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PHYS 600 – HW8 SOLUTIONS

PROBLEM 1 using the Rodrigues formula $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$ obviously

$$H_0(x) = 1 \;,\; H_1(x) = 2x \;,\; H_2(x) = 4x^2 - 2,\; H_3(x) = 8x^3 - 12x$$

PROBLEM 2 using the generating function $e^{2tx-t^2} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x)$

$$H_0(x) = \left(e^{2tx-t^2}\right)_{t=0} = 1,\; H_1(x) = \left(\frac{d\, e^{2tx-t^2}}{dt}\right)_{t=0} = (-2\, e^{-t(t-2x)}(t-x))_{t=0} = 2x$$

$$H_2(x) = \left(\frac{d^2\, e^{2tx-t^2}}{dt^2}\right)_{t=0} = (\, e^{-t(t-2x)}(-2 + 4t^2 - 8tx + 4x^2)\,)_{t=0} = 4x^2 - 2$$

$$H_3(x) = \left(\frac{d^3\, e^{2tx-t^2}}{dt^3}\right)_{t=0} = (\, -4\, e^{-t(t-2x)}(2t^3 + 3x - 6t^2x - 2x^3 + t(-3 + 6x^2))\,)_{t=0} = 8x^3 - 12x$$

PROBLEM 3 using the Rodrigues formula $L_n(x) = e^x \frac{d^n}{dx^n}(x^n e^{-x})$ obviously

$$L_0(x) = 1 \;,\; L_1(x) = 1 - x \;,\; L_2(x) = x^2 - 4x + 2,\; L_3(x) = -x^3 + 9x^2 - 18x + 6$$

PROBLEM 4 using the generating function $\frac{e^{-\frac{tx}{1-t}}}{1-t} = \sum_{n=0}^{\infty} \frac{t^n}{n!} L_n(x)$

$$L_0(x) = \left(\frac{e^{-\frac{tx}{1-t}}}{1-t}\right)_{t=0} = 1,\; L_1(x) = \left(\frac{d\, \frac{e^{-\frac{tx}{1-t}}}{1-t}}{dt}\right)_{t=0} = \left(\frac{e^{\frac{tx}{-1+t}}(-1 + t + x)}{(-1+t)^3}\right)_{t=0} = 1 - x$$

$$L_2(x) = \left(\frac{d^2\, \frac{e^{-\frac{tx}{1-t}}}{1-t}}{dt^2}\right)_{t=0} = \left(-\frac{e^{\frac{tx}{-1+t}}\,(2 + 2t^2 + 4t(-1+x) - 4x + x^2)}{(-1+t)^5}\right)_{t=0} = x^2 - 4x + 2$$

$$L_3(x) = \left(\frac{d^3\, \frac{e^{-\frac{tx}{1-t}}}{1-t}}{dt^3}\right)_{t=0} = \left(\frac{e^{\frac{tx}{-1+t}}}{(-1+t)^7}(-6 + 6t^3 + 18t^2(-1+x) + 18x - 9x^2 + x^3 + 9t(2 - 4x + x^2))\right)_{t=0} = -x^3 + 9x^2 - 18x + 6$$