0.296967,\ yc \to 0.118202\}$$

$$\text{EstimatedVariance} \to 9.5354\times10^{-13}$$

ParameterCITable →

	Estimate	Asymptotic SE	CI
a	7.3443×10^{-6}	2.29756×10^{-8}	$\{7.27608\times10^{-6}, 7.41252\times10^{-6}\}$
b	1.1224	0.0270796	{1.042, 1.20281}
r	13.4677	0.0124331	{13.4308, 13.5046}
xc	−0.296967	0.016552	{−0.346113, −0.24782}
yc	0.118202	0.016552	{0.0690554, 0.167349}

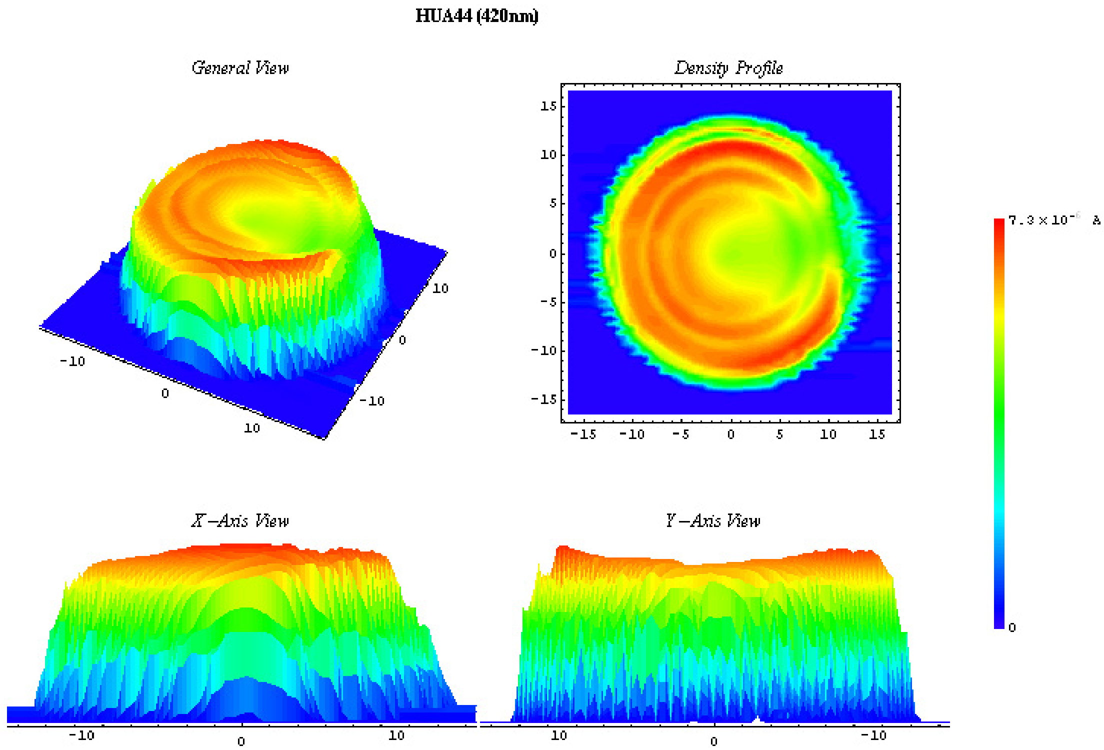
ParameterTable →

	Estimate	Asymp. SE	TStat	PValue
a	7.3443×10^{-6}	2.29756×10^{-8}	319.657	$3.7113935879\times10^{-3282}$
b	1.1224	0.0270796	41.4484	$3.9734329621\times10^{-322}$
r	13.4677	0.0124331	1083.22	$2.3361149386\times10^{-5831}$
xc	−0.296967	0.016552	−17.9414	0.
yc	0.118202	0.016552	7.14124	1.06071×10^{-12}

$$\text{AsymptoticCovarianceMatrix} \to \begin{pmatrix} 5.27877\times10^{-16} & -1.75203\times10^{-10} & -9.58452\times10^{-11} & -3.66711\times10^{-16} & -5.90373\times10^{-16} \\ -1.75203\times10^{-10} & 0.000733302 & 0.0000432431 & 2.22889\times10^{-9} & 3.64262\times10^{-9} \\ -9.58452\times10^{-11} & 0.0000432431 & 0.000154581 & -1.01798\times10^{-10} & -1.74343\times10^{-10} \\ -3.66711\times10^{-16} & 2.22889\times10^{-9} & -1.01798\times10^{-10} & 0.000273969 & -6.87121\times10^{-13} \\ -5.90373\times10^{-16} & 3.64262\times10^{-9} & -1.74343\times10^{-10} & -6.87121\times10^{-13} & 0.00027397 \end{pmatrix}$$

$$\text{AsymptoticCorrelationMatrix} \to \begin{pmatrix} 1. & -0.281601 & -0.335526 & -9.64289\times10^{-7} & -1.55242\times10^{-6} \\ -0.281601 & 1. & 0.128439 & 4.97276\times10^{-6} & 8.12683\times10^{-6} \\ -0.335526 & 0.128439 & 1. & -4.94665\times10^{-7} & -8.47179\times10^{-7} \\ -9.64289\times10^{-7} & 4.97276\times10^{-6} & -4.94665\times10^{-7} & 1. & -2.50802\times10^{-9} \\ -1.55242\times10^{-6} & 8.12683\times10^{-6} & -8.47179\times10^{-7} & -2.50802\times10^{-9} & 1. \end{pmatrix}$$

Hamamatsu UA0044 (λ = 420nm)



BestFitParameters → {a → 5.81452×10^{-6}, b → 1.08616, r → 13.5145, xc → 0.432555, yc → −0.469294}

EstimatedVariance → 4.30946×10^{-13}

ParameterCITable →

	Estimate	Asymptotic SE	CI
a	5.81452×10^{-6}	1.54522×10^{-8}	{5.76864×10^{-6}, 5.8604×10^{-6}}
b	1.08616	0.0218813	{1.02119, 1.15113}
r	13.5145	0.0107365	{13.4826, 13.5464}
xc	0.432555	0.0142628	{0.390206, 0.474905}
yc	−0.469294	0.0142628	{−0.511644, −0.426945}

ParameterTable →

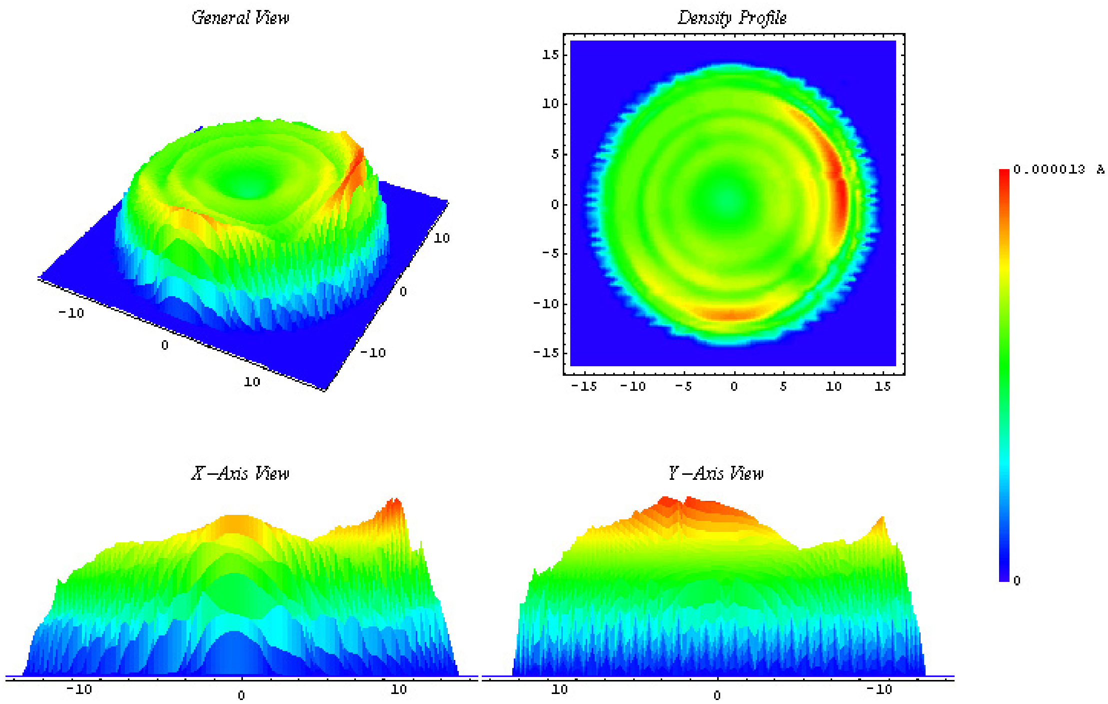
	Estimate	Asymp. SE	TStat	PValue
a	5.81452×10^{-6}	1.54522×10^{-8}	376.292	$2.6075982223\times10^{-3615}$
b	1.08616	0.0218813	49.6387	$8.4258228756\times10^{-436}$
r	13.5145	0.0107365	1258.74	$1.82984368587\times10^{-6149}$
xc	0.432555	0.0142628	30.3275	0.
yc	−0.469294	0.0142628	−32.9034	0.

$$\text{AsymptoticCovarianceMatrix} \to \begin{pmatrix} 2.38769\times10^{-16} & -9.66601\times10^{-11} & -5.65788\times10^{-11} & 3.70341\times10^{-17} & -4.15\times10^{-17} \\ -9.66601\times10^{-11} & 0.000478792 & 0.0000310926 & -7.63709\times10^{-10} & 4.81444\times10^{-10} \\ -5.65788\times10^{-11} & 0.0000310926 & 0.000115273 & 2.01977\times10^{-10} & -9.35784\times10^{-11} \\ 3.70341\times10^{-17} & -7.63709\times10^{-10} & 2.01977\times10^{-10} & 0.000203427 & 1.8355\times10^{-13} \\ -4.15\times10^{-17} & 4.81444\times10^{-10} & -9.35784\times10^{-11} & 1.8355\times10^{-13} & 0.000203428 \end{pmatrix}$$

$$\text{AsymptoticCorrelationMatrix} \to \begin{pmatrix} 1. & -0.28588 & -0.341036 & 1.68038\times10^{-7} & -1.88302\times10^{-7} \\ -0.28588 & 1. & 0.132348 & -2.44709\times10^{-6} & 1.54265\times10^{-6} \\ -0.341036 & 0.132348 & 1. & 1.31896\times10^{-6} & -6.11092\times10^{-7} \\ 1.68038\times10^{-7} & -2.44709\times10^{-6} & 1.31896\times10^{-6} & 1. & 9.02287\times10^{-10} \\ -1.88302\times10^{-7} & 1.54265\times10^{-6} & -6.11092\times10^{-7} & 9.02287\times10^{-10} & 1. \end{pmatrix}$$

Hamamatsu UA0044 (λ = 550nm)

HUA44 (550nm)



BestFitParameters → {a → 7.69048×10^{-6}, b → 1.12347, r → 13.6, xc → −0.351639, yc → −0.662289}

EstimatedVariance → 9.0845×10^{-13}

ParameterCITable →

	Estimate	Asymptotic SE	CI
a	7.69048×10^{-6}	2.21767×10^{-8}	{7.62463×10^{-6}, 7.75633×10^{-6}}
b	1.12347	0.0251422	{1.04882, 1.19812}
r	13.6	0.0115185	{13.5658, 13.6342}
xc	−0.351639	0.0153462	{−0.397206, −0.306073}
yc	−0.662289	0.0153462	{−0.707856, −0.616723}

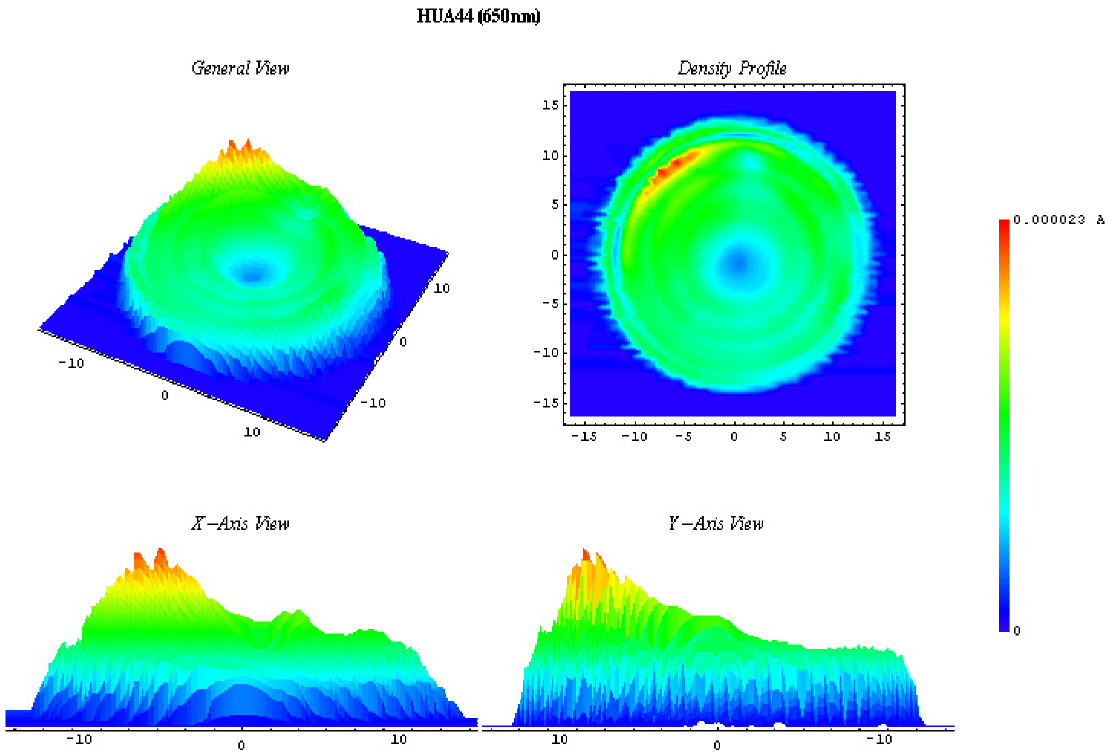
ParameterTable →

	Estimate	Asymp. SE	TStat	PValue
a	7.69048×10^{-6}	2.21767×10^{-8}	346.782	$5.6519319160 \times 10^{-3448}$
b	1.12347	0.0251422	44.6847	$4.8754541981 \times 10^{-366}$
r	13.6	0.0115185	1180.72	$7.1027969314 \times 10^{-6014}$
xc	−0.351639	0.0153462	−22.9138	0.
yc	−0.662289	0.0153462	−43.1566	$4.0372848777 \times 10^{-345}$

$$\text{AsymptoticCovarianceMatrix} \rightarrow \begin{pmatrix} 4.91807 \times 10^{-16} & -1.56149 \times 10^{-10} & -8.51994 \times 10^{-11} & 7.79343 \times 10^{-16} & -8.34929 \times 10^{-16} \\ -1.56149 \times 10^{-10} & 0.00063213 & 0.0000367902 & -1.93316 \times 10^{-9} & 2.36963 \times 10^{-9} \\ -8.51994 \times 10^{-11} & 0.0000367902 & 0.000132675 & -7.12814 \times 10^{-10} & 6.22447 \times 10^{-10} \\ 7.79343 \times 10^{-16} & -1.93316 \times 10^{-9} & -7.12814 \times 10^{-10} & 0.000235505 & 3.5041 \times 10^{-13} \\ -8.34929 \times 10^{-16} & 2.36963 \times 10^{-9} & 6.22447 \times 10^{-10} & 3.5041 \times 10^{-13} & 0.000235505 \end{pmatrix}$$

$$\text{AsymptoticCorrelationMatrix} \rightarrow \begin{pmatrix} 1. & -0.280052 & -0.333537 & 2.28998 \times 10^{-6} & -2.45331 \times 10^{-6} \\ -0.280052 & 1. & 0.127038 & -5.01031 \times 10^{-6} & 6.14154 \times 10^{-6} \\ -0.333537 & 0.127038 & 1. & -4.03256 \times 10^{-6} & 3.52133 \times 10^{-6} \\ 2.28998 \times 10^{-6} & -5.01031 \times 10^{-6} & -4.03256 \times 10^{-6} & 1. & 1.48791 \times 10^{-9} \\ -2.45331 \times 10^{-6} & 6.14154 \times 10^{-6} & 3.52133 \times 10^{-6} & 1.48791 \times 10^{-9} & 1. \end{pmatrix}$$

Hamamatsu UA0044 (λ = 650nm)



BestFitParameters → {a → 9.62176×10^{-6}, b → 1.36219, r → 13.6896, xc → −0.625815, yc → −0.354343}

EstimatedVariance → 3.66477×10^{-12}

ParameterCITable →

	Estimate	Asymptotic SE	CI
a	9.62176×10^{-6}	4.32394×10^{-8}	{9.49337×10^{-6}, 9.75015×10^{-6}}
b	1.36219	0.0532802	{1.20399, 1.52039}
r	13.6896	0.0165319	{13.6405, 13.7386}
xc	−0.625815	0.0223003	{−0.69203, −0.559601}
yc	−0.354343	0.0223004	{−0.420558, −0.288128}

ParameterTable →

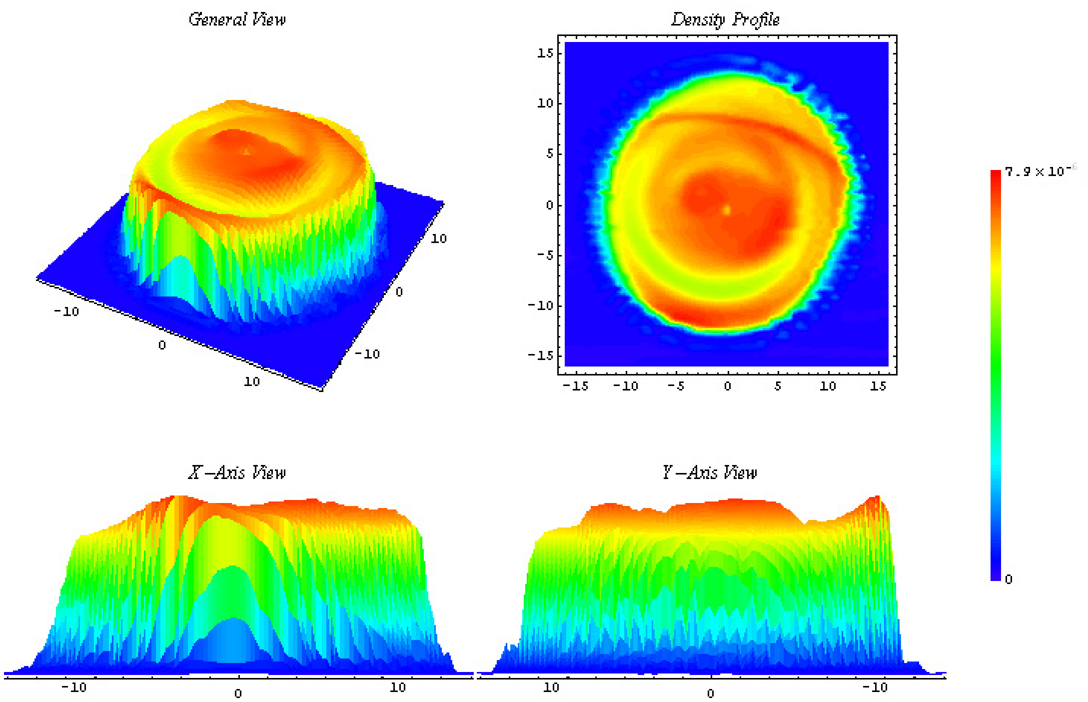
	Estimate	Asymp. SE	TStat	PValue
a	9.62176×10^{-6}	4.32394×10^{-8}	222.523	$1.39752572268\times10^{-2562}$
b	1.36219	0.0532802	25.5665	0.
r	13.6896	0.0165319	828.071	$1.68859252579\times10^{-5263}$
xc	−0.625815	0.0223003	−28.0631	0.
yc	−0.354343	0.0223004	−15.8895	0.

AsymptoticCovarianceMatrix →
$$\begin{pmatrix} 1.86965\times10^{-15} & -5.8256\times10^{-10} & -2.13668\times10^{-10} & -2.29716\times10^{-14} & 2.25157\times10^{-14} \\ -5.8256\times10^{-10} & 0.00283878 & 0.0000913427 & 1.34008\times10^{-7} & -1.31332\times10^{-7} \\ -2.13668\times10^{-10} & 0.0000913427 & 0.000273303 & 1.62071\times10^{-9} & -1.60198\times10^{-9} \\ -2.29716\times10^{-14} & 1.34008\times10^{-7} & 1.62071\times10^{-9} & 0.000497305 & -1.90532\times10^{-10} \\ 2.25157\times10^{-14} & -1.31332\times10^{-7} & -1.60198\times10^{-9} & -1.90532\times10^{-10} & 0.000497309 \end{pmatrix}$$

AsymptoticCorrelationMatrix →
$$\begin{pmatrix} 1. & -0.252869 & -0.298908 & -0.0000238232 & 0.0000233503 \\ -0.252869 & 1. & 0.103702 & 0.000112786 & -0.000110533 \\ -0.298908 & 0.103702 & 1. & 4.39615\times10^{-6} & -4.34531\times10^{-6} \\ -0.0000238232 & 0.000112786 & 4.39615\times10^{-6} & 1. & -3.83128\times10^{-7} \\ 0.0000233503 & -0.000110533 & -4.34531\times10^{-6} & -3.83128\times10^{-7} & 1. \end{pmatrix}$$

Photonis 25216

Ph25216



BestFitParameters → {a → 6.56731×10^{-6}, b → 1.16685, r → 12.5365, xc → 0.718384, yc → −1.03213}

EstimatedVariance → 4.38701×10^{-13}

ParameterCITable →

	Estimate	Asymptotic SE	CI
a	6.56731×10^{-6}	1.68147×10^{-8}	{6.51738×10^{-6}, 6.61724×10^{-6}}
b	1.16685	0.0226017	{1.09975, 1.23396}
r	12.5365	0.00961006	{12.5079, 12.565}
xc	0.718384	0.012763	{0.680487, 0.75628}
yc	−1.03213	0.0127631	{−1.07003, −0.994237}

ParameterTable →

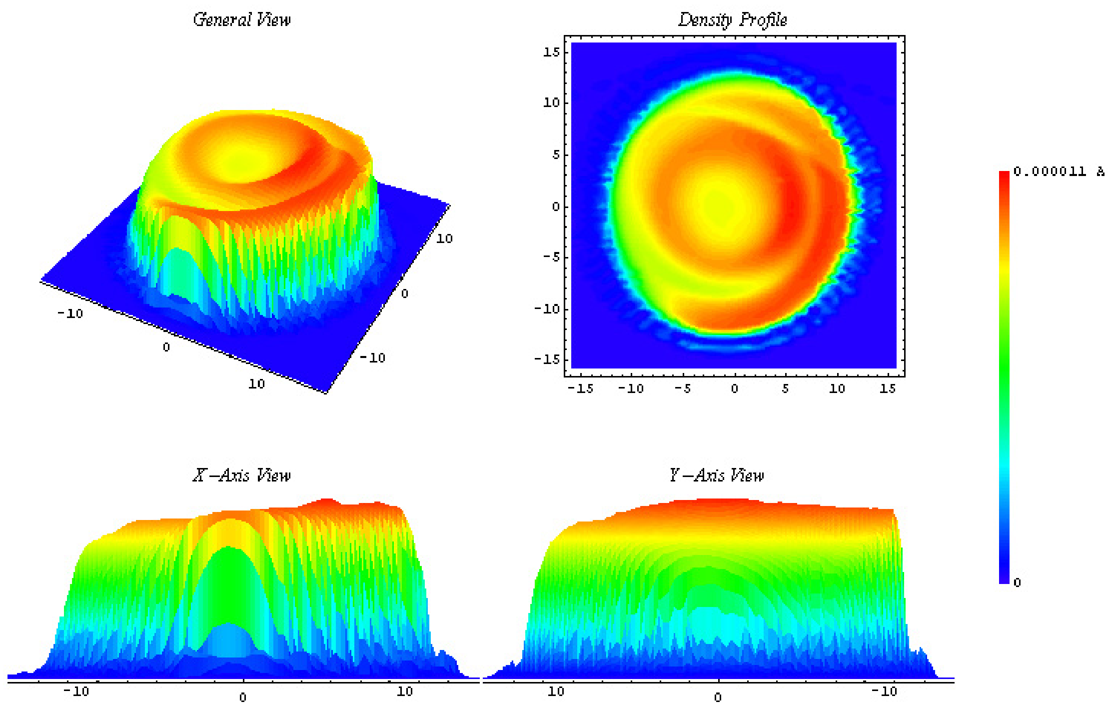
	Estimate	Asymp. SE	TStat	PValue
a	6.56731×10^{-6}	1.68147×10^{-8}	390.569	$6.3046855610\times10^{-3692}$
b	1.16685	0.0226017	51.6268	$1.67726349529\times10^{-464}$
r	12.5365	0.00961006	1304.51	$3.5727557639\times10^{-6225}$
xc	0.718384	0.012763	56.2862	$5.7596683688\times10^{-533}$
yc	−1.03213	0.0127631	−80.8686	$2.18555935414\times10^{-904}$

$$\text{AsymptoticCovarianceMatrix} \to \begin{pmatrix} 2.82735\times10^{-16} & -1.08839\times10^{-10} & -5.52139\times10^{-11} & 5.36657\times10^{-17} & -8.95813\times10^{-17} \\ -1.08839\times10^{-10} & 0.000510837 & 0.0000288468 & 4.41489\times10^{-10} & -7.14298\times10^{-10} \\ -5.52139\times10^{-11} & 0.0000288468 & 0.0000923533 & -2.42695\times10^{-10} & 3.92079\times10^{-10} \\ 5.36657\times10^{-17} & 4.41489\times10^{-10} & -2.42695\times10^{-10} & 0.000162895 & 2.16897\times10^{-13} \\ -8.95813\times10^{-17} & -7.14298\times10^{-10} & 3.92079\times10^{-10} & 2.16897\times10^{-13} & 0.000162896 \end{pmatrix}$$

$$\text{AsymptoticCorrelationMatrix} \to \begin{pmatrix} 1. & -0.286387 & -0.34169 & 2.50065\times10^{-7} & -4.17419\times10^{-7} \\ -0.286387 & 1. & 0.13281 & 1.53047\times10^{-6} & -2.47618\times10^{-6} \\ -0.34169 & 0.13281 & 1. & -1.9787\times10^{-6} & 3.19663\times10^{-6} \\ 2.50065\times10^{-7} & 1.53047\times10^{-6} & -1.9787\times10^{-6} & 1. & 1.33151\times10^{-9} \\ -4.17419\times10^{-7} & -2.47618\times10^{-6} & 3.19663\times10^{-6} & 1.33151\times10^{-9} & 1. \end{pmatrix}$$

Photonis 25832

Ph25832



BestFitParameters → {a → 9.44996×10^{-6}, b → 1.169, r → 12.3168, xc → −0.112235, yc → 1.15981}

EstimatedVariance → 7.51272×10^{-13}

ParameterCITable →

	Estimate	Asymptotic SE	CI
a	9.44996×10^{-6}	2.24449×10^{-8}	{9.38332×10^{-6}, 9.51661×10^{-6}}
b	1.169	0.0208102	{1.10721, 1.23079}
r	12.3168	0.00881982	{12.2906, 12.343}
xc	−0.112235	0.0116995	{−0.146973, −0.0774961}
yc	1.15981	0.0116995	{1.12507, 1.19454}

ParameterTable →

	Estimate	Asymp. SE	TStat	PValue
a	9.44996×10^{-6}	2.24449×10^{-8}	421.029	$5.9094299854\times10^{-3847}$
b	1.169	0.0208102	56.1742	$2.6454195229\times10^{-531}$
r	12.3168	0.00881982	1396.49	$1.26815089005\times10^{-6369}$
xc	−0.112235	0.0116995	−9.59311	0.
yc	1.15981	0.0116995	99.1329	$4.9124624287\times10^{-1173}$

AsymptoticCovarianceMatrix →

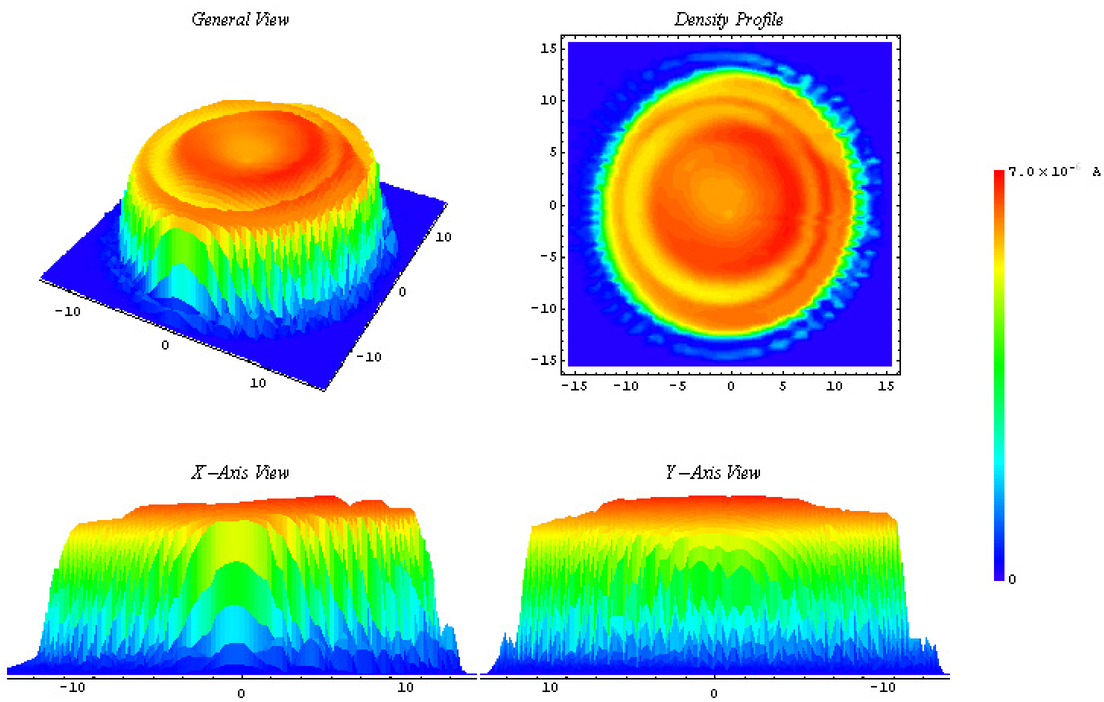
$$\begin{pmatrix} 5.03775\times10^{-16} & -1.34861\times10^{-10} & -6.82394\times10^{-11} & 6.84798\times10^{-16} & -5.76874\times10^{-16} \\ -1.34861\times10^{-10} & 0.000433064 & 0.0000247743 & -2.51975\times10^{-10} & 1.96547\times10^{-10} \\ -6.82394\times10^{-11} & 0.0000247743 & 0.0000777892 & -8.05724\times10^{-10} & 6.79461\times10^{-10} \\ 6.84798\times10^{-16} & -2.51975\times10^{-10} & -8.05724\times10^{-10} & 0.000136878 & -2.27875\times10^{-12} \\ -5.76874\times10^{-16} & 1.96547\times10^{-10} & 6.79461\times10^{-10} & -2.27875\times10^{-12} & 0.000136878 \end{pmatrix}$$

AsymptoticCorrelationMatrix →

$$\begin{pmatrix} 1. & -0.28873 & -0.344713 & 2.60782\times10^{-6} & -2.19683\times10^{-6} \\ -0.28873 & 1. & 0.134979 & -1.03494\times10^{-6} & 8.07276\times10^{-7} \\ -0.344713 & 0.134979 & 1. & -7.80835\times10^{-6} & 6.58473\times10^{-6} \\ 2.60782\times10^{-6} & -1.03494\times10^{-6} & -7.80835\times10^{-6} & 1. & -1.6648\times10^{-8} \\ -2.19683\times10^{-6} & 8.07276\times10^{-7} & 6.58473\times10^{-6} & -1.6648\times10^{-8} & 1. \end{pmatrix}$$

Photonis 25892

Ph25892



BestFitParameters → {a → 6.22495×10^{-6}, b → 1.21714, r → 12.6155, xc → −0.50153, yc → 1.48685}

EstimatedVariance → 2.49078×10^{-13}

ParameterCITable →

	Estimate	Asymptotic SE	CI
a	6.22495×10^{-6}	1.25106×10^{-8}	{6.18781×10^{-6}, 6.2621×10^{-6}}
b	1.21714	0.0190388	{1.1606, 1.27367}
r	12.6155	0.00743031	{12.5934, 12.6375}
xc	−0.50153	0.00990288	{−0.530934, −0.472126}
yc	1.48685	0.00990288	{1.45745, 1.51626}

ParameterTable →

	Estimate	Asymp. SE	TStat	PValue
a	6.22495×10^{-6}	1.25106×10^{-8}	497.575	$6.2526281395 \times 10^{-4194}$
b	1.21714	0.0190388	63.9294	$1.05070977057 \times 10^{-647}$
r	12.6155	0.00743031	1697.84	$3.8017808297 \times 10^{-6784}$
xc	−0.50153	0.00990288	−50.6449	$2.7796817512 \times 10^{-450}$
yc	1.48685	0.00990288	150.143	$7.9420468815 \times 10^{-1835}$

AsymptoticCovarianceMatrix →

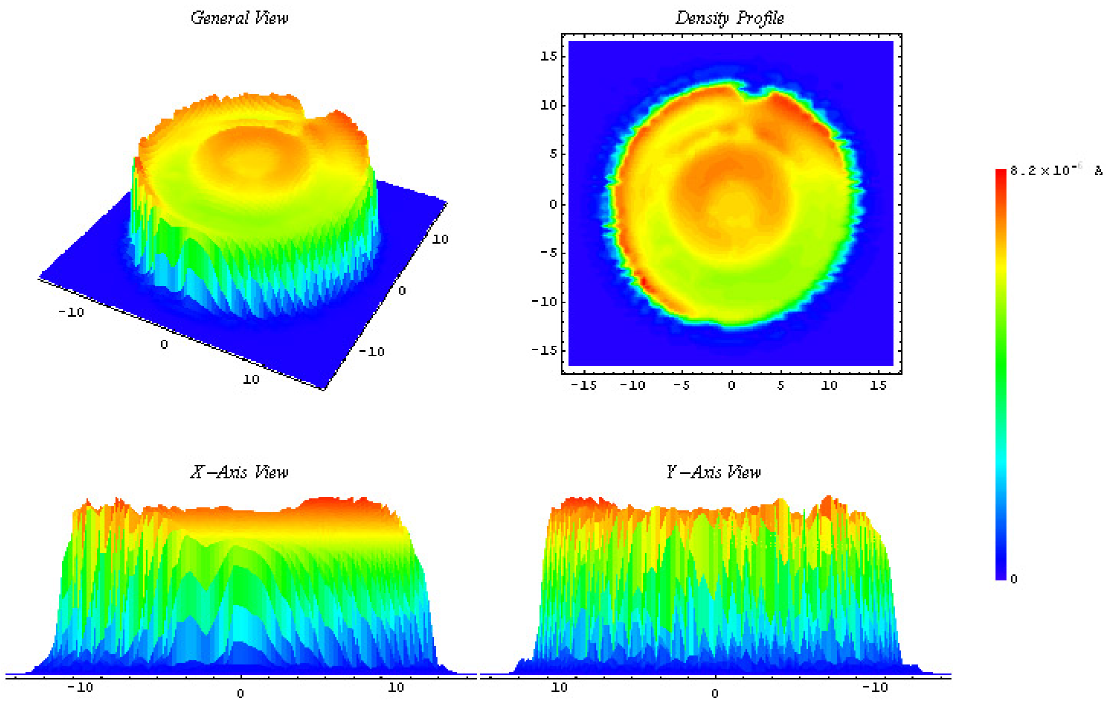
$$\begin{pmatrix} 1.56514 \times 10^{-16} & -6.65401 \times 10^{-11} & -3.09211 \times 10^{-11} & -7.91687 \times 10^{-17} & 1.9315 \times 10^{-17} \\ -6.65401 \times 10^{-11} & 0.000362474 & 0.0000178874 & 2.89603 \times 10^{-10} & -8.91422 \times 10^{-11} \\ -3.09211 \times 10^{-11} & 0.0000178874 & 0.0000552096 & 7.47348 \times 10^{-11} & -1.34751 \times 10^{-11} \\ -7.91687 \times 10^{-17} & 2.89603 \times 10^{-10} & 7.47348 \times 10^{-11} & 0.000098067 & -3.77506 \times 10^{-15} \\ 1.9315 \times 10^{-17} & -8.91422 \times 10^{-11} & -1.34751 \times 10^{-11} & -3.77506 \times 10^{-15} & 0.000098067 \end{pmatrix}$$

AsymptoticCorrelationMatrix →

$$\begin{pmatrix} 1. & -0.279362 & -0.332637 & -6.39021 \times 10^{-7} & 1.55903 \times 10^{-7} \\ -0.279362 & 1. & 0.126445 & 1.53604 \times 10^{-6} & -4.72806 \times 10^{-7} \\ -0.332637 & 0.126445 & 1. & 1.01567 \times 10^{-6} & -1.83131 \times 10^{-7} \\ -6.39021 \times 10^{-7} & 1.53604 \times 10^{-6} & 1.01567 \times 10^{-6} & 1. & -3.84947 \times 10^{-11} \\ 1.55903 \times 10^{-7} & -4.72806 \times 10^{-7} & -1.83131 \times 10^{-7} & -3.84947 \times 10^{-11} & 1. \end{pmatrix}$$

Electron Tubes 122

ET122



BestFitParameters → {a → 6.44762×10^{-6}, b → 1.79178, r → 12.5047, xc → −0.18639, yc → −0.51994}

EstimatedVariance → 4.69567×10^{-13}

ParameterCITable →

	Estimate	Asymptotic SE	CI
a	6.44762×10^{-6}	1.66522×10^{-8}	{6.39817×10^{-6}, 6.49706×10^{-6}}
b	1.79178	0.044652	{1.6592, 1.92436}
r	12.5047	0.00798503	{12.4809, 12.5284}
xc	−0.18639	0.0108643	{−0.218649, −0.154132}
yc	−0.51994	0.0108714	{−0.55222, −0.487661}

ParameterTable →

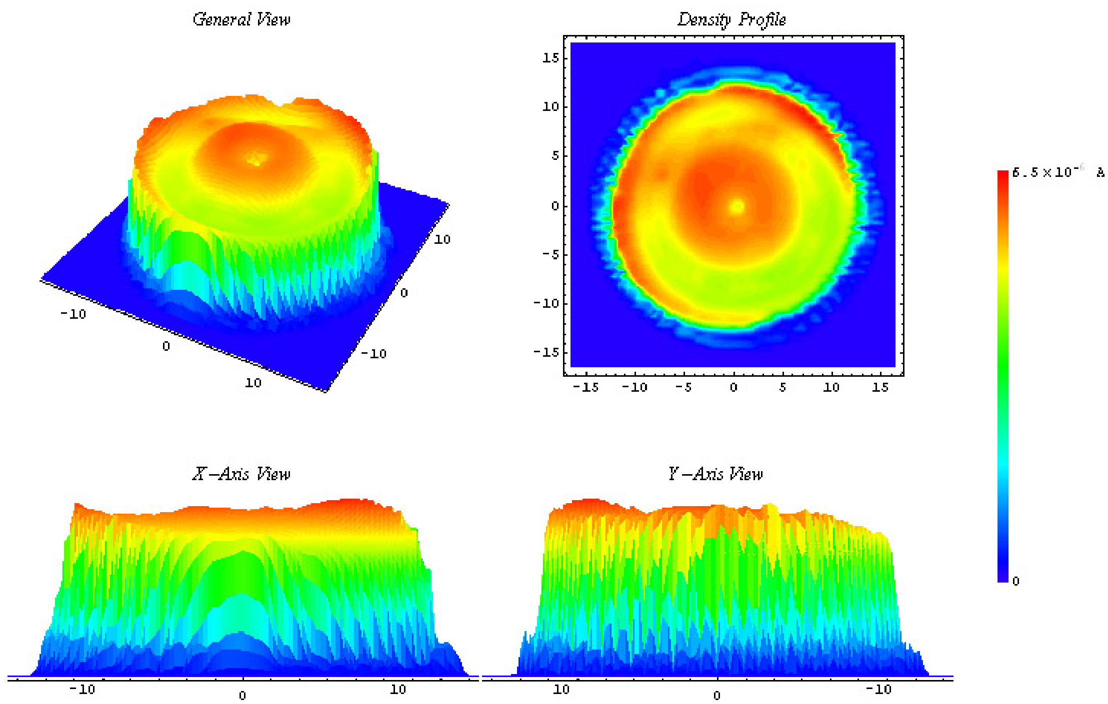
	Estimate	Asymp. SE	TStat	PValue
a	6.44762×10^{-6}	1.66522×10^{-8}	387.193	$4.7595214414 \times 10^{-3674}$
b	1.79178	0.044652	40.1276	0.
r	12.5047	0.00798503	1566.01	$1.21787379562 \times 10^{-6612}$
xc	−0.18639	0.0108643	−17.1562	0.
yc	−0.51994	0.0108714	−47.8265	$5.7483392792 \times 10^{-410}$

$$\text{AsymptoticCovarianceMatrix} \rightarrow \begin{pmatrix} 2.77296 \times 10^{-16} & -1.71213 \times 10^{-10} & -3.59753 \times 10^{-11} & 1.15136 \times 10^{-14} & 4.39639 \times 10^{-15} \\ -1.71213 \times 10^{-10} & 0.0019938 & 0.0000307362 & -4.00017 \times 10^{-7} & -1.63206 \times 10^{-7} \\ -3.59753 \times 10^{-11} & 0.0000307362 & 0.0000637607 & 3.07464 \times 10^{-8} & 1.30801 \times 10^{-8} \\ 1.15136 \times 10^{-14} & -4.00017 \times 10^{-7} & 3.07464 \times 10^{-8} & 0.000118033 & -5.2483 \times 10^{-10} \\ 4.39639 \times 10^{-15} & -1.63206 \times 10^{-7} & 1.30801 \times 10^{-8} & -5.2483 \times 10^{-10} & 0.000118187 \end{pmatrix}$$

$$\text{AsymptoticCorrelationMatrix} \rightarrow \begin{pmatrix} 1. & -0.230263 & -0.270555 & 0.000063641 & 0.0000242851 \\ -0.230263 & 1. & 0.0862051 & -0.000824587 & -0.00033621 \\ -0.270555 & 0.0862051 & 1. & 0.000354419 & 0.000150678 \\ 0.000063641 & -0.000824587 & 0.000354419 & 1. & -4.44358 \times 10^{-6} \\ 0.0000242851 & -0.00033621 & 0.000150678 & -4.44358 \times 10^{-6} & 1. \end{pmatrix}$$

Electron Tubes 125

ET125



BestFitParameters → {a → 5.2622×10^{-6}, b → 1.40911, r → 12.7079, xc → −0.554931, yc → −0.483056}

EstimatedVariance → 3.36119×10^{-13}

ParameterCITable →

	Estimate	Asymptotic SE	CI
a	5.2622×10^{-6}	1.41675×10^{-8}	{5.22014×10^{-6}, 5.30427×10^{-6}}
b	1.40911	0.0322737	{1.31328, 1.50494}
r	12.7079	0.00936259	{12.6801, 12.7357}
xc	−0.554931	0.012601	{−0.592346, −0.517516}
yc	−0.483056	0.0126009	{−0.520471, −0.445641}

ParameterTable →

	Estimate	Asymp. SE	TStat	PValue
a	5.2622×10^{-6}	1.41675×10^{-8}	371.427	$1.38889210745 \times 10^{-3588}$
b	1.40911	0.0322737	43.6612	$5.4131658899 \times 10^{-352}$
r	12.7079	0.00936259	1357.31	$2.7757569259 \times 10^{-6309}$
xc	−0.554931	0.012601	−44.0385	$3.7353674728 \times 10^{-357}$
yc	−0.483056	0.0126009	−38.3351	0.

$$\text{AsymptoticCovarianceMatrix} \to \begin{pmatrix} 2.00719\times10^{-16} & -1.18058\times10^{-10} & -4.05358\times10^{-11} & -4.1013\times10^{-15} & 9.24236\times10^{-16} \\ -1.18058\times10^{-10} & 0.00104159 & 0.0000326446 & 4.70195\times10^{-8} & -1.05848\times10^{-8} \\ -4.05358\times10^{-11} & 0.0000326446 & 0.000087658 & -2.22744\times10^{-10} & 4.62182\times10^{-11} \\ -4.1013\times10^{-15} & 4.70195\times10^{-8} & -2.22744\times10^{-10} & 0.000158786 & -6.5981\times10^{-12} \\ 9.24236\times10^{-16} & -1.05848\times10^{-8} & 4.62182\times10^{-11} & -6.5981\times10^{-12} & 0.000158782 \end{pmatrix}$$

$$\text{AsymptoticCorrelationMatrix} \to \begin{pmatrix} 1. & -0.258199 & -0.305597 & -0.0000229732 & 5.17712\times10^{-6} \\ -0.258199 & 1. & 0.108036 & 0.000115617 & -0.0000260276 \\ -0.305597 & 0.108036 & 1. & -1.88801\times10^{-6} & 3.91757\times10^{-7} \\ -0.0000229732 & 0.000115617 & -1.88801\times10^{-6} & 1. & -4.15539\times10^{-8} \\ 5.17712\times10^{-6} & -0.0000260276 & 3.91757\times10^{-7} & -4.15539\times10^{-8} & 1. \end{pmatrix}$$